EXAM-2 MATH 261: Elementary Differential Equations MATH 261
FALL 2006 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____________________________________________ (______________________)
Last Name, First Name MI (What you wish to be called)

ID # ______________________________________ EXAM DATE Friday, October 6, 2006

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

______________________________ _______________
SIGNATURE DATE

INSTRUCTIONS

1. Besides this cover page, there are 12 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-12 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. GOOD LUCK!!

| REQUEST FOR REGRADE |
|---|
| Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.) |
| |
| |
| (Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.) |
| Date _______ Signature______________________________ |

Scores

| page | points | score |
|---|---|---|
| 1 | 10 | |
| 2 | 8 | |
| 3 | 5 | |
| 4 | 10 | |
| 5 | 6 | |
| 6 | 8 | |
| 7 | 12 | |
| 8 | 8 | |
| 9 | 8 | |
| 10 | 8 | |
| 11 | 8 | |
| 12 | 11 | |
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| 18 | | |
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| 20 | | |
| 21 | | |
| 22 | | |
| Total | 102 | |

PRINT NAME ________________________(________________) ID No. __________________
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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer from the list below. Using Euler's Method with h = 0.1, you are to find the first two iterates (i.e. $y_1$ and $y_2$) to obtain a numerical approximation of the solution of the Initial Value Problem (IVP) for x = 1.2; that is, if y = φ(x) is the solution to the IVP, you are to find an approximation for φ(1.2). We use the standard notation used in class (attendance is mandatory).

IVP ODE $y' = x+y$ IC $y(1) = 1$

Work the problem using a table as discussed in class.

1. (2 pts.) The general formula for Euler's method is ______________________. ____ A B C D E

2. (1 pt.) $x_0$ = ______. _____ A B C D E

3. (1 pt.) $x_1$ = ______. _____ A B C D E

4. (1 pt.) $x_2$ = ______. _____ A B C D E

5. (1 pt.) $y_0$ = __________. _____ A B C D E

6. (2 pts.) $y_1$ = __________. _____ A B C D E

7. (2 pts.) $y_2$ = __________. ______ A B C D E

Possible answers.
A) $y_{k+1} = y_{k+1} + h\,f(x_k,y_k)$ B) $y_{k+1} = y_k - h\,f(x_k,y_k)$ C) $y_{k+1} = y_k + h\,f(x_{k+1},y_{k+1})$
D) $y_{k+1} = y_k - h\,f(x_{k+1},y_{k+1})$ E) $y_{k+1} = y_k + h\,f(x_k,y_k)$ AB) $y_{k+1} = y_{k-1} + h\,f(x_k,y_k)$
AC) $y_{k+1} = y_k + h\,f'(x_k,y_k)$ AD) 0.0 AE) 0.1 BC) 0.2 BD) 0.3 BE) 1.0 CD) 1.1
CE) 1.2 DE) 1.3 ABC) 1.21 ABD) 1.22 ABE) 1.23 ACD) 1.43 ACE) 2.1 ADE) 2.2
BCD) 2.3 BCE) 2.4 BDE) 2.31 CDE) 2.32 ABCD) 2.33 ABCE) 2.41 ACDE) 2.42
BCDE) 2.43 AC 2.44 AD 2.45 AE. None of the above.

Possible points this page = 10. POINTS EARNED THIS PAGE = _______

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

Solve $\underset{2x2}{A}\,\underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ where $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ i \end{bmatrix}$. Write your answer according to the directions given in class (attendance is mandatory).

8. (4 pts.) If $\left[A \middle| \vec{b}\right]$ is reduced to $\left[U \middle| \vec{c}\right]$ using Gauss elimination, then

$\left[U \middle| \vec{c}\right]$ = ______________________________________________. _____A B C D E

A) $\left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$ B) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ C) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$ D) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ E) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & i \end{array}\right]$

AB) None of the above are possible.

9. ( 4 pts.) The solution of $\underset{2x2}{A}\,\underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ is ____________________________._____A B C D E

A) No Solution B) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ C) $\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $\vec{x} = \begin{bmatrix} 0 \\ 1 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ E). $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$

AB) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AC) $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ AD) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

BC) None of the above correctly describes the solution or set of solutions.

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**True or false**. Solution of Linear Algebraic Equations having possibly complex coefficients. Assume A is an m×n matrix of possibly complex numbers, that $\vec{x}$ is an n×1 column vector of possibly complex unknowns, and that $\vec{b}$ is an m×1 possibly complex-valued column vector. Now consider

$$\underset{m\times n}{A}\ \underset{n\times 1}{\vec{x}} = \underset{m\times 1}{\vec{b}}\,. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

10.(1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) may have no solutions.

11.(1 pt.) A)True or B)False The vector equation (*) always has at least one solution.

12. (1 pt.) A)True or B)False If A is square (n=m) and nonsingular, then (*) always has a unique solution.

13. (1 pt.) A)True or B)False The equation (*) can be considered as a mapping problem from one vector space to another.

14. (1 pt.) A)True or B)False If $A = \begin{bmatrix} 1 & i \\ i & 1 \end{bmatrix}$ then (*) has a unique solution for any $\vec{b} \in \mathbf{R}^m$.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

15. ( 2 pts.) Definition. The set S = $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \subseteq V$ where V is a vector space is linearly

independent if _________________________________________________. _____A B C D E

A) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has an infinite number of solutions.

B) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has only the trivial solution $c_1 = c_2 = \cdots = c_n = 0$.

C) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has a solution other than the trivial solution.

D) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has at least two solutions.

E) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has no solution.

AB) the associated matrix is nonsingular. AC. The associated matrix is singular

Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, determine the appropriate answer that gives an appropriate method to prove that your results are correct (attendance is mandatory). Be careful. If you get them backwards, you miss them both.

16. ( 4 pts.) Let S = $\{[2, 2, 6]^T, [3, 3, 9]^T\}$. Then S is ________________________________

_____________________________________________________________. _____A B C D E

A) linearly independent as $c_1[2, 2, 6]^T + c_2\,[3, 3, 9]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $3[2, 2, 6]^T + (-2)\,[3, 3, 9]^T = [0,0,0]$.

C) linearly dependent as $c_1[2, 2, 6]^T + c_2\,[3, 3, 9]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $3[2, 2, 6]^T + (-2)\,[3, 3, 9]^T = [0,0,0]$.

E) is neither linearly independent or linearly dependent as the definition does not apply.

17. ( 4 pts.) Let S =.$\{[2, 4, 8]^T, [3, 6, 11]^T\}$. Then S is _______________________________

_____________________________________________________________. _____A B C D E

A) linearly independent as $c_1[2, 4, 8]^T + c_2\,[3, 6, 11]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $3[2, 4, 8]^T + (-2)\,[3, 6, 11]^T = [0,0,0]$.

C) linearly dependent as $c_1[2, 4, 8]^T + c_2\,[3, 6, 11]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $3[2, 4, 8]^T + (-2)\,[3, 6, 11]^T = [0,0,0]$.

E) is neither linearly independent or linearly dependent as the definition does not apply.

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ________

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( 6 pts.) True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a vector space V to a vector space W. Now consider the mapping problem

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

18. A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has an infinite number of solutions.

19. A)True or B)False The vector equation (*) may have exactly two distinct solutions.

20. A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} = \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where $c_1, c_2, \ldots, c_n$ are arbitrary constants.

21. A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

22. A)True or B)False The equation (*) can be considered as a mapping problem from one vector space to another.

23. A)True or B)False If the null space of T is $N(T) = \{\vec{0}\}$, $\vec{b} \neq \vec{0}$, and $\vec{b}$ is in the range space of T, then (*) has the unique solution $\vec{x} = \vec{0}$.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(__________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let the operator $T:\mathbf{R}^2\rightarrow\mathbf{R}^2$ be defined by $T(\vec{x}) = \underset{2x2}{A}\ \underset{2x1}{\vec{x}}$ where $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ and $\vec{x} = [x,y]^T$. Solve $T(\vec{x}) = \vec{0}$. The form of the answer need not be unique. To obtain the answer listed, be sure you work the problem following the directions given in class and write your answer accordingly (attendance is mandatory).

24. ( 4pts.) If A is reduced to U using Gauss elimination, then

U =_____________________________________________________________. _____ A B C D E

A) $\begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$ D) $\begin{bmatrix} 3 & 12 \\ 3 & 12 \end{bmatrix}$ E) $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$ AB) None of the above

25. ( 4pts.) The solution of $T(\vec{x}) = \vec{0}$

is ______________________________________________________________. _____A B C D E

A) No Solution B) $\vec{x} = y\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ C) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -4 \\ 1 \end{bmatrix}$ D) $\vec{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$ E) $\vec{x} = y\begin{bmatrix} 4 \\ 1 \end{bmatrix}$

AB) $\vec{x} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$ AC) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 4 \\ 1 \end{bmatrix}$ AD) None of the above.

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Solve the ODE $x^2y'' + 2xy' - 2 = 0$ $I = (0,\infty)$ (i.e. $x>0$) on the back of the previous sheet. It is a second order ODE with y missing. Let $v = y'$ and note that the resulting first order ODE in v is linear. Also let $L: A((0,\infty),\mathbf{R}) \to A((0,\infty),\mathbf{R})$ be the linear operator defined by $L[y[ = y'' + (2/x)y'$. Fill in the blanks and circle the correct answers for the questions below.

26. (1 pt) The standard form (for solving first order linear ODE's) for the resulting first order ODE

in v is_______________________. ______ A B C D E A) $x^2 v' + 2x v - 2 = 0$
B) $v' + (2/x) v - 2/x^2 = 0$ C) $v' + (2/x) v = 2/x^2$ D) $y'' = -(2/x) v + 2/x^2$
E) $v' + (2/x) v = 2/x^2$ AB) $v' + 2v = 2/x$ AC) None of the above

27 (2 pts.) An integrating factor for the resulting first order linear ODE in v

is_____________. _____A B C D E A) $e^{-1}$ B) $e^{-x}$ C) $e^{-2x}$ D) $e^x$ E) e AB) $e^{2x}$
AC) $x^{-1}$ AD) $x^{-2x}$ AE) $\pi^{-x}$ BC) x BD) 2x BE) $x^2$ CD)None of the above

28. (2 pts.) In solving the resulting first linear ODE in v, the following steps occurs

_____________________________________________________. ______ A B C D E
A) $d(vx)/dx = 2$ B) $d(vx^2)/dx = 2x$ C) $d(2vx)/dx = x$ D) $d(vx^2)/dx = x$ E) $d(vx)/dx = x$
AB) $d(2v)/dx = x$ AC) $d(ve^x)/dx = x$ AD) $d(vx^2)/dx = 2$ AE) $d(vx^2)/dx = x^2$
AF) None of the above steps ever appears in any solution of this problem.

29. (2 pt) The general solution of the resulting ODE in v

is__________________________________. ______A B C D E A) $v(x) = (2/x^2) + c_1/x$
B) $v(x) = (2/x) + c_1/x^2$ C) $v(x) = (1/x^2) + c_1/x^2$ D) $v(x) = (2/x^2) + c_1/x^2$
E) $v(x) = (1/x^2) + c_1/x$ AB) $v(x) = (2/x) + c_1/x$ AC) None of the above.

30. (2 pt) The general solution of $x^2y'' + 2xy' - 2 = 0$, $I = (0,\infty)$

is __________________________________. ____A B C D E A) $y(x) = \ell n(x) + c_1/x^2 + c_2$ B) $y(x) = 2\ell n(x) + c_1/x^2 + c_2x$ C) $y(x) = 2\ell n(x) + c_1/x + c_2$ D) $y(x) = \ell n(x) + c_1/x^2 + c_2x$
E) $y(x) = 2\ell n(x) + c_1/x + c_2$ AB) $y(x) = 2\ell n(x) + c_1/x^2 + c_2x$ AC) None of the above.

31. (1 pt) The dimension of the null space for L is __________________. ______ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC)7 AD) None of the above.

32. (2 pts) A basis for the null space of L is _________________________. _____ A B C D E
A) $\{1/x, 1\}$ B) $\{1/x^2, 1\}$ C) $\{1/x, 1/x^2\}$ D) $\{1, e^{-x}\}$ E) $\{1/x, e^{-x}\}$ AB) $\{1/x^2, e^x\}$
AC) $\{1, x\}$ AD) $\{1, x^2\}$ AE) $\{x, x^2\}$ BC) None of the above.

Possible points this page = 12. POINTS EARNED THIS PAGE = _______

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Solve the ODE $y'' - 6y' + 9y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ so that $y' = dy/dx$ on the back of the previous sheet. Also let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the linear operator defined by $L[y] = y'' - 6y' + 9y$. Then circle the correct answers for the questions below Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

33.(1 pt). The order of this ODE is ____________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

34. (1 pt) . The dimension of the null space for L is ___________. ______ A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

35. (1 pts). The auxiliary equation for this ODE is ____________________. ______A B C D E
A) $r^2 - 6r + 9r = 0$ B) $r^4 - 6r^2 + 9 = 0$ C) $r^2 + 6r + 9 = 0$ D) $r^2 - 6r + 9 = 0$ E) $r^2 + 6r + 9r = 0$ AB) None of the above.

36. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation

are _________________. _____ A B C D E A) $r = 0, 3$ B) $r = 0, 0, 3$
C) $r = 3, 3$ D) $r = -3, -3$ E) $r = 3, -3$ AB) $r = 0, -3, 3$ AC) None of the above.

37. (2 pts). A basis for the null space of L is _______________________. ______A B C D E
A) $\{1, e^{3x}\}$ B) $\{1, x, e^{3x}\}$ C) $\{ e^{3x}, xe^{3x} \}$ D) $\{ e^{-3x}, xe^{-3x} \}$ E) $\{ e^{3x}, e^{-3x} \}$
AB) $\{ 1, e^{-3x}, e^{3x} \}$ AC) $\{1, x\}$ AD) $\{1, e^{-3x}\}$ AE) $\{1, xe^{-3x}\}$ BC) $\{1, e^{3x}\}$
BD) None of the above.

38. (1 pt). The general solution of this ODE is:__________________________. ____A B C D E
A) $y(x) = c_1 + c_2 e^{3x}$ B) $y(x) = c_1 + c_2 x + c_3 e^{3x}$ C) $y)x) = c_1 e^{3x} + c_2 x e^{3x}$
D) $y(x) = c_1 e^{-3x} + c_2 x e^{-3x}$ E) $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ AB) $y(x) = c_1 + c_2 e^{-3x} + c_3 e^{3x}$
AC) $y(x) = c_1 + c_2 x$ AD) $y(x) = c_1 + c_2 e^{-3x}$ AE) $y(x) = c_1 + c_2 x e^{-3x}$
BC) $y(x) = c_1 + c_2 e^{3x}$ BD) None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE _________

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Solve the ODE $y'' - 6y' + 18y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ so that $y' = dy/dx$ on the back of the previous sheet. Also let $L:\mathcal{A}(\mathbf{R},\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ be the linear operator defined by $L[y] = y'' - 6y' + 18y = 0$. Then circle the correct answers for the questions below Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

39. (1 pt) The order of the ODE given above is ___________. ______ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

40. (1 pt) The dimension of the null space for L is ___________. _____ A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

41. (1 pts) The auxiliary equation for this ODE is ________________________. ______ A B C D E
A) $r^2 + 6r + 18 = 0$ B) $r^4 - 6r^2 + 18 = 0$ C) $r^2 - 6r + 18 = 0$ D) $r^2 - 6r + 18r = 0$
E) $r^2 + 6r + 18r = 0$ AB) None of the above

42. (2 pts) <u>Listing repeated roots</u>, the roots of the auxiliary equation

are ________________. _______A B C D E A) $r = 3+3i, 3-3i$ B) $r = 3+3i, 3+3i$
C) $r = 3-3i, 3-3i$ D) $r = -3, -3$ E) $r = 3, -3$ AB) $r = 3, 3$ AC) $r = -3, -9$
AD) $r = 3, -9$ AE) $r = 3, 9$ BC) None of the above.

43. (2 pts) A basis for the null space of L is________________________________. _____ A B C D E
A)$\{e^{3x}\cos(3x), e^{3x}\sin(3x)\}$ B)$\{e^{3x}\cos(3x), x\,e^{3x}\cos(3x)\}$ C)$\{e^{3x}\sin(3x), x\,e^{3x}\sin(3x)\}$
D)$\{ e^{-3x}, xe^{-3x} \}$ E)$\{ e^{3x}, e^{-3x} \}$ AB)$\{ e^{3x}, x\,e^{3x} \}$ AC)$\{ e^{-3x}, e^{-9x} \}$ AD) $\{ e^{3x}, e^{-9x} \}$
AE)$\{ e^{3x}, e^{9x} \}$ BC) None of the above.

44. (1 pt) The general solution of this ODE is ____________________________. ____A B C D E
A) $y(x) = c_1 e^{3x}\cos(3x) + c_2 e^{3x}\sin(3x)$ B) $y(x) = c_1 e^{3x}\cos(3x) + c_2\, x\, e^{3x}\cos(3x)$
C) $y(x) = c_1 e^{3x}\sin(3x) + c_2\, x\, e^{3x}\sin(3x)$ D) $y(x) = c_1 e^{-3x} + c_2\, x\, e^{-3x}$
E) $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ AB) $y(x) = c_1 e^{3x} + c_2\, x\, e\, 3x$ AC) $y(x) = c_1 e^{-3x} + c_2 e^{-9x}$
AD) $y(x) = c_1 e^{3x} + c_2 e^{-9x}$ AE) $y(x) = c_1 e^{3x} + c_2 e^{9x}$ BC) None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE ___________

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Solve the ODE $y'' - 4y' + 2y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ so that $y' = dy/dx$ on the back of the previous sheet. Let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be the linear operator defined by $L[y] = y'' - 4y' + 2y$. Then circle the correct answers for the questions below Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

45. (1 pt) The order of the ODE given above is ________________. _____ A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC) 7 AD) None of the above.

46. (1 pt) The dimension of the null space for L is ____________. _____A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

47. (1 pts) The auxiliary equation for this ODE is ______________________. ____A B C D E
A) $r^2 - 4r + 2r = 0$ B) $r^4 - 4r^2 + 2 = 0$ C) $r^2 + 4r + 2 = 0$ D) $r^2 - 4r + 2 = 0$
E) $r^2 + 4r + 2r = 0$ AB) None of the above.

48. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation

are _______________________. _____ A B C D E A) $r = 2+2i,\ 2-2i$ B) $r = 2+2i,\ 2+2i$
C) $r = 2-2i,\ 2-2i$ D) $r = -2+\sqrt{2},\ -2-\sqrt{2}$ E) $r = 2+\sqrt{2},\ 2-\sqrt{2}$ AB) $r = -2+\sqrt{2},\ -2+\sqrt{2}$
AC) $r = -2,\ -2$ AD) $r = 2,\ -2$ AE) $r = 2,\ 2$ BC) None of the above.

49. (2 pts). A basis for the null space of L is __________________________. _____A B C D E
A) $\{\cos(\sqrt{2}\,x), \sin(\sqrt{2}\,x)\}$ B) $\{\cos(\sqrt{2}\,x), x\cos(\sqrt{2}\,x)\}$ C) $\{\sin(\sqrt{2}\,x), x\sin(\sqrt{2}\,x)\}$
D) $\{e^{(-2+\sqrt{2})x}, e^{(-2-\sqrt{2})x}\}$ E) $\{e^{(2+\sqrt{2})x}, e^{(2-\sqrt{2})x}\}$ AB) $\{e^{(-2+\sqrt{2})x}, e^{(-2+\sqrt{2})x}\}$
AC) $\{e^{-2x}, x e^{-2x}\}$ AD) $\{e^{2x}, e^{-2x}\}$ AE) $\{e^{2x}, x e^{2x}\}$ BC) None of the above.

50. (1 pt). The general solution of this ODE is____________________________.____A B C D E
A) $y(x) = c_1\cos(\sqrt{2}\,x) + c_2\sin(\sqrt{2}\,x)$ B) $y(x) = c_1\cos(\sqrt{2}\,x) + c_2\,x\cos(\sqrt{2}\,x)$
C) $y(x) = c_1\sin(\sqrt{2}\,x) + c_2\,x\sin(\sqrt{2}\,x)$ D) $y(x) = c_1 e^{(2+\sqrt{2})x} + c_2 e^{(-2-\sqrt{2})x}$
E) $y(x) = c_1 e^{(2+\sqrt{2})x} + c_2 e^{(2-\sqrt{2})x}$ AB) $y(x) = c_1 e^{2x} + c_2\,x e^{2x}$ AC) $y(x) = c_1 e^{-2x} + c_2 e^{-2x}$
AD) $y(x) = c_1 e^{2x} + c_2 e^{-2x}$ AE) $y(x) = c_1 e^{2x} + c_2 e^{2x}$ BC) None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME _________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Solve the ODE $2y'' - 5y' + 3y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ so that $y' = dy/dx$ on the back of the previous sheet. Also let $L:\mathcal{A}(\mathbf{R},\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ be the linear operator defined by $L[y] = 2y'' - 5y' + 3y$. Then circle the correct answers for the questions below Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

51. (1 pt) The order of this ODE is __________________. ____ A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

52. (1 pt) The dimension of the null space for L is ________________. _____A B C D E
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

53. (1 pts). The auxiliary equation for this ODE is _______________________. ____A B C D E
A) $2r^2 - 5r + 3 = 0$ B) $2r^4 - 5r^2 + 3 = 0$ C) $2r^2 + 5r + 3 = 0$ D) $2r^2 - 5r + 3r = 0$
E) $2r^2 + 5r + 3r = 0$ AB) None of the above.

54. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation

are ____________________________. _____A B C D E A) $r = 1+i, 1-i$
B) $r = 2 + i, 2 - i$ C) $r = 3/2, 1$ D) $r = -3/2, -1$ E) $r = 3/2, -1$ AB) $r = -3/2, 1$
AC) $r = -2, -2$ AD) $r = 2, -2$ AE) $r = 2, 2$ BC) None of the above.

55. (2 pts). A basis for the null space of L is ________________________. _____A B C D E
A) $\{e^x \cos(x), e^x \sin(x)\}$ B) $\{ e^{\frac{3}{2}x}, e^{-x} \}$ C) $\{e^{2x} \cos(x), e^{2x} \cos(x)\}$ D) $\{ e^{\frac{3}{2}x}, e^x \}$
E) $\{ e^{-\frac{3}{2}x}, e^{-x} \}$ AB) $\{ e^{-\frac{3}{2}x}, e^x \}$ AC) $\{ e^{-2x}, x e^{-2x} \}$ AD) $\{ e^{2x}, e^{-2x} \}$
AE) $\{ e^{2x}, x e^{2x} \}$ BC) None of the above.

56. (1 pt) The general solution of the ODE given above is __________________. ____A B C D E
A) $y(x) = c_1 e^x \cos(x) + c_2 e^x \sin(x)$ B) $y(x) = c_1 e^{2x} \cos(x) + c_2 e^{2x} \cos(x)$
C) $y(x) = c_1 e^{\frac{3}{2}x} + c_2 e^x$ D) $y(x) = c_1 e^{-\frac{3}{2}x} + c_2 e^{-x}$ E) $y(x) = c_1 e^{\frac{3}{2}x} + c_2 e^{-x}$
AB) $y(x) = c_1 e^{-\frac{3}{2}x} + c_2 e^x$ AC) $y(x) = c_1 e^{-2x} + c_2 e^{-2x}$ AD) $y(x) = c_1 e^{2x} + c_2 e^{-2x}$
AE) $y(x) = c_1 e^{2x} + c_2 e^{2x}$ BC) None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME _________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Write in your choice to fill in the blanks from those listed below . Then circle the letter or letters to the right or below the blanks that correspond to your answer.

**DEFINITION**. An operator $T:V\to W$ where V and W are vector spaces over the same field **K** is

linear if for all $\alpha,\beta\in\mathbf{K}$ and $\vec{v}_1, \vec{v}_2 \in V$, we have _________________ 57. (2 pts.) A B C D E

**THEOREM.** The operator $L:\mathcal{A}(\mathbf{R},\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ defined by $L[y] = y'' + 2y' + 6y$.is a linear operator.
Proof. By the above definition, to show that the operator L is a linear operator, we must show that if $c_1$ and $c_2$ are constants in **R** and $\varphi_1(x)$ and $\varphi_2(x)$ are functions in $\mathcal{A}(\mathbf{R},\mathbf{R})$, then

__________________________ 58.(2 pts.) A B C D E. Since this is an identity, we can use the standard format for proving identities.

STATEMENT REASON

______________ $= [c_1\varphi_1(x)+c_2\varphi_2(x)]''+2[c_1\varphi_1(x)+c_2\varphi_2(x)]'+6[c_1\varphi_1(x)+c_2\varphi_2(x)]$ __________
59. (2 pts) A B C D E. 60. (2 pts.) A B C D E

= _______________________ 61. (2 pts) A B C D E. Calculus theorems

= _______________________ 62 (1 pt) A B C D E. Definition of L.

Since we have shown the appropriate identity, we have shown that L is a linear operator.
QED

Possible answers to fill in the blanks.
A) $T(\vec{v}_1+\vec{v}_2) = T(\vec{v}_1)+T(\vec{v}_2)$ B) $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$ C) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = T(\vec{v}_1)+T(\vec{v}_2)$
D) $T(\alpha\vec{v}_1) = T(\vec{v}_1)$ E) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = \alpha T(\vec{v}_1)+\beta T(\vec{v}_2)$ AB) $T(\alpha\vec{v}_1+\beta\vec{v}_2) = T(\alpha\vec{v}_1)+T(\beta\vec{v}_2)$ AC) $L[\varphi_1(x)+\varphi_2(x)] = L[\varphi_1(x)]+L[\varphi_2(x)]$ AD) $L[c_1\varphi_1(x)] = c_1 L[\varphi_1(x)]$
AE) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[\varphi_1(x)]+L[\varphi_2(x)]$ BC). $L[c_1\varphi_1(x)] = L[\varphi_1(x)]$
BD) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$ BE) $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[c_1\varphi_1(x)]+L[c_2\varphi_2(x)]$
CD) $L[c_1\varphi_1(x)] = c_1 L[\varphi_1(x)]$ CE) $L[\varphi_1(x)]+L[\varphi_2(x)]$ DE). $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$
ABC) $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$ ABD) $[c_1\varphi_1(x)+c_2\varphi_2(x)]''+[c_1\varphi_1(x) + c_2\varphi_2(x)]'+[c_1\varphi_1(x) + c_2\varphi_2(x)$
ABE) $c_1[\varphi_1''(x)+2\varphi_1'(x)+6\varphi_1(x)]+c_2[\varphi_2''(x)+2\varphi_2'(x)+6\varphi_2(x)]$ ACD) $c_1[\varphi_1''(x)+2\varphi_1'(x)+6\varphi_1(x)]$
ACE) $c_2[\varphi_2''(x)+2\varphi_2'(x)+6\varphi_2(x)]$ ADE) Definition of L BCD).Theorems from Calculus,
BCE) Definition of T BDE) Definition of $\mathcal{A}(\mathbf{R},\mathbf{R})$ CDE) None of the above.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE ________