EXAM-2 -B2 | MATH 261: Elementary Differential Equations | MATH 261

FALL 2012 | EXAMINATION COVER PAGE | Professor Moseley

PRINT NAME____(________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, Oct. 8, 2012 11:30am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

________________________________ ________________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 11 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-12 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit may be given if deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

Scores

page	points	score
1	10	
2	7	
3	10	
4	10	
5	11	
6	6	
7	10	
8	9	
9	9	
10	9	
11	9	
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
Total	100	

REQUEST FOR REGRADE

Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature________________________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let $y = \varphi(x)$ be the solution of the IVP given below. Using Euler's Method with $h = 0.1$ you are to find a numerical approximation for $\varphi(0.2)$ (i.e.find y_1 and y_2). Use a table and the standard notation used in class (attendance is mandatory).

IVP ODE $y' = y - x$ IC $y(1) = 2$

1. (2 pts.) The general formula for Euler's method may be written

as ____. _______ A B C D E

2. (1 pt.) x_0 = _______. ______ A B C D E 5. (1 pt.) y_0 = _________. _______ A B C D E

3. (1 pt.) x_1 = _______. _______A B C D E 6. (2 pts.) y_1 = ________. ______ A B C D E

4. (1 pt.) x_2 = ______. _______ A B C D E 7. (2 pts.) y_2 = ______. _________ A B C D E

Possible answers this page.

A) $y_{k+1} = y_k + h\, f(x_k,y_k)$ B) $y_{k+1} = y_{k+1} + h\, f(x_k,y_k)$ C) $y_{k+1} = y_k + h\, f(x_{k+1},y_{k+1})$
D) $y_{k+1} = y_k - h\, f(x_{k+1},y_{k+1})$ E) $y_{k+1} = y_k - h\, f(x_k,y_k)$ AB) $y_{k+1} = y_k + f(x_k,y_k)$
AC) $y_{k+1} = y_k + h\, f'(x_k,y_k)$ AD) 0.0 AE) 0.01 BC) 0.02 BD) 0.1 BE) 0.2 CD) 1.0
CE) 1.1 DE) 1.2 ABC) 2.0 ABD) 2.1 ABE) 2.2 ACD) 3.0 ACE) 3.1 ADE) 3.2
BCD) 4.0 BCE) 4.1 BDE) 4.2 CDE) 2.32 ABCD) 2.41 ABCE) 2.43 ABDE) 3.3
ACDE) 3.4 BCDE)3.64 ABCDE)None of the above.

Possible points this page = 10. POINTS EARNED THIS PAGE = _______

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True or false. Solution of Linear Algebraic Equations having possibly complex coefficients. Assume A is an m×n matrix of possibly complex numbers, that $\vec{x}$ is an n×1 column vector of possibly complex unknowns, and that $\vec{b}$ is an m×1 possibly complex-valued column vector. Now consider the problem Prob($\mathbf{C}^n$, $A\vec{x} = \vec{b}$); that is, the problem of solving the vector equation

$$\underset{m\times n}{A}\ \underset{n\times 1}{\vec{x}} = \underset{m\times 1}{\vec{b}}\ . \qquad (*)$$

where we look for solutions in $\mathbf{C}^n$. Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

8.(1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has exactly one solution.

9.(1 pt.) A)True or B)False The vector equation (*) may have exactly two distinct solutions.

10.(1 pt.) A)True or B)False The vector equation (*) may have an infinite number of solutions.

11. (1 pt.) A)True or B)False If A is square and singular, then (*) always has a unique solution.

12. (1 pt.) A)True or B)False If $A = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$ then (*) has a unique solution for any $\vec{b} \in \mathbf{C}^m$.

13. (1 pt.) A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

14. (1 pt.) A)True or B)False The equation (*) can be considered as a linear mapping problem from one vector space to another.

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

15. (2 pts.) Definition. Let $S = \{ \vec{v}_1, \vec{v}_2, ..., \vec{v}_k \} \subseteq V$ where V is a vector space and the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0}$ be (*). Then S is linearly independent if

____. _______A B C D E

A) the vector equation (*) has only the trivial solution $c_1 = c_2 = \cdots = c_k = 0$.

B) the vector equation (*) has an infinite number of solutions.

C) the vector equation (*) has a solution other than the trivial solution.

D) the vector equation (*)has at least two solutions. E) the vector equation (*) has no solution.

AB) the associated matrix is nonsingular. AC. The associated matrix is singular

ABCDE) None of the above statements are correct.

Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, determine the appropriate answer that gives an appropriate method to prove that your results are correct (attendance is mandatory). Be careful. If you get them backwards, you miss them both.

16. (4 pts.) Let $S = \{ \vec{v}_1, \vec{v}_2 \} \subseteq \mathbf{R}^3$ where $\vec{v}_1 = [2, 2, 6]^T$ and $\vec{v}_2 = [3, 3,9]^T$. Then S is

____. _____A B C D E

A) linearly independent as $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$.

C) linearly dependent as $c_1\vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $3\vec{v}_1 + (-2) \vec{v}_2 = [0,0,0]$.

E) neither linearly independent or linearly dependent as the definition does not apply.

17. (4 pts.) Let $S = \{ \vec{v}_1, \vec{v}_2 \} \subseteq \mathbf{R}^3$ where $\vec{v}_1 = [2, 4, 7]^T$ and $\vec{v}_2 = [3, 6, 12]^T$. Then S is

____. _____A B C D E

A) linearly independent as $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$.

C) linearly dependent as $c_1\vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $3\vec{v}_1 + (-2) \vec{v}_2 = [0,0,0]$.

E) neither linearly independent or linearly dependent as the definition does not apply.

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Let $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ i \end{bmatrix}$. Solve $A\vec{x} = \vec{b}$.

Write your answer according to the directions given in class (attendance is mandatory).

18. (4 pts.) If $\left[A \middle| \vec{b} \right]$ is reduced to $\left[U \middle| \vec{c} \right]$ using Gauss elimination, then

$\left[U \middle| \vec{c} \right]$ =____. _______A B C D E

A) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ B) $\left[\begin{array}{cc|c} 1 & i & -1 \\ 0 & 0 & 0 \end{array}\right]$ C) $\left[\begin{array}{cc|c} 1 & -i & 1 \\ 0 & 0 & 0 \end{array}\right]$ D) $\left[\begin{array}{cc|c} 1 & -i & -1 \\ 0 & 0 & 0 \end{array}\right]$ E) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$ AB) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$

ABCDE) None of the above

19. (4 pts.) The general solution of $A\vec{x} = \vec{b}$ can be written

as $\begin{bmatrix} x \\ y \end{bmatrix}$ = ____. _______A B C D E

A) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ C) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AB) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AC) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

AD) No solution AE) Exactly one solution, but none of the above is correct

BC) More that one solution, but a finite number and none of the above is correct.

BD) An infinite number of solutions, but none of the above is a correct description

ABCDE)None of the above correctly describes the answer to the question

20. (1 pt.) The solution set for this problem may be written as

S =________________________ .______A B C D E A) $\emptyset$ B) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ C) $\left\{\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right\}$ D) $\left\{ y\begin{bmatrix} -i \\ 1 \end{bmatrix} : y \in \mathbf{R} \right\}$

E) $\left\{ \vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$ AB) $\left\{ \vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$

AC) $\left\{ \vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$ AD) $\left\{ \vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$

ABCDE) None of the above correctly describes the solution set

21. (1 pt.) The number of solutions to this problem is __Infinite__________._AC____A B C D E

A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions ABCDE) None of the above

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

DEFINITION. An operator T:V→W where V and W are vector spaces over the same field **K** is linear if for all α,β∈**K** and $\vec{v}_1, \vec{v}_2 \in$V, we have

22. (3 pts.) ____. ________A B C D E

THEOREM. The operator L:$\mathcal{A}$(**R**,**R**)→$\mathcal{A}$(**R**,**R**) defined by L[y] = y" + 2y is a linear operator.

Proof. By the above definition, to show that the operator L is a linear operator, we must show that if c_1 and c_2 are constants in the field **R** and $\varphi_1(x)$ and $\varphi_2(x)$ are functions in $\mathcal{A}$(**R**,**R**), then

23.(2 pts.) ____. _______A B C D E Since this is an identity, we can use the statement/reason format for proving identities.

STATEMENT	REASON
$L[c_1\varphi_1(x)+c_2\varphi_2(x)] = [c_1\varphi_1(x)+c_2\varphi_2(x)]'' + 2[c_1\varphi_1(x)+c_2\varphi_2(x)]$	24. (2 pts.)_________________ _________ A B C D E
= 25. (2 pts) ___________________________________ . _________ A B C D E	Calculus theorems
= 26. (2 pt) ____________________________________ . __________ A B C D E	Definition of L.

Since we have shown the appropriate identity, we have shown that L is a linear operator.

QED

Possible answers this page.

A) $T(\vec{v}_1 + \vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$ B) $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$ C) $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$
D) $T(\alpha\vec{v}_1) = T(\vec{v}_1)$ E) $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = \alpha T(\vec{v}_1) + \beta T(\vec{v}_2)$ AB) $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = T(\alpha\vec{v}_1) + T(\beta\vec{v}_2)$
AC) $L[\varphi_1(x)+\varphi_2(x)] = L[\varphi_1(x)] + L[\varphi_2(x)]$ AD) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$ AE) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[\varphi_1(x)]+L[\varphi_2(x)]$ BC) $L[c_1\varphi_1(x)] = L[\varphi_1(x)]$
BD) $L[c_1\varphi_1(x)] = c_1L[\varphi_1(x)]$ BE) $L[c_1\varphi_1(x)] = c_1L[\varphi_1(x)]$ CD) $L[\varphi_1(x)]+L[\varphi_2(x)]$
CE) $L[c_1\varphi_1(x)] + L[c_2\varphi_2(x)]$ DE) $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$ ABC) $L[c_1\varphi_1(x)]$
ABD) $c_1[\varphi_1''(x) +\varphi_1(x)]+c_2[\varphi_2''(x) +\varphi_2(x)]$ ABE) $c_1[\varphi_1''(x) - 2\varphi_1(x)]+c_2[\varphi_2''(x) - 2\varphi_2(x)]$
ACD) $c_1[\varphi_1''(x) + 3\varphi_1(x)]+c_2[\varphi_2''(x) + 3\varphi_2(x)]$ ADE) $c_1[\varphi_1''(x) + 4\varphi_1(x)]+c_2[\varphi_2''(x) + 4\varphi_2(x)]$
BCD) Definition of L BCE) Theorems from Calculus, BDE) Definition of T CDE) Definition of $\mathcal{A}$(**R**,**R**) ABCDE) None of the above.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let $x^2y'' + 2xy' + 4 = 0$ $I = (0,\infty)$ be (*). Also let $L: \mathcal{A}((0,\infty),\mathbf{R}) \to \mathcal{A}((0,\infty),\mathbf{R})$ be the operator defined by $L[y] = y'' + (2/x)y'$ and N_L be the null space of L. On the back of the previous page provide sufficient steps in the solution of (*) to answer the following questions..

27. (1 pt) Let (**) be the resulting first order linear ODE in v and x after making the substitution $v = y'$ in (*). The standard form for (**) is_______________________. _______ A B C D E
A) $x^2v'+xv+4x^2 = 0$ B) $x^2v'+2xv-4x^2 = 0$ C) $x^2v'+x v + 4x = 0$ D) $x^2v' + x v-4x = 0$
AB) $x^2v'+xv + 4 = 0$ AC) $x^2v'+xv - 4 = 0$ AD) $x^2v'+xv = 4/x$ AE) $x^2v'+xv = -4/x$
BC) $x^2v'+xv= 4 x$ BD) $x^2v'+xv= -4x$ BE) $x^2v'+xv = 4x^2$ CD) $x^2v'+xv = -4x^2$
CE) $v'+(1/x)v = 4x^2$ DE) $v'+(1/x)v = -4x^{-1}$ ABC) $v'+(2/x)v = -4x^{-2}$ ABD) $v'+(3/x)v = -4x^{-3}$
ABE) $v' + (4/x) v = -4x^{-4}$ ACD) $v'+(4/x) v = -4$ ACE) $v'+(1/x)v =4/x$ BCD) $v' + (1/x) v = -4/x$
ABCDE) None of the above

28. (2 pts.) An integrating factor for (**) is μ =____________. _______A B C D E A) e^x B) e^{-x} C) e^{2x} D) e^{-2x} E) x AB) x^2 AC) x^3 AD) x^4 AE) $1/x$ BC) $-1/x$ ABCDE) None of the above

29. (3 pts.) In solving (**), the following step occurs:

____. _________ A B C D E
A) $d(ve^x)/dx = 4$ B) $d(ve^x)/dx = 4$ C) $d(ve^x)/dx = 4x$ D) $d(ve^x)/dx = -4x$ E) $d(vx)/dx = 4$
AB) $d(vx)/dx = 4x^2$ AC) $d(vx)/dx = 4x$ AD) $d(vx)/dx = -4$ AE) $d(vx^2)/dx = -4$
BC) $d(vx^3)/dx = -4$ BD) $d(vx^4)/dx = -4$ BE) $d(vx^2)/dx = -4x^2$ CD) $d(vx^2)/dx = 4x^3$
ABCDE) None of the above steps ever appears in any solution of (**).

Possible points this page = 10. POINTS EARNED THIS PAGE = _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

This problem is a continuation of the problem type from the previous page, but with different functions. Consider the ODE $y'' + p(x)\,y' = g(x)$ (with $x>0$) which we call (*). Let $L:\mathcal{A}((0,\infty),\mathbf{R})\to\mathcal{A}((0,\infty),\mathbf{R})$ be the operator defined by $L[y] = y'' + p(x)\,y'$ and N_L be the null space of L. Suppose by letting $v = y'$, we can obtain the ODE $d(vx)/dx = 2$ which we call (**). On the back of the previous sheet, you are to solve (**) and then (*) and then answer the following questions.

30. (4 pt) The general solution of (**) may be written as

$v =$ ___________________________________. _______A B C D E A) $1 + c/x$ B) $2 + c/x$ C) $3+c/x$ D) $4 +c/x$ E) $(4/x) + c\ln(x)$ AB) $(4/x) + c\ln(x)$ AC) $2x + c/x^2$ AD) $-2 + c/x^2$ AE) $2+c/x$ BC) $-2+c/x$ BD) $2+ c\ln(x)$ BE) $-2+c\ln(x)$ CD) $2x +c/x^2$ CE) $-2x+c/x$ DE) $2x+c/x$ ABC) $2x + c/x^2$ ABD) $2x + c/x^2$ ABCDE) None of the above.

31. (3 pt) The general solution of (*) may be written as

$y(x) =$ ____. _______A B C D E A) $4 +c_1/x^2 + c_2$ B) $-4+c_1/x^2 +c_2$ C) $4x+c_1/x +c_2$ D) $-4x+(c_1/x) + c_2$ E) $4x+(c_1/x^2)+c_2$ AB) $-4x+c_1/x^2 + c_2$ AC) $x + c_1\ln x + c_2$ AD) $2x+ c_1\ln x+c_2$ AE) $3x+(c_1/x)+c_2$ BC) $4x +(c_1/x)+c_2$ BD) $x^2+ c_1\ln x +c_2$ BE) $-x^2 + c_1\ln(x) + c_2$ CD) $x^2+ c_1\ln x +c_2$ CE) $-x^2+ c_1\ln x +c_2$ DE) $x+(c_1/x) +c_2$ ABC) $-x+(c_1/x)+c_2$ ABD) $x+(c_1/x^2) +c_2$ ABE) $-x + (c_1/x^2) + c_2$ ABCDE) None of the above.

32. (1 pt) The dimension of the null space for L is ____________________. _______ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 ABCDE) None of the above.

33. (2 pts) A basis for the null space of L is B = ______________________. ______ A B C D E A) $\{1/x, 1\}$ B) $\{1/x^2, 1\}$ C) $\{ 1/x, 1/x^2 \}$ D) $\{ 1, e^{-x} \}$ E) $\{ 1/x, e^{-x} \}$ AB) $\{ 1/x^2, e^x \}$ AC) $\{ 1, x \}$ AD) $\{ 1, x^2 \}$ AE) $\{ x, x^2 \}$ BC) $\{1, \ln(x)\}$ ABCDE) None of the above.

Possible points this page = 10. POINTS EARNED THIS PAGE = _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answers.

Let y = y(x) so that y′ = dy/dx. Consider the ODE $y''+4y'+4y = 0 \quad \forall x \in \mathbf{R}$ which we call (*). Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = y''+4y'+4y$ and let N_L be the null space of L. On the back of the previous page solve (*) and then answer the questions below. Be careful! Once you make a mistake, the rest is wrong.

34. (1 pt) The dimension of N_L is __________. ______ A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 ABCDE) None of the above.

35. (1 pts) The auxiliary equation for (*) is _________________________. ______A B C D E
A)$r^2+2r+1 = 0$ B)$r^2+4r+4 = 0$ C) $r^2+6r+9 = 0$ D) $r^2+8r+16 = 0$ E) $r^2-2r+1 = 0$ AB) $r^2-4r+4 = 0$ AC)$r^2-6r+9 = 0$ AD)$r^2-8r+16 = 0$ AE) $r^2-2r^2-1 = 0$ BC)$r^4+2r^2-1 = 0$ BD) $r^2-2r^2+1= 0$ BE)AE)$r^2-8r-16 = 0$ CD)$r^2+10r+25 = 0$ CE)$r^2+10r-25 = 0$ DE)$r^2-10r+25 = 0$ ABC)$r^2-10r-25 = 0$ ABCDE) None of the above.

36. (2 pts) Listing repeated roots, the roots of the auxiliary equation

are r =__________________. ________ A B C D E A)0,0 B)0,2 C)−1,−1 D)2,2 E) −2,−2 AB)2,−2 AC)0,3 AD)0,−3 AE)3,3 BC) −3,−3 BD)3,−3 BE)2,3 CD)−4,−4 ABCDE)None of the above.

37. (3 pts) A basis for N_L is B = _________________. _______A B C D E A)$\{1,x\}$ B)$\{1,e^{2x}\}$ C)$\{1,e^{-2x}\}$ D)$\{e^{-x},xe^{-x}\}$ E)$\{e^{-2x},x e^{-2x}\}$ AB)$\{e^{2x},e^{-2x}\}$ AC)$\{1,e^{3x}\}$ AD)$\{1,e^{-3x}\}$ AE)$\{e^{3x},xe^{3x}\}$ BC) $\{e^{-3x},xe^{-3x}\}$ BD)$\{e^{3x},e^{-3x}\}$ BE)$\{e^{2x},e^{3x}\}$ CD)$\{e^{-4x}, xe^{-4x}\}$ CE)$\{1,e^{-2x},e^{-3x}\}$ DE)$\{1,e^{2x}, e^{-2x}\}$ ABC)$\{1, e^{2x},xe^{2x}\}$ ABD)$\{1,x,e^{-2x}\}$ ABE) $\{1,x, e^{2x}\}$ BCD) $\{1,x, e^{3x}\}$ ABCDE) None of the above.

38. (2 pt) The general solution of (*) is y =_____________________________. ____A B C D E
A)c_1+c_2x B)$c_1e^{-x}+c_2xe^{-x}$ C)$c_1x+c_2xe^{-2x}$ D)$c_1x+c_2e^{-2x}$ E) $c_1e^{-2x}+c_2xe^{-2x}$ AB)$c_1e^{2x}+c_2e^{-2x}$ AC)$c_1+c_2e^{3x}$ AD)$c_1+c_2e^{-2x}$ AE)$c_1e^{2x}+c_2xe^{2x}$ BC)$c_1e^{-2x}+c_2xe^{-2x}$ BD)$c_1e^{2x}+c_2e^{-2x}$ BE) $c_1e^{-3x}+c_2xe^{-3x}$ CD)$c_1e^{3x}+c_2e^{-2x}$ CE)$c_1e^{-4x}+c_2xe^{-4x}$ DE)$c_1+c_2e^{2x}+c_3e^{-2x}$ ABC)$c_1+c_2e^{2x}+c_3xe^{2x}$ ABD)$c_1+c_2x+c_3e^{-2x}$ ABE)$c_1+c_2x+c_3e^{2x}$ BCD)$c_1+c_2x+c_3e^{3x}$ ABCDE) None of the above.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answers.

Let $y = y(x)$ so that $y' = dy/dx$. Consider the ODE $y'' - 4y' - y = 0 \ \forall x \in \mathbf{R}$, which we call (*). Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = y'' - 4y' - y$, and let N_L be the null space of L. On the back of the previous page solve (*) and then answer the questions below. Be careful! Once you make a mistake, the rest is wrong.

39. (1 pt) The dimension of N_L is ___________. ________ A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

40. (1 pts) The auxiliary equation for (*) is ____________________________. ______ A B C D E
A) $r^2+4r+5 = 0$ B) $r^2-4r+5 = 0$ C) $r^2+4r+6 = 0$ D) $r^2-4r+6 = 0$ E) $r^2+4r+7 = 0$
AB) $r^2-4r+7 = 0$ AC) $r^2+6r+10 = 0$ AD) $r^2-6r+10 = 0$ AE) $r^2+6r+11 = 0$ BC) $r^2-6r+11 = 0$
ABCDE) None of the above.

41. (2 pts) Listing repeated roots, the roots of the auxiliary equation are

r = _________________________. ______A B C D E A)1,3 B) −1, −3 C)2,4 D) −2, −4
E) $1+\sqrt{2}, 1-\sqrt{2}$ AB) $2+\sqrt{5}, 2-\sqrt{5}$ AC) $3+\sqrt{10}, 3-\sqrt{10}$ AD) $4+\sqrt{17}, 4-\sqrt{17}$ AE) $1+i\sqrt{2}, 1-i\sqrt{2}$
BC) $2+i\sqrt{5}, 2-i\sqrt{5}$ BD) $3+i\sqrt{10}\, i, 3-\sqrt{10}\, i$ BE) $4+i\sqrt{17}\, i, 4-i\sqrt{17}\, i$ CD) $3+i\sqrt{2}, 3-i\sqrt{2}$
CE) $-3+i\sqrt{2}, -3-i\sqrt{2}$ ABCDE) None of the above.

42. (3 pts) A basis for N_L is B =______________________________. ________ A B C D E A) $\{e^x, e^{3x}\}$
B) $\{e^{-x}, e^{-3x}\}$ C) $\{e^{2x}, e^{4x}\}$ D) $\{e^{-2x}, e^{-4x}\}$ E) $\{ e^{(1+\sqrt{2})x}, e^{(1-\sqrt{2})x} \}$ AB) $\{ e^{(2+\sqrt{5})x}, e^{(2-\sqrt{5})x} \}$
AC) $\{ e^{(3+\sqrt{10})x}, e^{(3-\sqrt{10})x} \}$ AD) $\{ e^{(4+\sqrt{17})x}, e^{(4-\sqrt{17})x} \}$ AE) $\{e^x\cos(\sqrt{2}\,x), e^x\sin\sqrt{2}\,(x)\}$
BC) $\{e^{2x}\cos(\sqrt{5}\,x), e^{-2x}\sin(\sqrt{5}\,x)\}$ BD) $\{e^{3x}\cos(\sqrt{10}\,x), e^{3x}\sin(\sqrt{10}\,x)\}$
BE) $\{e^{4x}\cos(\sqrt{17}\,x), e^{4x}\sin(\sqrt{17}\,x)\}$ CD) $\{e^{3x}\cos(\sqrt{2}\,x), e^{3x}\sin(\sqrt{2}\,x)\}$
CE) $\{e^{-3x}\cos(\sqrt{2}\,x), e^{-3x}\sin(\sqrt{2}\,x)\}$ ABCDE) None of the above.

43. (2 pt) The general solution of (*) is y(x) =______________________________. ______A B C D E
A) $c_1e^x + c_2e^{3x}$ B) $c_1e^{-x} + c_2e^{-3x}$ C) $c_1e^{2x} + c_2 e^{4x}$ D) $c_1e^{-2x}+c_2e^{-4x}$ E) $c_1e^{(1+\sqrt{2})x} + c_2e^{(1-\sqrt{2})x}$
AB) $c_1e^{(2+\sqrt{5})x} + c_2e^{(2-\sqrt{5})x}$ } AC) $c_1e^{(3+\sqrt{10})x} + c_2e^{(3-\sqrt{10})x}$ AD) $c_1e^{(4+\sqrt{17})x} + c_2e^{(4-\sqrt{17})x}$
AE) $c_1e^x\cos(\sqrt{2}\,x)+c_2e^x\sin(\sqrt{2}\,x)$ BC) $c_1e^{2x}\cos\sqrt{5}\,(x)+c_2e^{2x}\sin(\sqrt{5}\,x)$
BD) $c_1e^{3x}\cos(\sqrt{10}\,x)+c_2e^{2x}\sin(\sqrt{10}\,x)$ BE) $c_1e^{4x}\cos(\sqrt{17}\,x) + c_2e^{4x}\sin(\sqrt{17}\,x)$
CD) $c_1e^{3x}\cos(\sqrt{2}\,x+ c_2e^{3x}\sin(\sqrt{2}\,x)$ CE) $c_1e^{-3x}\cos(\sqrt{2}\,x)+c_2e^{-3x}\sin(\sqrt{2}\,x)$
ABCDE) None of the above. .

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answers.

Let y = y(x) so that y′ = dy/dx. Consider the ODE $4y'' - 5y' + y = 0 \ \forall x \in \mathbf{R}$ which we call (*). Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = 4y'' - 5y' + y$, and let N_L be the null space of L. On the back of the previous page solve (*) and then answer the questions below. Be careful! Once you make a mistake, the rest is wrong.

44. (1 pt) The dimension of N_L is ________. _______A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

45. (1 pts) The auxiliary equation for (*) is ________________________. _______A B C D E
A) $r^2 - 2r + 1 = 0$ B) $r^2 - 4r + 1 = 0$ C) $r^2 + 4r + 2 = 0$ D) $r^2 - 4r + 2 = 0$
E) $r^2 + 4r + 3 = 0$ AB) $r^2 - 4r + 3 = 0$ AC) $r^2 + 6r + 6 = 0$ AD) $r^2 - 6r + 6 = 0$
AE) $r^2 + 6r + 7 = 0$ E) $r^2 - 6r + 7r = 0$ AB) None of the above.

46. (2 pts) Listing repeated roots, the roots of the auxiliary equation are

r = __________________. _________A B C D E A)1,1/2 B)1,1/4 C)1,1/6 D)1,1/8
E)$2+\sqrt{2}, 2-\sqrt{2}$ AB)$-2+\sqrt{2}, -2-\sqrt{2}$ AC)$3+\sqrt{2}, 3-\sqrt{2}$ AD)$-3+\sqrt{2}, -3-\sqrt{2}$
AE)$2+i, 2-i$ BC) $-2+i, -2-i$ BD)$2+\sqrt{2}\,i, 2-\sqrt{2}\,i$ BE)$-2+\sqrt{2}\,i, -2-\sqrt{2}\,i$ CD)$3+\sqrt{2}\,i, 3-\sqrt{2}\,i$
CE)$-3+\sqrt{2}\,i, -3-\sqrt{2}\,i$ ABCDE) None of the above.

47. (3 pts) A basis for N_L is B =________________________. ________ A B C D E A)$\{e^x, e^{x/2}\}$
B)$\{e^x, e^{x/4}\}$ C)$\{e^x, e^{x/6}\}$ D)$\{e^x, e^{x/8}\}$ E)$\{e^{(2+\sqrt{2})x}, e^{(2-\sqrt{2})x}\}$ AB)$\{e^{(-2+\sqrt{2})x}, e^{(-2-\sqrt{2})x}\}$
AC)$\{e^{(3+\sqrt{2})x}, e^{(3-\sqrt{2})x}\}$ AD)$\{e^{(-3+\sqrt{2})x}, e^{(-3-\sqrt{2})x}\}$ AE)$\{e^{2x}\cos(x), e^{2x}\sin(x)\}$ BC)$\{e^{-2x}\cos(x), e^{-2x}\sin(x)\}$
BD)$\{e^{2x}\cos(\sqrt{2}\,x), e^{2x}\sin(\sqrt{2}\,x)\}$ BE)$\{e^{-2x}\cos(\sqrt{2}\,x), e^{-2x}\sin(\sqrt{2}\,x)\}$
CD)$\{e^{3x}\cos(\sqrt{2}\,x), e^{3x}\sin(\sqrt{2}\,x)\}$ CE)$\{e^{-3x}\cos(\sqrt{2}\,x), e^{-3x}\sin(\sqrt{2}\,x)\}$ ABCDE) None of the above.

48. (2 pt) The general solution of (*) is y(x) =________________________. _______A B C D E
A)$c_1e^x + c_2e^{x/2}$ B)$c_1e^x + c_2e^{x/4}$ C)$c_1e^x + c_2 e^{x/6}$ D)$c_1e^x + c_2e^{x/8}$ E) $c_1e^{(2+\sqrt{2})x} + c_2e^{(2-\sqrt{2})x}$
AB) $c_1e^{(-2+\sqrt{2})x} + c_2e^{(-2-\sqrt{2})x}$ } AC) $c_1e^{(3+\sqrt{2})x} + c_2e^{(3-\sqrt{2})x}$ AD) $c_1e^{(-3+\sqrt{2})x} + c_2e^{(-3-\sqrt{2})x}$
AE)$c_1e^{2x}\cos(x) + c_2e^{2x}\sin(x)$} BC)$c_1e^{-2x}\cos(x) + c_2e^{-2x}\sin(x)$} BD)$c_1e^{2x}\cos(\sqrt{2}\,x) + c_2e^{2x}\sin(\sqrt{2}\,x)$}
BE)$c_1e^{-2x}\cos(\sqrt{2}\,x + c_2), e^{-2x}\sin(\sqrt{2}\,x)$} CD)$c_1e^{3x}\cos(\sqrt{2}\,x + c_2e^{3x}\sin(\sqrt{2}\,x)$
CE)$c_1e^{-3x}\cos(\sqrt{2}\,x) + c_2e^{-3x}\sin(\sqrt{2}\,x)$} ABCDE) None of the above. .

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Let $y = y(x)$ so that $y' = dy/dx$. Consider the ODE $y'' + 4y = 0 \ \forall x \in \mathbf{R}$ which we call (*). Let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = y'' + 4y$, and let N_L be the null space of L. On the back of the previous page solve (*) and then answer the questions below. Be careful! Once you make a mistake, the rest is wrong.

49. (1 pt) The dimension of N_L is _____________. _______A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

50. (1 pts). The auxiliary equation for (*) is ____________________________. ____A B C D E
A) $r^2 + 1 = 0$ B) $r^2 + 4 = 0$ C) $r^2 + 9 = 0$ D) $r^2 + 16 = 0$ E) $2r^2 + 3r + 1 = 0$
AB) $2r^2 + 3r - 1 = 0$ AC) $2r^2 - 3r + 1 = 0$ AD) $2r^2 - 3r - 1 = 0$ AE) $2r^2 + r + 3 = 0$ BC $2r^2 + r - 3 = 0$
BD) $r^2 - r + 3 = 0$ BE) $2r^2 - r - 3 = 0$ CD) $2r^2 + 5r + 3 = 0$ CE) $2r^2 + 5r - 3 = 0$ ABC) $2r^2 - 5r + 3 = 0$
ABD) $2r^2 - 5r - 3 = 0$ ABCDE) None of the above.

51. (2 pts). Listing repeated roots, the roots of the auxiliary equation are

r = __________________________. _______A B C D E A)1,1 B)2, 2 C) 3,3 D)4, 4
E) $i, -i$ AB) $2i, -2i$ AC) $3i, -3i$ AD) $4i, -4i$ AE) $1 + (1/2)i, 1 - (1/2)i$ BC) $-1 + (1/2)i, -1 - (1/2)i$
BD) $1 + (3/2)i, 1 - (3/2)i$ BE) $-1 + (3/2)i, -1 - (3/2)i$ CD) $1 + i, 1 - i$ CE) 2, 3 DE) 2, −3
ABC) −2, 3 ABD) −2, −3 ABE) None of the above.

52. (3 pts). A basis for N_L is B =_________________________________. _____A B C D E
A) $\{e^x, e^{(1/2)x}\}$ B) $\{e^x, e^{-(1/2)x}\}$ C) $\{e^{-x}, e^{(1/2)x}\}$ D) $\{e^{-x}, e^{-(1/2)x}\}$ E) $\{e^x, e^{(3/2)x}\}$ AB) $\{e^x, e^{-(3/2)x}\}$
AC) $\{e^{-x}, e^{(3/2)x}\}$ AD) $\{e^{-x}, e^{-(3/2)x}\}$ AE) $\{\cos(x), \sin(x)\}$ BC) $\{\cos(2x), \sin(2x)\}$
BD) $\{\cos(3x), \sin(3x)\}$ BE) $\{\cos(4x), \sin(4x)\}$ CD) $\{e^x\cos(x), e^x\sin(x)\}$ CE) $\{e^{2x}, e^{3x}\}$ DE) $\{e^{2x}, e^{-3x}\}$
ABC) $\{e^{-2x}, e^{3x}\}$ ABD) $\{e^{-2x}, e^{-3x}\}$ ABCDE)None of the above

53. (2 pt) The general solution of (*) is y =__________________________. ________A B C D E
A) $c_1e^x + c_2e^{(1/2)x}$ B) $c_1e^x + c_2e^{-(1/2)x}$ C) $c_1e^{-x} + c_2e^{(1/2)x}$ D) $c_1e^{-x} + c_2e^{-(1/2)x}$ E) $c_1e^x + c_2e^{(3/2)x}$
AB) $c_1e^x + c_2e^{-(3/2)x}$ AC) $c_1e^{-x} + c_2e^{(3/2)x}$ AD) $c_1e^{-x} + c_2e^{-(3/2)x}$ AE) $c_1\cos(x) + c_2\sin(x)$
BC) $c_1\cos(2x) + c_2\sin(2x)$ BD) $y = c_1\cos(3x) + c_2\sin(3x)$ BE) $c_1\cos(4x) + c_2\sin(4x)$
CD) $c_1e^{(1/2)x} + c_2e^{3x}$ CE) $c_1e^{(1/2)x} + c_2e^{-3x}$ DE) $c_1e^{-(1/2)x} + c_2e^{3x}$ ABC) $c_1e^{-2x} + c_2e^{3x}$
ABD) $c_1e^{-2x} + c_2e^{-3x}$ ABCDE)None of the above.

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