EXAM-2A-1 FALL 2009

## MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

PRINT NAME		(	,		)
Last Name,	First Name	MI	(What you	wish to be	called)
ID#		_ EXAM DATE _	Friday, Oct	9, 2009 2	:30pm
I swear and/or affirm that all of tand that I have neither given nor			page	Scores points	score
and that I have heliher given her	received any neap during	g the cham.	1	10	Score
SIGNATURE	<u> </u>	DATE	2	7	
INSTRUCTIONS: Besides this cover page, there are 11 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL THE			3	10	
PAGES. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! NO SCRATCH				10	
				11	
				6	
<b>PAPER!</b> Use the back of the ex	You may remove	7	10		
the staple if you wish. Print your in-the Blank/Multiple Choice or	0	8	9		
pages. For each Fill-in-the Blank answer in the blank provided. N	ion write your	9	9		
and write the corresponding lette	wer in the blank	10	9		
provided. Then circle this letter pages. However, to insure credit	_	11	9		
and carefully. Your entire solution	ust your final	12			
answer. <b>SHOW YOUR WORK</b> expressed in your best mathemat	13				
given if deemed appropriate. Procomputations as time allows.	nd check your	14			
		15			
REQUES	T FOR REGRADE		16		
Please regard the following pro (e.g., I do not understand what			17		
		,	18		
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(Regrades should be requested	within a week of the day	to the avam is	20		
returned. Attach additional she	ain your reasons.)	21			
I swear and/or affirm that upon <b>nothing on this exam</b> except o		22			
changing anything is considered	( · · · · · · · · · · · · · · · · · · ·	Tota	1 100		
Date Signature					

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PRINT NAME ( ) ID No.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let  $y = \varphi(x)$  be the solution of the IVP given below. Using Euler's Method with h = 0.1 you are to find a numerical approximation for  $\varphi(0.2)$  (i.e.find  $y_1$  and  $y_2$ ). Use a table and the standard notation used in

class (attendance is mandatory).

**IVP** ODE y' = x + y IC y(0) = 0

1. (2 pts.) The general formula for Euler's method may be written

as \_\_\_\_\_\_. \_\_\_ A B C D E

2. (1 pt.)  $x_0 =$ \_\_\_\_\_\_ A B C D E 5. (1 pt.)  $y_0 =$ \_\_\_\_\_ A B C D E

3. (1 pt.)  $x_1 =$ \_\_\_\_\_. A B C D E 6. (2 pts.)  $y_1 =$ \_\_\_\_. A B C D E

4. (1 pt.)  $x_2 =$  A B C D E 7. (2 pts.)  $y_2 =$  A B C D E

Possible answers this page.

A)  $y_{k+1} = y_k + h f(x_k, y_k)$  B)  $y_{k+1} = y_{k+1} + h f(x_k, y_k)$  C)  $y_{k+1} = y_k + h f(x_{k+1}, y_{k+1})$ 

D)  $y_{k+1} = y_k - h f(x_{k+1}, y_{k+1})$  E)  $y_{k+1} = y_k - h f(x_k, y_k)$  AB)  $y_{k+1} = y_k + f(x_k, y_k)$ 

AC)  $y_{k+1} = y_k + h f'(x_k, y_k)$  AD) 0.0 AE) 0.01 BC) 0.02 BD) 0.1 BE) 0.2 CD) 1.0

CE) 1.1 DE) 1.2 ABC) 1.21 ABD) 1.22 ABE) 1.23 ACD) 1.52 ACE) 2.0 ADE) 2.1

BCD) 2.2 BCE) 2.3 BDE) 2.31 CDE) 2.32 ABCD) 2.41 ABCE) 2.43 ABDE) 3.3

ACDE) 3.4 BCDE)3.64 ABCDE)None of the above.

Possible points this page = 10. POINTS EARNED THIS PAGE =

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PRINT NAME \_\_\_\_\_(\_\_\_\_\_) ID No. \_\_\_\_\_\_

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**True or false**. Solution of Linear Algebraic Equations having possibly complex coefficients. Assume A is an m×n matrix of possibly complex numbers, that  $\vec{x}$  is an n×1 column vector of possibly complex unknowns, and that  $\vec{b}$  is an m×1 possibly complex-valued column vector. Now consider the problem  $Prob(\mathbf{C}^n, A\vec{x} = \vec{b})$ ; that is, the problem of solving the vector equation

$$A \vec{x} = \vec{b} .$$

$$mxn nx1 = mx1 .$$
(\*)

where we look for solutions in  $\mathbb{C}^n$ . Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

8.(1 pt.) A)True or B)False If  $\vec{b} = \vec{0}$ , then (\*) always has at least one solution.

9.(1 pt.) A)True or B)False The vector equation (\*) may have exactly two distinct solutions.

10.(1 pt.) A)True or B)False The vector equation (\*) may have an infinite number of solutions.

11. (1 pt.) A)True or B)False If A is square and nonsingular, then (\*) always has a unique solution.

12. (1 pt.) A)True or B)False If  $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$  then (\*) has a unique solution for any  $\vec{b} \in \mathbb{C}^m$ .

13. (1 pt.) A)True or B)False Either (\*) has no solutions, exactly one solution, or an infinite number of solutions.

14. (1 pt.) A)True or B)False The equation (\*) can be considered as a linear mapping problem from one vector space to another.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

15. (2 pts.) <u>Definition</u>. Let  $S = \{\vec{v}_1, \vec{v}_2, ..., \vec{v}_k\} \subseteq V$  where V is a vector space and the vector equation  $c_1\vec{v}_1+c_2\vec{v}_2+...+c_k\vec{v}_k=\vec{0}$  be (\*). Then S is linearly independent if

A) the vector equation (\*) has only the trivial solution  $c_1 = c_2 = \cdots = c_k = 0$ .

- B) the vector equation (\*) has an infinite number of solutions.
- C) the vector equation (\*) has a solution other than the trivial solution.
- D) the vector equation (\*)has at least two solutions. E) the vector equation (\*) has no solution.
- AB) the associated matrix is nonsingular. AC. The associated matrix is singular

ABCDE) None of the above statements are correct.

Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, determine the appropriate answer that gives an appropriate method to prove that your results are correct (attendance is mandatory). Be careful. If you get them backwards, you miss them both.

16. (4 pts.) Let  $S = \{\vec{v}_1, \vec{v}_2\} \subseteq \mathbb{R}^3$  where  $\vec{v}_1 = [2, 2, 6]^T$  and  $\vec{v}_2 = [3, 3, 8]^T$ . Then S is

\_\_. \_\_\_\_A B C D E

- A) linearly independent as  $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$  implies  $c_1 = 0$  and  $c_2 = 0$ .
- B) linearly independent as  $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$ .
- C) linearly dependent as  $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$  implies  $c_1 = 0$  and  $c_2 = 0$ .
- D) linearly dependent as  $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$ .
- E) neither linearly independent or linearly dependent as the definition does not apply.

17. (4 pts.) Let  $S = \{\vec{v}_1, \vec{v}_2\} \subseteq \mathbb{R}^3$  where  $\vec{v}_1 = [2, 4, 8]^T$  and  $\vec{v}_2 = [3, 6, 12]^T$ . Then S is

A) linearly independent as  $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$  implies  $c_1 = 0$  and  $c_2 = 0$ .

- B) linearly independent as  $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$ .
- C) linearly dependent as  $c_1 \vec{v}_1 + c_2 \vec{v}_2 = [0,0,0]$  implies  $c_1 = 0$  and  $c_2 = 0$ .
- D) linearly dependent as  $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$ .
- E) neither linearly independent or linearly dependent as the definition does not apply.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let the operator  $T: \mathbb{R}^2 \to \mathbb{R}^2$  be defined by  $T(\vec{x}) = A_{2x22x1} \vec{x}$  where  $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$  and  $\vec{x} = [x,y]^T$ . On the

back of the previous sheet, solve the problem  $\text{Prob}(\mathbf{R}^2, T(\vec{x}) = \vec{0})$ ; that is, the problem of solving the vector equation  $T(\vec{x}) = \vec{0}$ . The form of the answer may not be unique. To obtain the answer listed follow the directions given in class (attendance is mandatory).

18. (4pts.) If A is reduced to U using Gauss elimination (recall that you are to make all pivots

positive), then U = \_\_\_\_\_\_. \_ A B C D E A) 
$$\begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$$
 B)  $\begin{bmatrix} 1 & -4 \\ -3 & 12 \end{bmatrix}$  C)  $\begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$  D)  $\begin{bmatrix} 1 & -4 \\ 0 & 0 \end{bmatrix}$  E)  $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$  ABCDE) None of the above

19. (4pts.) The solution of  $T(\vec{x}) = \vec{0}$  may be written

as 
$$\vec{x} =$$
 \_\_\_\_\_\_. A B C D E A) No Solution B)  $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$  C)  $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$  D)  $\begin{bmatrix} 4 \\ -1 \end{bmatrix}$  E)  $\begin{bmatrix} -4 \\ -1 \end{bmatrix}$  AB)  $y \begin{bmatrix} 4 \\ 1 \end{bmatrix}$  AC)  $y \begin{bmatrix} 4 \\ -1 \end{bmatrix}$  AD)  $y \begin{bmatrix} 4 \\ 0 \end{bmatrix}$ 

ABCDE) None of the above.

20. (1 pt.) The solution set for this problem may be written as

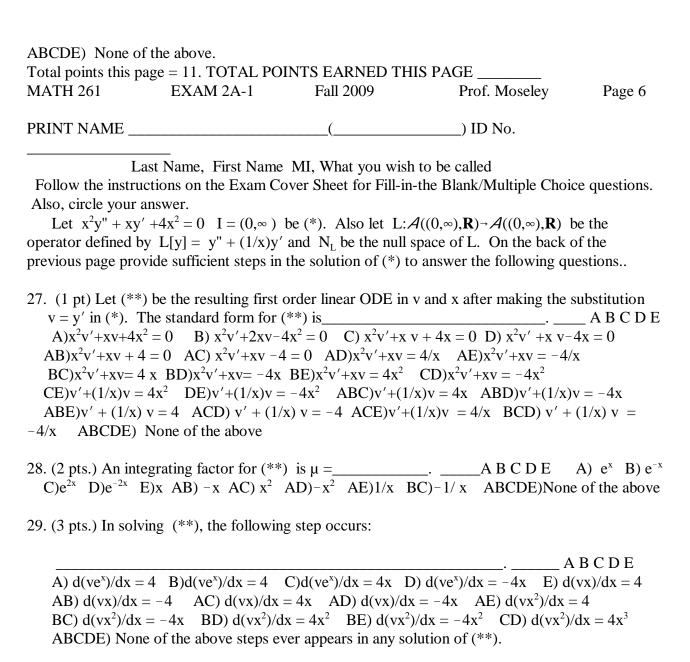
$$S = \underbrace{\qquad \qquad } A B C D E \qquad A) \varnothing \quad B) \left\{ \begin{bmatrix} 0 \\ 0 \end{bmatrix} \right\} \quad C) \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right\} \quad D) \left\{ \begin{bmatrix} 4 \\ -1 \end{bmatrix} \right\}$$

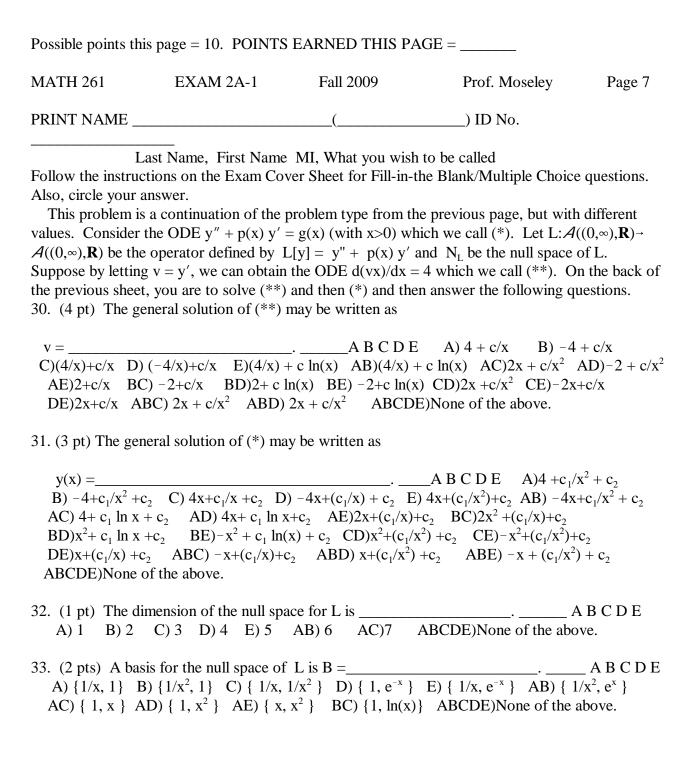
$$E) \left\{ \begin{bmatrix} 4 \\ 1 \end{bmatrix} \right\} \qquad AB) \left\{ \vec{x} = y \begin{bmatrix} 4 \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\} \qquad AC) \left\{ \vec{x} = y \begin{bmatrix} 4 \\ -1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$$

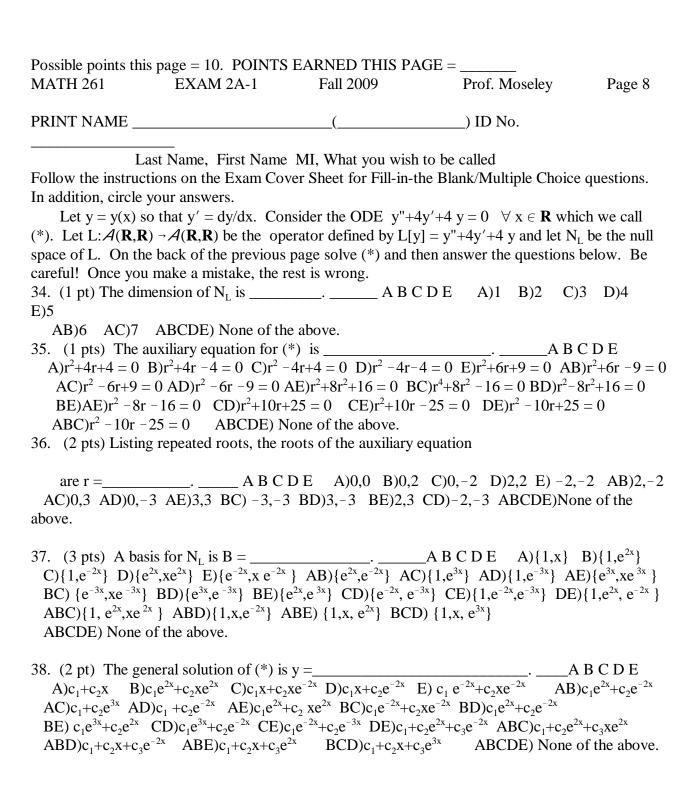
AD) 
$$\left\{ \vec{x} = y \begin{bmatrix} 4 \\ 0 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R} \right\}$$
 BC) None of the above correctly describes the solution set

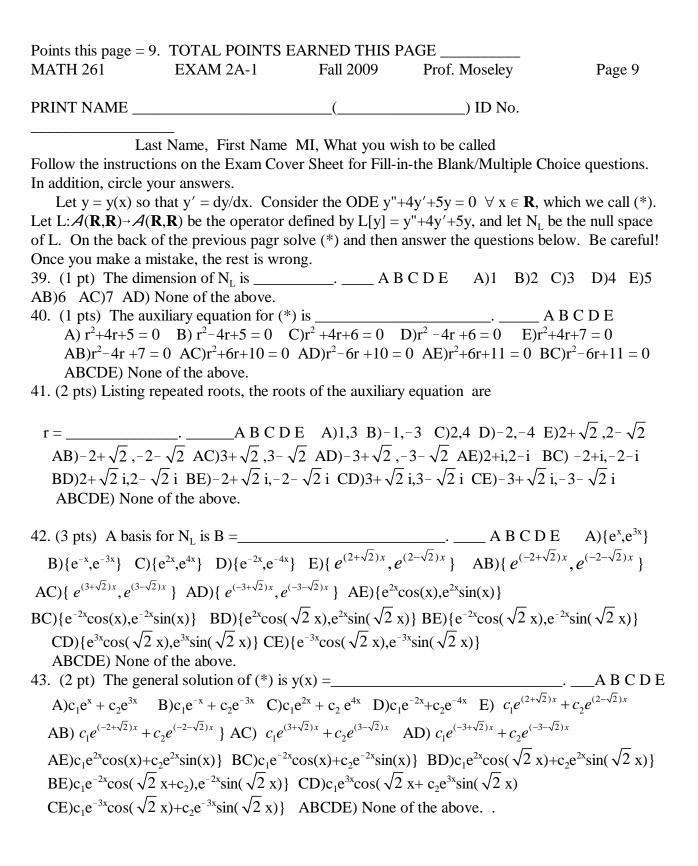
21. (1 pt.) The number of solutions to this problem is \_\_\_ \_\_A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

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Follow the instruction DEFINITION. A		ver Sheet for Fill-in-t nere V and W are vec	to be called he Blank/Multiple Choice etor spaces over the same	-
22. (3 pts.)			·	_A B C D E
Proof. By the abo	ove definition, to show	that the operator L i	$L[y] = y'' + 3y$ is a linear of s a linear operator, we mare functions in $A(\mathbf{R}, \mathbf{R})$	ust show that
23.(2 pts.)			_	ABCDE
	entity, we can use the s CATEMENT	statement/reason form	nat for proving identities. <u>REASON</u>	
$L[c_1\varphi_1(x)+c_2\varphi_2(x)]$	$] = [c_1 \varphi_1(x) + c_2 \varphi_2(x)]''$	$+3[c_1\phi_1(x)+c_2\phi_2(x)]$	24. (2 pts.)	
E	11100	- 1111 / 212 / -		A B C D
= 25	5. (2 pts)		Calculus the	eorems
		A B C D E		
= 26	. (2 pt)	A B C D E	Definition	of L.
Since we have sho			n that L is a linear operat QF	tor. ED
	to fill in the blanks. $R(\vec{y}) + T(\vec{y}) = R(\vec{y})$	$\Gamma(\alpha \vec{\mathbf{v}}) = \alpha \mathbf{T}(\vec{\mathbf{v}})$	C) $T(\alpha \vec{V}_1 + \beta \vec{v}_2) = T(\alpha \vec{V}_1 + \beta \vec{v}_2)$	$(\vec{\mathbf{v}}) + \mathbf{T}(\vec{\mathbf{v}})$
-	_		AB)T( $\alpha \vec{v}_1 + \beta \vec{v}_2$ ) = T( $\alpha$	-
•	·	_	<u>-</u>	-
AE) $L[c_1\phi_1(x)+c_2\phi_1(x)+c_2\phi_2]$ BD) $L[c_1\phi_1(x)+c_2\phi_2]$ CD) $L[\phi_1(x)]+L[\phi_1(x)]$ ABC) $L[c_1\phi_1(x)]$ ABE) $c_1[\phi_1''(x)+\phi_2(x)]$ ADE) $c_1[\phi_1''(x)+\phi_2(x)]$	$ \varphi_{2}(x) $ CE) L[c <sub>1</sub> ABD) [c <sub>1</sub> $\varphi_{1}(x)$ +c <sub>2</sub> $\varphi_{2}(x)$ $3\varphi_{1}(x)$ ]+c <sub>2</sub> [ $\varphi_{2}''(x)$ + $3\varphi_{2}(x)$ $3\varphi_{1}(x)$ ]ACE) c <sub>2</sub> [ $\varphi_{2}''(x)$	$\begin{array}{cccccccccccccccccccccccccccccccccccc$	C) $L[c_1\phi_1(x)] = L[\phi_1(x)]$ E) $L[c_1\phi_1(x)] = c_1L[\phi_1(x)]$ DE) $c_1 L[\phi_1(x)] + c_2L$ $\phi_2(x)$ $\phi_2(x) - 3\phi_1(x)] + c_2[\phi_2''(x)] - c_2[\phi_2''(x)]$	$ \begin{bmatrix} \phi_2(x) \\ \phi_2(x) \end{bmatrix} $

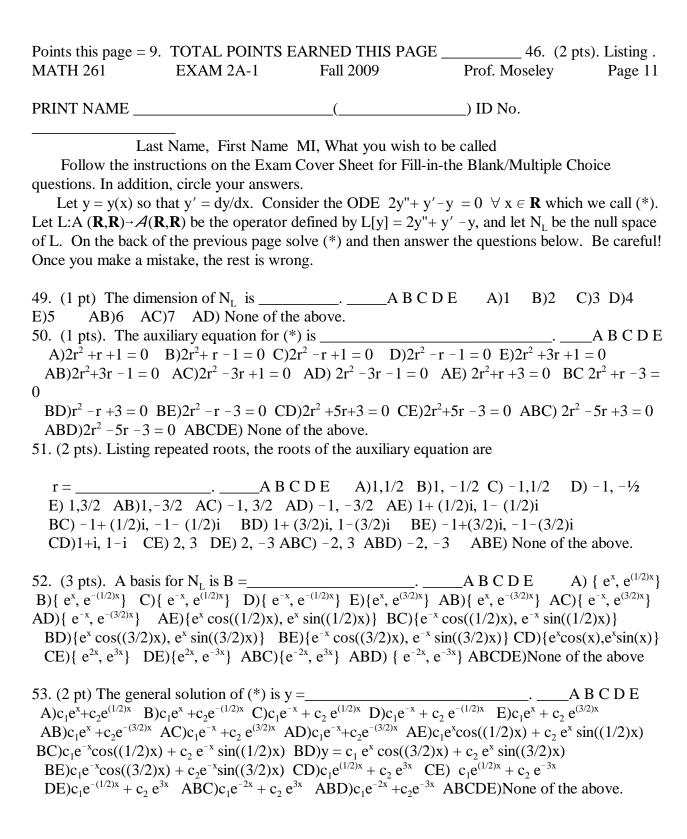












Points this page = 9. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_