EXAM-2 MATH 261: Elementary Differential Equations MATH 261
FALL 2008 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, October 3, 2008

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

_________________________________ _________________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 12 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-12 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit may be given if deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

REQUEST FOR REGRADE
Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)
(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)
Date _______ Signature________________________________

Scores

page	points	score
1	10	
2	10	
3	6	
4	10	
5	7	
6	10	
7	9	
8	8	
9	6	
10	7	
11	7	
12	7	
13	7	
14		
15		
16		
17		
18		
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21		
22		
Total	104	

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Using Euler's Method with h = 0.1, you are to find the first two iterates (i.e. y_1 and y_2) to obtain a numerical approximation of the solution of the Initial Value Problem (IVP) given below for x = 0.2; that is, if y = φ(x) is the solution to the IVP, you are to find an approximation for φ(0.2). Use a table and the standard notation used in class (attendance is mandatory).

IVP ODE $y' = x+y$ IC $y(0) = 1$

1. (2 pts.) The general formula for Euler's method may be written

as ____. ____ A B C D E

2. (1 pt.) x_0 = ______. _____ A B C D E
3. (1 pt.) x_1 = ______. _____ A B C D E
4. (1 pt.) x_2 = ______. _____ A B C D E
5. (1 pt.) y_0 = __________. _____ A B C D E
6. (2 pts.) y_1 = __________. _____ A B C D E
7. (2 pts.) y_2 = __________. ______ A B C D E

Possible answers this page.

A) $y_{k+1} = y_{k+1} + h\,f(x_k,y_k)$ B) $y_{k+1} = y_k + h\,f(x_k,y_k)$ C) $y_{k+1} = y_k + h\,f(x_{k+1},y_{k+1})$
D) $y_{k+1} = y_k - h\,f(x_{k+1},y_{k+1})$ E) $y_{k+1} = y_k - h\,f(x_k,y_k)$ AB) $y_{k+1} = y_{k-1} + h\,f(x_k,y_k)$
AC) $y_{k+1} = y_k + h\,f'(x_k,y_k)$ AD) 0.0 AE) 0.1 BC) 0.2 BD) 0.3 BE) 0.4 CD) 1.0
CE) 1.1 DE) 1.2 ABC) 1.21 ABD) 1.22 ABE) 1.23 ACD) 1.43 ACE) 2.0 ADE) 2.1
BCD) 2.2 BCE) 2.3 BDE) 2.31 CDE) 2.32 ABCD) 2.33 ABCE) 2.41 ABDE) 2.42
ACDE) 2.43 BCDE) 2.64 ABCDE)None of the above.

Possible points this page = 10. POINTS EARNED THIS PAGE = _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$. Below or on the back of the previous page solve Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$); that is, solve the vector equation $A\vec{x} = \vec{b}$ where we look for solutions in $\mathbf{C}^2$. The form of the answer may not be unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory).

8. (4 pts.) If $\left[A \middle| \vec{b} \right]$ is reduced to $\left[U \middle| \vec{c} \right]$ using Gauss elimination we obtain

$\left[U \middle| \vec{c} \right]$ =______________________________. _____A B C D E A) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ B) $\left[\begin{array}{cc|c} 1 & i & -1 \\ 0 & 0 & 0 \end{array}\right]$

C) $\left[\begin{array}{cc|c} 1 & -i & 1 \\ 0 & 0 & 1 \end{array}\right]$ D) $\left[\begin{array}{cc|c} 1 & -i & -1 \\ 0 & 0 & i \end{array}\right]$ E) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$ AB) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ AC)None of the above.

9. (4 pts.) The solution of $A\vec{x} = \vec{b}$ may be written as

$\vec{x}$ = ________________________________._____A B C D E A) No Solution B) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AB) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AC) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AD) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ BC) None of the above correctly describes the solution or collection of solutions.

10. (1 pt.) The solution set S for Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$) may be written as

S =____________________.______A B C D E A) $\varnothing$ B) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ C) $\left\{\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

D) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ E) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AB) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

AC) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AD) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

BC) None of the above correctly describes the solution set for this problem.

11. (1 pt.) The number of solutions to Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$) is _________._____A B C D E A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE _______

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True or false. Solution of Linear Algebraic Equations having possibly complex coefficients.
Assume A is an m×n matrix of possibly complex numbers, that $\vec{x}$ is an n×1 column vector of possibly complex unknowns, and that $\vec{b}$ is an m×1 possibly complex-valued column vector. Now consider the problem Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$); that is, the problem of solving the vector equation

$$\underset{m\times n}{A}\ \underset{n\times 1}{\vec{x}} = \underset{m\times 1}{\vec{b}}\,. \qquad (*)$$

where we look for solutions in $\mathbf{C}^2$. Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

12.(1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has at least one solution.

13.(1 pt.) A)True or B)False The vector equation (*) may have exactly two distinct solutions.

14.(1 pt.) A)True or B)False The vector equation (*) may have an infinite number of solutions.

15. (1 pt.) A)True or B)False If A is square and nonsingular, then (*) always has a unique solution.

16. (1 pt.) A)True or B)False If $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ then (*) has a unique solution for any $\vec{b} \in \mathbf{C}^m$.

17. (1 pt.) A)True or B)False The equation (*) can be considered as a mapping problem from one vector space to another.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

18. (2 pts.) Definition. Let S = { $\vec{v}_1, \vec{v}_2, ..., \vec{v}_k$ }$\subseteq$V where V is a vector space and the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + ... + c_k\vec{v}_k = \vec{0}$ be (*). Then S is linearly independent if

____. _____A B C D E

A) the vector equation (*) has an infinite number of solutions.
B) the vector equation (*) has a solution other than the trivial solution.
C) the vector equation (*) has only the trivial solution $c_1 = c_2 = \cdots = c_k = 0$.
D) the vector equation (*)has at least two solutions. E) the vector equation (*) has no solution.
AB) the associated matrix is nonsingular. AC. The associated matrix is singular
AD) None of the above statements are correct.

Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, determine the appropriate answer that gives an appropriate method to prove that your results are correct (attendance is mandatory). Be careful. If you get them backwards, you miss them both.

19. (4 pts.) Let S = { $\vec{v}_1, \vec{v}_2$ }$\subseteq\mathbf{R}^3$ where $\vec{v}_1 = [2, 2, 6]^T$ and $\vec{v}_2 = [3, 3, 9]^T$. Then S is

____. _____A B C D E

A) linearly independent as $c_1\ \vec{v}_1 + c_2\ \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
B) linearly independent as $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$.
C) linearly dependent as $c_1\vec{v}_1 + c_2\ \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
D) linearly dependent as $3\vec{v}_1 + (-2)\ \vec{v}_2 = [0,0,0]$.
E) neither linearly independent or linearly dependent as the definition does not apply.

20. (4 pts.) Let S = { $\vec{v}_1, \vec{v}_2$ }$\subseteq\mathbf{R}^3$ where $\vec{v}_1 = [2, 4, 8]^T$ and $\vec{v}_2 = [3, 6, 11]^T$. Then S is

____. _____A B C D E

A) linearly independent as $c_1\ \vec{v}_1 + c_2\ \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
B) linearly independent as $3\vec{v}_1 + (-2)\vec{v}_2 = [0,0,0]$.
C) linearly dependent as $c_1\vec{v}_1 + c_2\ \vec{v}_2 = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
D) linearly dependent as $3\vec{v}_1 + (-2)\ \vec{v}_2 = [0,0,0]$.
E) neither linearly independent or linearly dependent as the definition does not apply.

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME _________________________(_________________) ID No. __________________
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True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a (real or complex) vector space V to a (real or complex) vector space W. Now consider the mapping problem

$$T(\vec{x}) = \vec{b} \qquad (*)$$

which we denote by Prob(V, $T(\vec{x}) = \vec{b}$) to indicate that the vector space V is the set where we look for solutions. Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

21. (1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has at least one solution.

22. (1 pt.) A)True or B)False The vector equation (*) may have exactly two distinct solutions.

23. (1 pt.) A)True or B)False The vector equation (*) may have an infinite number of solutions.

24. (1 pt.) A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} = \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where c_1, c_2, ... , c_n are arbitrary scalars.

25. (1 pt.) A)True or B)False If the null space of T is $N_T = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has a unique solution.

26. (1 pt.) A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

27. (1 pt.) A)True or B)False The equation (*) can be considered as a mapping problem from one vector space to another.

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let the operator T:$\mathbf{R}^2 \to \mathbf{R}^2$ be defined by $T(\vec{x}) = \underset{2x2}{A}\,\underset{2x1}{\vec{x}}$ where $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ and $\vec{x}$ = $[x,y]^T$.

Consider the problem Prob($\mathbf{R}^2$, $T(\vec{x}) = \vec{0}$); that is, the problem of solving the vector equation $T(\vec{x}) = \vec{0}$. The form of the solution need not be unique. To obtain the answer listed follow the directions given in class (attendance is mandatory).

28. (4pts.) If A is reduced to U using Gauss elimination, then

U =____. _____ A B C D E

A) $\begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$ D) $\begin{bmatrix} 1 & -4 \\ 0 & 0 \end{bmatrix}$ E) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ AB) None of the above

29. (4pts.) The solution of $T(\vec{x}) = \vec{0}$ may be written

as $\vec{x}$ =____. _____A B C D E

A) No Solution B) $\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ E) $y\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ AB) $y\begin{bmatrix} -4 \\ -1 \end{bmatrix}$ AB) $\vec{x} = y\begin{bmatrix} 4 \\ 0 \end{bmatrix}$
AC) None of the above.

30. (1 pt.) The solution set for this problem may be written as

S =__________________.______A B C D E A) $\varnothing$ B) $\left\{\begin{bmatrix} 4 \\ 1 \end{bmatrix}\right\}$ C) $\left\{\vec{x} = y\begin{bmatrix} 4 \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R}\right\}$

D) $\left\{\begin{bmatrix} -4 \\ 1 \end{bmatrix}\right\}$ E) $\left\{\vec{x} = y\begin{bmatrix} 4 \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R}\right\}$ AB) $\left\{\vec{x} = y\begin{bmatrix} -4 \\ 1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R}\right\}$

AC) $\left\{\vec{x} = y\begin{bmatrix} -4 \\ -1 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R}\right\}$ AD) $\left\{\vec{x} = y\begin{bmatrix} 4 \\ 0 \end{bmatrix} \in \mathbf{R}^2 : y \in \mathbf{R}\right\}$

BC) None of the above correctly describes the solution set for this problem

31. (1 pt.) The number of solutions to this problem is ______________._____A B C D E
A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ___________________
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

DEFINITION. An operator $T:V\to W$ where V and W are vector spaces over the same field **K** is linear if for all $\alpha,\beta\in\mathbf{K}$ and $\vec{v}_1, \vec{v}_2 \in V$, we have

32. (2 pts.) ____. ____A B C D E

THEOREM. The operator $L:\mathcal{A}(\mathbf{R},\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ defined by $L[y] = y'' + 3y$ is a linear operator.
Proof. By the above definition, to show that the operator L is a linear operator, we must show that if c_1 and c_2 are constants in the field **R** and $\varphi_1(x)$ and $\varphi_2(x)$ are functions in $\mathcal{A}(\mathbf{R},\mathbf{R})$, then

33.(2 pts.) ____. ______A B C D E Since this is an identity, we can use the statement/reason format for proving identities.

STATEMENT	REASON
$L[c_1\varphi_1(x)+c_2\varphi_2(x)] = [c_1\varphi_1(x)+c_2\varphi_2(x)]'' + 3[c_1\varphi_1(x)+c_2\varphi_2(x)]$	34. (2 pts.)___________________ _____ A B C D E
= 35. (2 pts) __________________________ . _____ A B C D E	Calculus theorems
= 36. (1 pt) __________________________ . _____ A B C D E	Definition of L.

Since we have shown the appropriate identity, we have shown that L is a linear operator.
QED

Possible answers to fill in the blanks.

A) $T(\vec{v}_1+\vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$ B) $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$ C) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$
D) $T(\alpha\vec{v}_1) = T(\vec{v}_1)$ E) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = \alpha T(\vec{v}_1) + \beta T(\vec{v}_2)$ AB) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = T(\alpha\vec{v}_1) + T(\beta\vec{v}_2)$
AC) $L[\varphi_1(x)+\varphi_2(x)] = L[\varphi_1(x)]+ L[\varphi_2(x)]$ AD) $L[c_1\varphi_1(x)] = c_1L[\varphi_1(x)]$
AE) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[\varphi_1(x)]+L[\varphi_2(x)]$ BC) $L[c_1\varphi_1(x)] = L[\varphi_1(x)]$
BD) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$ BE) $L[c_1\varphi_1(x)] = c_1L[\varphi_1(x)]$
CD) $L[\varphi_1(x)]+L[\varphi_2(x)]$ CE) $L[\varphi_1(x)] + L[\varphi_2(x)]$ DE) $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$
ABC) $[c_1\varphi_1(x)+c_2\varphi_2(x)]'' + 6[c_1\varphi_1(x) + c_2\varphi_2(x)$ ABD) $c_1[\varphi_1''(x) + 3\varphi_1(x)]+c_2[\varphi_2''(x) + 3\varphi_2(x)]$ ABE) $c_1[\varphi_1''(x) - 3\varphi_1(x)]+c_2[\varphi_2''(x) - 3\varphi_2(x)]$ ACD) $c_1[\varphi_1''(x) + 3\varphi_1(x)]$
ACE) $c_2[\varphi_2''(x) + 3\varphi_2(x)]$ ADE) Definition of L BCD) Theorems from Calculus,
BCE) Definition of T BDE) Definition of $\mathcal{A}(\mathbf{R},\mathbf{R})$ CDE) None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let $x^2y'' + 2xy' + 4 = 0$ $I = (0,\infty)$ (i.e. $x>0$) be (*), let $L: \mathcal{A}((0,\infty),\mathbf{R}) \to \mathcal{A}((0,\infty),\mathbf{R})$ be the operator defined by $L[y] = y'' + (2/x)y'$, and let N_L be the null space of L. Solve (*) below or on the back of the previous page.

37. (1 pt) Let (**) be the resulting first order linear ODE in v and x after making the substitution $v = y'$ in (*). The standard form for (**) is___________________________. ____ A B C D E
A) $x^2 v' + 2x v + 4 = 0$ B) $x^2 v' + 2x v - 4 = 0$ C) $x^2 v' + 2x v + 4x = 0$
D) $x^2 v' + 2x v - 4x = 0$ AB) $v' + (2/x) v + 4/x^2 = 0$ AC) $v' + (2/x) v - 4/x^2 = 0$
AD) $v' + (2/x) v = 4/x^2$ AE) $v' + (2/x) v = -4/x^2$ BC) $v' + (2/x) v = 4/x$
BD) $v' + (2/x) v = -4/x$ AC) None of the above

38. (2 pts.) An integrating factor for (**) is μ =_____________. _____A B C D E A) e^{-1} B) e^{-x}
C) e^{-2x} D) e^x E) e^{2x} AB) x^{-2} AC) x^{-1} AD) x AE) 2x BC) x^2 BD)None of the above

39. (2 pts.) In solving (**), the following step occurs:

____. ______ A B C D E
A) $d(ve^x)/dx = 4$ B) $d(vx)/dx = 4$ C) $d(vx)/dx = -4$ D) $d(vx^2)/dx = 4$ E) $d(vx^2)/dx = -4$
AB) $d(vx)/dx = 4x$ AC) $d(vx)/dx = -4x$ AD) $d(vx^2)/dx = 4x$ AE) $d(vx^2)/dx = -4x$
BC) $d(vx)/dx = 4x^2$ BD) $d(vx)/dx = -4x^2$ BE) $d(vx^2)/dx = 4x^2$ CD) $d(vx^2)/dx = -4x^2$
CE) None of the above steps ever appears in any solution of (**).

40. (3 pt) The general solution of (**) may be written as

$v(x) =$ ___________________________________. ______A B C D E A) $(4/x) + c/x^2$
B) $(-4/x) + c/x^2$ C) $(4/x^2) + c/x$ D) $(4/x^2) + c/x$ E) $(4/x^2) + c/x^2$ AB) $(4/x^2) + c/x^2$
AC) $2 + c/x^2$ AD) $2 + c/x^2$ AE) $2 + c/x$ BC) $2 + c/x$ BD) $2 + c/x^2$
BE) $2 + c/x^2$ CD)None of the above.

Possible points this page = 8. POINTS EARNED THIS PAGE = _______

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We continue with the solution of $x^2y'' + 2xy' + 4 = 0$ $I = (0,\infty)$ (i.e. $x>0$) which we call (*), where we let $L:\mathcal{A}((0,\infty),\mathbf{R})\rightarrow\mathcal{A}((0,\infty),\mathbf{R})$ be the operator defined by $L[y] = y'' + (2/x)y'$, and N_L be the null space of L.

$v = y' =$

$y =$

41. (3 pt) The general solution of (*) may be written as

$y(x) =$_____________________________________. ____A B C D E A) $4\ell n(x) + c_1/x^2 + c_2$
B) $-4\ell n(x) + c_1/x^2 + c_2$ C) $4\ell n(x) + c_1/x + c_2$ D) $-4\ell n(x) + c_1/x + c_2$ E) $4\ell n(x) + c_1/x^2 + c_2$
AB) $-4\ell n(x) + c_1/x^2 + c_2$ AC) $2x + c_1/x^2 + c_2$ AD) $2x + c_1/x^2 + c_2$ AE) $2x + c_1/x + c_2$
BC) $2x + c_1/x + c_2$ BD) $2x + c_1/x^2 + c_2$ BE) $2x + c_1/x^2 + c_2$ CD) None of the above.

42. (1 pt) The dimension of the null space for L is ___________________. ______ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC)7 AD) None of the above.

43. (2 pts) A basis for the null space of L is B =________________________. _____ A B C D E
A) $\{1/x, 1\}$ B) $\{1/x^2, 1\}$ C) $\{1/x, 1/x^2\}$ D) $\{1, e^{-x}\}$ E) $\{1/x, e^{-x}\}$ AB) $\{1/x^2, e^x\}$
AC) $\{1, x\}$ AD) $\{1, x^2\}$ AE) $\{x, x^2\}$ BC) None of the above.

Possible points this page = 6. POINTS EARNED THIS PAGE = _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer. Let y = y(x) so that $y' = dy/dx$.

Let $y'' + 6y' + 9y = 0 \ \forall x \in \mathbf{R}$ be (*), let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = y'' + 6y' + 9y$, and let N_L be the null space of L. Solve (*) below or on the back of the previous page. Be careful! Once you make a mistake, the rest is wrong.

44. (1 pt) The dimension of N_L is __________. ______ A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

45. (1 pts) The auxiliary equation for (*) is ______________________. ______A B C D E
A) $r^2 - 6r + 9r = 0$ B) $r^4 - 6r^2 + 9 = 0$ C) $r^2 + 6r + 9 = 0$ D) $r^2 - 6r + 9 = 0$
E) $r^2 + 6r + 9r = 0$ AB) $r^2 - 4r + 4r = 0$ AC) $r^4 - 4r^2 + 4 = 0$ AD) $r^2 + 4r + 4 = 0$
AE) $r^2 - 4r + 4 = 0$ BC) $r^2 + 4r + 4r = 0$ BD) None of the above.

46. (2 pts) Listing repeated roots, the roots of the auxiliary equation

are r =_________________. _____ A B C D E A) 0, 3 B) 0, 0, 3 C) 3, 3 D) −3, −3 E) 3, −3 AB) −3, 3 AC) 0, 2 B) 0, 0, 2 C) 2, 2 D) −2, −2 E) 2, −2 AB) 0, −2, 2 AC) None of the above.

47. (2 pts) A basis for N_L is B = ______________________. ______A B C D E A) $\{1, e^{3x}\}$ B) $\{1, x, e^{3x}\}$ C) $\{e^{3x}, xe^{3x}\}$ D) $\{e^{-3x}, xe^{-3x}\}$ E) $\{e^{3x}, e^{-3x}\}$ AB) $\{1, e^{-3x}, e^{3x}\}$ AC) $\{1, x\}$ AD) $\{1, e^{-3x}\}$ AE) $\{1, xe^{-3x}\}$ BC) $\{1, e^{3x}\}$ BD) $\{1, e^{2x}\}$ BE) $\{1, x, e^{2x}\}$ CD) $\{e^{2x}, xe^{2x}\}$ CE) $\{e^{-2x}, xe^{-2x}\}$ DE) $\{e^{2x}, e^{-2x}\}$ ABC) $\{1, e^{-2x}, e^{2x}\}$ ABD) $\{1, e^{-2x}\}$ ABE) $\{1, xe^{-2x}\}$ BCD) $\{1, e^{2x}\}$ BCE) None of the above.

48. (1 pt) The general solution of (*) is y(x) =____________________________. ____A B C D E
A) $y = c_1 + c_2 e^{3x}$ B) $y = c_1 + c_2 x + c_3 e^{3x}$ C) $y = c_1 e^{3x} + c_2 x e^{3x}$
D) $y = c_1 e^{-3x} + c_2 x e^{-3x}$ E) $y = c_1 e^{3x} + c_2 e^{-3x}$ AB) $y = c_1 + c_2 e^{-3x} + c_3 e^{3x}$
AC) $y = c_1 + c_2 x$ AD) $y = c_1 + c_2 e^{-3x}$ AE) $y = c_1 + c_2 x e^{-3x}$
BC) $y = c_1 + c_2 e^{3x}$ BD) $y = c_1 + c_2 e^{2x}$ BE) $y = c_1 + c_2 x + c_3 e^{2x}$
CD) $y = c_1 e^{2x} + c_2 x e^{2x}$ CE) $y = c_1 e^{-2x} + c_2 x e^{-2x}$ DE) $y = c_1 e^{2x} + c_2 e^{-2x}$
ABC) $y = c_1 + c_2 e^{-2x} + c_3 e^{2x}$ ABD) $y = c_1 + c_2 e^{-2x}$ ABE) $y = c_1 + c_2 x e^{-2x}$
BCD) $y = c_1 + c_2 e^{2x}$ BCE) None of the above.

Points this page = 7. TOTAL POINTS EARNED THIS PAGE __________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer. Let y = y(x) so that y′ = dy/dx

Let $y'' + 6y' + 18y = 0 \quad \forall x \in \mathbf{R}$ be (*), let $L:\mathcal{A}(\mathbf{R},\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = y'' + 6y' + 18y$, and let N_L be the null space of L. Solve (*) below or on the back of the previous page. Be careful! Once you make a mistake, the rest is wrong.

49. (1 pt) The dimension of N_L is ___________. ____ A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

50. (1 pts) The auxiliary equation for (*) is ________________________. _____ A B C D E
A) $r^4 + 6r^2 + 18 = 0$ B) $r^4 - 6r^2 + 18 = 0$ C) $r^2 + 6r + 18 = 0$ D) $r^2 - 6r + 18 = 0$
E) $r^2 + 6r + 18r = 0$ AB) $r^4 + 4r^2 + 8 = 0$ AC) $r^4 - 4r^2 + 8 = 0$ AD) $r^2 + 4r + 8 = 0$
AE) $r^2 - 4r + 8 = 0$ BC) $r^2 + 4r + 8r = 0$ BD) None of the above

51. (2 pts) Listing repeated roots, the roots of the auxiliary equation are

r = _______________. ______A B C D E A) 0,6 B)0,8 C)6, 6 D) 6, −6 E) −6, −6
AB) 8, 8 AC) 8, −8 AD)−8, −8 AE) 3, 9 BC) 3,−9 BD)−3, 9 BE)−3,−9 CD)2,6 CE)2,−6
DE)−2,6 ABC)−2,−6 ABD) 2+2i, 2−2i ABE) −2+2i, −2−2i ACD)3+3i, 3−3i
ACE)−3+3i, −3−3i ADE) None of the above.

52. (2 pts) A basis for N_L is B =___________________________. ____ A B C D E A)$\{1,e^{6x}\}$
B)$\{1,e^{8x}\}$ C)$\{e^{6x},xe^{6x}\}$ D)$\{e^{6x},e^{-6x}\}$ E)$\{e^{-6x},xe^{-6x}\}$ AB) $\{e^{8x},xe^{8x}\}$ AC)$\{e^{8x},e^{-8x}\}$
AD)$\{e^{-8x},xe^{-8x}\}$ AE)$\{e^{3x},e^{9x}\}$ BC)$\{e^{3x},e^{-9x}\}$ BD)$\{e^{-3x},xe^{9x}\}$ BE)$\{e^{-3x},xe^{-9x}\}$ CD)$\{e^{2x},e^{6x}\}$
CE)$\{e^{2x},e^{-6x}\}$ DE)$\{e^{-2x},e^{6x}\}$ ABC)$\{e^{-2x},xe^{-6x}\}$ ABD)$\{e^{2x}\cos(2x),\ e^{2x}\sin(2x)\}$
ABE)$\{e^{-2x}\cos(2x),\ e^{-2x}\sin(2x)\}$ ACD)$\{e^{3x}\cos(3x),\ e^{3x}\sin(3x)\}$
ACE)$\{e^{-3x}\cos(3x),\ e^{-3x}\sin(3x)\}$ ADE)None of the above.

53. (1 pt) The general solution of (*) is y(x) =_______________________________. ___A B C D E
A)$c_1 + c_2 e^{6x}$ B)$c_1 + c_2 e^{8x}$ C)$c_1e^{6x} + c_2 xe^{6x}$ D)$c_1e^{6x} + c_2 e^{-6x}$ E) $c_1e^{-6x} + c_2 xe^{-6x}$
AB)$c_1e^{8x} + c_2 xe^{8x}$ AC) $c_1e^{8x} + c_2 e^{-8x}$ AD)$c_1e^{-8x} + c_2xe^{-8x}$ AE)$c_1e^{3x} + c_2e9x$ BC)$c_1e^{3x}+ c_2e^{-9x}$
BD)$c_1 e^{-3x} + c_2xe^{9x}$ BE)$c_1e^{-3x} + c_2xe^{-9x}$ CD) $c_1e^{2x} + c_2 e^{6x}$ CE) $c_1e^{2x} + c_2e^{-6x}$ DE)$c_1e^{-2x} + c_2e^{6x}$ ABC)$c_1e^{-2x} + c_2xe^{-6x}$ ABD)$c_1e^{2x}\cos(2x) + c_2e^{2x}\sin(2x)\}$ ABE)$c_1e^{-2x}\cos(2x) + c_2 e^{-2x}\sin(2x)$
ACD)$c_1e^{3x}\cos(3x) + c_2e^{3x}\sin(3x)$ ACE)$c_1e^{-3x}\cos(3x) + c_2\ e^{-3x}\sin(3x)$ ADE)None of the above.

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Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer. Let y = y(x) so that y′ = dy/dx.

Let y" + 4y′ + 2y = 0 $\forall$ x $\in$ **R** be (*), let L:$\mathcal{A}$(**R**,**R**)→$\mathcal{A}$(**R**,**R**) be the operator defined by L[y] = y" + 4y′ + 2y, and let N_L be the null space of L. Solve (*) below or on the back of the previous page. Be careful! Once you make a mistake, the rest is wrong.

54. (1 pt) The dimension of N_L is ________. _____A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

55. (1 pts) The auxiliary equation for (*) is ____________________________. ____A B C D E
A) $r^4 + 4r^2 + 2 = 0$ B) $r^4 - 4r^2 + 2 = 0$ C) $r^2 + 4r + 2 = 0$ D) $r^2 - 4r + 2 = 0$
E) $r^2 + 4r + 2r = 0$ AB) $r^4 + 6r^2 + 4 = 0$ AC) $r^4 - 6r^2 + 4 = 0$ AD) $r^2 + 6r + 4 = 0$
AE) $r^2 - 6r + 4 = 0$ E) $r^2 + 6r + 4r = 0$ AB) None of the above.

56. (2 pts). Listing repeated roots, the roots of the auxiliary equation are

r =_____________________. _____ A B C D E A) 2, 2 B) r = 2, −2 C) −2, −2 D) 3, 3
E) 3,−3 AB) −3,−3 AC) $2+\sqrt{2}$, $2+\sqrt{2}$ AD) $2+\sqrt{2}$, $2-\sqrt{2}$ AE) $-2+\sqrt{2}$, $-2+\sqrt{2}$
BC) $-2+\sqrt{2}$, $-2-\sqrt{2}$ BD) $2+\sqrt{2}\,i$, $2-\sqrt{2}\,i$ BE) $-2+\sqrt{2}\,i$, $-2-\sqrt{2}\,i$ CD) $3+\sqrt{5}$, $3+\sqrt{5}$
CE) $3+\sqrt{5}$, $3-\sqrt{5}$ DE) $-3+\sqrt{5}$, $3+\sqrt{5}$ ABC) $-3+\sqrt{5}$, $-3-\sqrt{5}$ ABD) $3+\sqrt{5}\,i$, $3-\sqrt{5}\,i$
ABE) $-3+\sqrt{5}\,i$, $-3-\sqrt{5}\,i$ ACD) None of the above.

57. (2 pts). A basis for N_L is B =______________________. _____A B C D E A) $\{e^{2x}, xe^{2x}\}$
B) $\{e^{2x}, e^{-2x}\}$ C) $\{e^{-2x}, xe^{-2x}\}$ D) $\{e^{3x}, xe^{3x}\}$ E) $\{e^{3x}, e^{-3x}\}$ AB) $\{e^{-3x}, xe^{-3x}\}$
AC) $\{e^{(2+\sqrt{2})x}, xe^{(2+\sqrt{2})x}\}$ AD) $\{e^{(2+\sqrt{2})x}, e^{(2-\sqrt{2})x}\}$ AE) $\{e^{(-2+\sqrt{2})x}, xe^{(-2+\sqrt{2})x}\}$ BC) $\{e^{(-2+\sqrt{2})x}, e^{(-2-\sqrt{2})x}\}$
BD) $\{e^{2x}\cos(\sqrt{2}x), e^{2x}\sin(\sqrt{2}x)\}$ BE) $\{e^{-2x}\cos(\sqrt{2}x), e^{-2x}\sin(\sqrt{2}x)\}$ CD) $\{e^{(3+\sqrt{5})x}, xe^{(3+\sqrt{5})x}\}$
CE) $\{e^{(3+\sqrt{5})x}, e^{(3-\sqrt{5})x}\}$ DE) $\{e^{(-3+\sqrt{5})x}, xe^{(-3+\sqrt{5})x}\}$ ABC) $\{e^{(-3+\sqrt{5})x}, e^{(-3-\sqrt{5})x}\}$ ABD) $\{e^{3x}\cos(\sqrt{5}x), e^{3x}\sin(\sqrt{5}x)\}$
ABE) $\{e^{-3x}\cos(\sqrt{5}x), e^{-3x}\sin(\sqrt{5}x)\}$ ACD) None of the above.

58. (1 pt). The general solution of (*) is y(x) =___________________________.____A B C D E
A) $c_1e^{2x} + c_2xe^{2x}$ B) $c_1 e^{2x} + c_2 e^{-2x}$ C) $c_1 e^{-2x} + c_2 xe^{-2x}$ D) $c_1 e^{3x} + c_2 xe^{3x}$
E) $c_1 e^{3x} + c_2 e^{-3x}$ AB) $c_1 e^{-3x} + c_2 xe^{-3x}$ AC) $c_1 e^{(-2+\sqrt{2})x} + c_2 e^{(-2-\sqrt{2})x}$ AD) $c_1 e^{(2+\sqrt{2})x} + c_2 e^{(2-\sqrt{2})x}$
AE) $c_1 e^{(-2+\sqrt{2})x} + c_2 xe^{(-2+\sqrt{2})x}$ BC) $c_1 e^{(-2+\sqrt{2})x} + c_2 e^{(-2-\sqrt{2})x}$ BD) $c_1 e^{2x}\cos(\sqrt{2}x) + c_2 e^{2x}\sin(\sqrt{2}x)$
BE) $c_1 e^{-2x}\cos(\sqrt{2}x) + c_2 e^{-2x}\sin(\sqrt{2}x)$ CD) $c_1 e^{(3+\sqrt{5})x} + c_2 xe^{(3+\sqrt{5})x}$ CE) $c_1 e^{(3+\sqrt{5})x} + c_2 e^{(3-\sqrt{5})x}$
DE) $c_1 e^{(-3+\sqrt{5})x} + c_2 xe^{(-3+\sqrt{5})x}$ ABC) $c_1 e^{(-3+\sqrt{5})x} + c_2 e^{(-3-\sqrt{5})x}$ ABD) $c_1 e^{3x}\cos(\sqrt{5}x) + c_2 e^{3x}\sin(\sqrt{5}x)$
ABE) $c_1 e^{-3x}\cos(\sqrt{5}x) + c_2 e^{-3x}\sin(\sqrt{5}x)$ ACD) None of the above.

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Last Name, First Name MI, What you wish to be called

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Let $2y'' + 5y' + 3y = 0$ $\forall x \in \mathbf{R}$ be (*), let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the operator defined by $L[y] = 2y'' + 5y' + 3y$, and let N_L be the null space of L. Solve (*) below or on the back of the previous page. Be careful! Once you make a mistake, the rest is wrong.

59. (1 pt) The dimension of N_L is ________________. _____A B C D E A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

60. (1 pts). The auxiliary equation for (*) is ______________________________. ____A B C D E
A) $2r^4 + 5r^2 + 3 = 0$ B) $2r^4 - 5r^2 + 3 = 0$ C) $2r^2 + 5r + 3 = 0$ D) $2r^2 - 5r + 3 = 0$
E) $2r^2 + 5r + 3r = 0$ AB) $2r^4 + 7r^2 + 6 = 0$ AC) $2r^4 - 7r^2 + 6 = 0$ AD) $2r^2 + 7r + 6 = 0$
AE) $2r^2 - 7r + 6 = 0$ BC) $2r^2 + 7r + 6r = 0$ BD) None of the above.

61. (2 pts). Listing repeated roots, the roots of the auxiliary equation are

r = _____________________________. _____A B C D E A) 1, 3/2 B) 1, −3/2
C) −1, 3/2 D) −1, −3/2 E) 2, 3/2 AB) 2, −3/2 AC) −2, 3/2 AD) r = −2, −3/2
AE) 1+ (3/2)i, 1− (3/2)i BC) −1+ (3/2)i, −1− (3/2)i BD) 2+ (3/2)i, 2− (3/2)i
BE) −2+ (3/2)i, −2− (3/2)i CD) 2+i, 2−i CE) None of the above.

62. (2 pts). A basis for N_L is B =_____________________. _____A B C D E A) $\{ e^x, e^{(3/2)x}\}$
B) $\{ e^x, e^{-(3/2)x}\}$ C) $\{ e^{-x}, e^{(3/2)x}\}$ D) $\{ e^{-x}, e^{-(3/2)x}\}$ E) $\{ e^{2x}, e^{(3/2)x}\}$ AB) $\{ e^{2x}, e^{-(3/2)x}\}$
AC) $\{ e^{-2x}, e^{(3/2)x}\}$ AD) $\{ e^{-2x}, e^{-(3/2)x}\}$ AE) $\{e^x \cos((3/2)x), e^x \sin((3/2)x)\}$
BC) $\{e^{-x} \cos((3/2)x), e^{-x} \sin((3/2)x)\}$ BD) $\{e^x \cos((3/2)x), e^x \sin((3/2)x)\}$
BE) $\{e^{-x} \cos((3/2)x), e^{-x} \sin((3/2)x)\}$ CD) None of the above

63. (1 pt) The general solution of (*) is y(x) =_____________________________. ____A B C D E
A) $c_1e^x + c_2 e^{(3/2)x}$ B) $c_1e^x + c_2 e^{-(3/2)x}$ C) $c_1e^{-x} + c_2 e^{(3/2)x}$ D) $c_1e^{-x} + c_2 e^{-(3/2)x}$ E) $c_1e^{2x} + c_2 e^{(3/2)x}$
AB) $c_1e^{2x} + c_2 e^{-(3/2)x}$ AC) $c_1e^{-2x} + c_2 e^{(3/2)x}$ AD) $c_1e^{-2x} + c_2 e^{-(3/2)x}$
AE) $c_1e^x\cos((3/2)x) + c_2 e^x \sin((3/2)x)$ BC) $c_1e^{-x}\cos((3/2)x) + c_2 e^{-x} \sin((3/2)x)$
BD) $y = c_1 e^{2x} \cos((3/2)x) + c_2 e^{2x} \sin((3/2)x)$ BC) $c_1e^{-2x}\cos((3/2)x) + c_2 e^{-2x} \sin((3/2)x)$
BD) None of the above.

Points this page = 7. TOTAL POINTS EARNED THIS PAGE __________