EXAM-2 MATH 261: Elementary Differential Equations MATH 261
FALL 2005I EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(___________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, October 7, 2005

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

__________________________________ __________________
SIGNATURE DATE

INSTRUCTIONS

1. Besides this cover page, there are 12 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-12 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. GOOD LUCK!!

Scores

page	points	score
1	10	
2	8	
3	5	
4	10	
5	6	
6	8	
7	10	
8	8	
9	8	
10	8	
11	8	
12	11	
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
Total	100	

REQUEST FOR REGRADE

Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature________________________________

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Using Euler's Method with h = 0.1, you are to find the first two iterates (i.e. y_1 and y_2) to obtain a numerical approximation of the solution of the Initial Value Problem (IVP) for x = 0.2; that is, if $y = \varphi(x)$ is the solution to the IVP, you are to find an approximation for $\varphi(0.2)$.

IVP ODE $y' = x+y$ IC $y(0) = 2$

1. (2 pt.) Using the standard notation used in class, the general formula for Euler's method is:
A. $y_{k+1} = y_{k+1} + h\, f(x_k,y_k)$, B. $y_{k+1} = y_k + h\, f(x_k,y_k)$, C. $y_{k+1} = y_k + h\, f(x_{k+1},y_{k+1})$
D. $y_{k+1} = y_k - h\, f(x_{k+1},y_{k+1})$, E. $y_{k+1} = y_k - h\, f(x_k,y_k)$, AB. $y_{k+1} = y_{k-1} + h\, f(x_k,y_k)$
AC. $y_{k+1} = y_k + h\, f'(x_k,y_k)$ AD. None of the above.

2. (1 pt.) Using the standard notation used in class, x_0 = _____.
A. 0 B. 1 C. 2 D. 3 E. 4 AB. 0.1 AC. 0.2 AD. 0.3 AE. None of the above.

3. (1 pt.) Using the standard notation used in class, x_1 = _____.
A. 0 B. 1 C. 2 D. 3 E. 4 AB. 0.1 AC. 0.2 AD. 0.3 AE. None of the above.

4. (1 pt.) Using the standard notation used in class, x_2 = _____.
A. 0 B. 1 C. 2 D. 3 E. 4 AB. 0.1 AC. 0.2 AD. 0.3 AE. None of the above.

5. (1 pt.) Using the standard notation used in class, y_0 = _____.
A. 0 B. 1 C. 2 D. 3 E. 4 AB. 0.1 AC. 0.2 AD. 0.3 AE. None of the above.

6. (2 pt.) Using the standard notation used in class, y_1 = _____.
A. 2.1 B. 2.2 C. 2.3 D. 2.4 E. 3.1 AB. 3.2 AC. 3.3 AD. 3.4 AE.None of the above.

7. (2 pt.) Using the standard notation used in class, y_2 = _____.
A. 2.31 B. 2.32 C. 2.33 D. 2.41 E.2.42 AB.2.43 AC. 2.44 AD. 2.45 AE. None of the above.

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You are to solve $\underset{2x2}{A}\,\underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ where $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ i \end{bmatrix}$. Be sure you write your answer according to the directions given in class (attendance is mandatory) for these kinds of problems.

8. (4 pts.) If $\left[A \middle| \vec{b}\right]$ is reduced to $\left[U \middle| \vec{c}\right]$ using Gauss elimination, then $\left[U \middle| \vec{c}\right]$ =______.

A. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$, B. $\left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$, C. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$, D. $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$, E. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & i \end{array}\right]$,

AB. None of the above are possible.

9. (4 pts.) The solution of $\underset{2x2}{A}\,\underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ is ________.

A. No Solution, B. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, C. $\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix}$, D. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$, E. $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$,

AB. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$, AC. $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, AD. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$,

BC. None of the above correctly describes the solution or set of solutions.

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(5 pts.) **True or false**. Solution of Linear Algebraic Equations having possibly complex coefficients. Assume A is an m×n matrix of possibly complex numbers, that $\vec{x}$ is an n×1 column vector of (possibly complex) unknowns, and that $\vec{b}$ is an m×1 (possibly complex valued) column vector. Now consider

$$\underset{m\times n}{A}\ \underset{n\times 1}{\vec{x}} = \underset{m\times 1}{\vec{b}}\,. \tag{*}$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False. If I can not read your answer, it is wrong.

True or False 10. If $\vec{b} = \vec{0}$, then (*) always has at least one solution.

True or False 11. The vector equation (*) always has exactly one solution.

True or False 12. If A is square (n=m) and singular, then (*) always has a unique solution.

True or False 13. The equation (*) can not be considered as a mapping problem from one vector space to another.

True or False 14. If $A = \begin{bmatrix} 1 & -i \\ i & 1 \end{bmatrix}$ then (*) has a unique solution for any $\vec{b} \in \mathbf{R}^m$.

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15. (2 pts.) Let S = $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \subseteq$V where V is a vector space. Choose the completion of the following definition of what it means for S to be linearly independent.

Definition. The set S = $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \subseteq$V where V is a vector space is linearly independent if

A. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n = \vec{0}$ has an infinite number of solutions.

B. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n = \vec{0}$ has only the trivial solution $c_1 = c_2 = \cdots = c_n = 0$.

C. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n = \vec{0}$ has a solution other than the trivial solution.

D. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n = \vec{0}$ has at least two solutions.

E. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n = \vec{0}$ has no solution.

AB. The associated matrix is nonsingular.

AC. The associated matrix is singular

(8 pts.) Determine Directly Using the Definition (DUD) if the following sets of vectors are linearly independent. As explained in class, circle the appropriate answer that gives an appropriate method to **prove** that your results are correct (Attendance is mandatory). Be careful. If you get them backwards, your grade is **zero**.

16. Let S = $\{[2, 2, 6]^T, [3, 3, 9]^T\}$. Circle the correct answer
A. S is linearly independent as $c_1[2, 2, 6]^T + c_2 [3, 3, 9]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
B. S is linearly independent as $3[2, 2, 6]^T + (-2) [3, 3, 9]^T = [0,0,0]$. C. S is linearly dependent as $c_1[2, 2, 6]^T + c_2 [3, 3, 9]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$. D. S is linearly dependent as $3[2, 2, 6]^T + (-2) [3, 3, 9]^T = [0,0,0]$. E. S is neither linearly independent or linearly dependent as the definition does not apply.

17. Let S =.$\{[2, 4, 8]^T, [3, 6, 11]^T\}$. Circle the correct answer
A. S is linearly independent as $c_1[2, 4, 8]^T + c_2 [3, 6, 11]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$.
B. S is linearly independent as $3[2, 4, 8]^T + (-2) [3, 6, 11]^T = [0,0,0]$.
C. S is linearly dependent as $c_1[2, 4, 8]^T + c_2 [3, 6, 11]^T = [0,0,0]$ implies $c_1 = 0$ and $c_2 = 0$. D. S is linearly dependent as $3[2, 4, 8]^T + (-2) [3, 6, 11]^T = [0,0,0]$.
E. S is neither linearly independent or linearly dependent as the definition does not apply.

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(6 pts.) True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a vector space V to a vector space W. Now consider the mapping problem

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false.
If true, circle True. If false, circle False.

True or False 18. If $\vec{b} = \vec{0}$, then (*) always has an infinite number of solutions.

True or False 19. The vector equation (*) may have exactly two distinct solutions.

True or False 20. If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} = \vec{0}$, then the general solution of (*) is not given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where c_1, c_2, ..., c_n are arbitrary constants.

True or False 21. Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

True or False 22. The equation (*) can be considered as a mapping problem from one vector space to another.

True or False 23. If the null space of T is not $N(T) = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has the unique solution $\vec{x} = \vec{0}$.

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(8pts.) Let the operator $T:\mathbf{R}^2 \to \mathbf{R}^2$ be defined by $T(\vec{x}) = \underset{2\times2}{A}\ \underset{2\times1}{\vec{x}}$ where $A = \begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$ and $\vec{x}$ = $[x,y]^T$. Solve $T(\vec{x}) = \vec{0}$. **SHOW YOUR WORK**. No credit will be given if you do not explain how you obtained your solution. Be sure you write your answer according to the directions given in class for these kinds of problems. (Attendance is mandatory.)

24. If A is reduced to U using Gauss elimination, then U =______.

A. $\begin{bmatrix} 1 & 4 \\ 3 & 12 \end{bmatrix}$, B. $\begin{bmatrix} 1 & 4 \\ 0 & 0 \end{bmatrix}$, C. $\begin{bmatrix} 1 & 4 \\ 1 & 4 \end{bmatrix}$, D. $\begin{bmatrix} 3 & 12 \\ 3 & 12 \end{bmatrix}$, E. $\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$, AB. None of the above.

25. The solution of $T(\vec{x}) = \vec{0}$ is ________.

A. No Solution, B. $\vec{x} = y\begin{bmatrix} -4 \\ 1 \end{bmatrix}$, C. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -4 \\ 1 \end{bmatrix}$, D. $\vec{x} = \begin{bmatrix} 4 \\ 1 \end{bmatrix}$, E. $\vec{x} = y\begin{bmatrix} 4 \\ 1 \end{bmatrix}$,

AB. $\vec{x} = \begin{bmatrix} -4 \\ 1 \end{bmatrix}$, AC. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} 4 \\ 1 \end{bmatrix}$, AD. None of the above.

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _______

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To solve $x^2y'' + 2xy' - 2 = 0$ $I = (0,\infty)$ (i.e. $x>0$) as a first order ODE, we note that y is missing, let $v = y'$ and note that the resulting first order ODE in v is linear.

26. (1 pt) The standard form (for solving 1st order linear ODE's) for the resulting 1st order ODE

in v is_____. A. $x^2 v' + 2x v - 2 = 0$, B. $v' + (2/x) v - 2/x^2 = 0$, C. $v' + (2/x) v = 2/x^2$
D. $y'' = -(2/x) v + 2/x^2$ E. $v' + (2/x) v = 2/x^2$ AB. $v' + 2v = 2/x$
AC. None of the above.

27 (1 pts.) An integrating factor needed to solve the resulting first order linear ODE in v

is:_____. A. e^{-1} B. e^{-x} C. e^{-2x} D. e^x E. e AB. e^{2x}
AC. x^{-1} AD. x^{-2x} AE. π^{-x} BC. x BD. 2x BE. x^2
CD. None of the above is an integrating factor.

28. (1 pts.) In solving the resulting first linear ODE in v, which of the following steps occurs?
A. $d(vx)/dx = 2$, B. $d(vx^2)/dx = 2x$, C. $d(2vx)/dx = x$, D. $d(vx^2)/dx = x$, E. $d(vx)/dx = x$,
AB. $d(2v)/dx = x$, AC. $d(ve^x)/dx = x$, AD. $d(vx^2)/dx = 2$, AE. $d(vx^2)/dx = x^2$, ,
AF. None of the above steps ever appears in any solution of this problem.

29. (2 pt) The general solution of the resulting ODE in v is: A. $v(x) = (2/x^2) + c_1/x$
B. $v(x) = (2/x) + c_1/x^2$ C. $v(x) = (1/x^2) + c_1/x^2$ D. $v(x) = (2/x^2) + c_1/x^2$
E. $v(x) = (1/x^2) + c_1/x$ AB. $v(x) = (2/x) + c_1/x$ AC. None of the above.

30. (2 pt) The general solution of $x^2y'' + 2xy' - 2 = 0$ $I = (0,\infty)$ is: A. $y(x) = \ell n(x) + c_1/x^2 + c_2$
B. $y(x) = 2\ell n(x) + c_1/x^2 + c_2x$ C. $y(x) = 2\ell n(x) + c_1/x + c_2$ D. $y(x) = \ell n(x) + c_1/x^2 + c_2x$
E. $y(x) = 2\ell n(x) + c_1/x + c_2$ AB. $y(x) = 2\ell n(x) + c_1/x^2 + c_2x$ AC. None of the above.

31. (1 pt) The dimension of the null space for the linear operator $L[y] = y'' + (2/x)y'$ is:
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC)7 AD) None of the above.

32. (2 pts) A basis for the null space of the linear operator $L: A((0,\infty),\mathbf{R}) \to A((0,\infty),\mathbf{R})$ defined by $L[y] = y'' + (2/x)y'$ is : A) $\{1/x, 1\}$, B) $\{1/x^2, 1\}$, C) $\{1/x, 1/x^2\}$ D) $\{1, e^{-x}\}$
E) $\{1/x, e^{-x}\}$, AB) $\{1/x^2, e^x\}$, AC) $\{1, x\}$, AD) $\{1, x^2\}$ AE) $\{x, x^2\}$
BC) None of the above.

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(8 pts.) You are to solve $y'' - 6y' + 9y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ so that $y' = \dfrac{dy}{dx}$ in steps by **circling the correct answers** to the following questions. Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

33.(1 pt). The order of the ODE is A)1 B)2 C)3 D)4 E)5 AB) 6 AC) 7
AD) None of the above.

34. (1 pt) . The dimension of the null space for the linear operator $L[y] = y'' - 6y' + 9y$ is:
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

35. (1 pts). The auxiliary equation for this equation is: A) $r^2 - 6r + 9r = 0$, B) $r^4 - 6r^2 + 9 = 0$,
C) $r^2 + 6r + 9 = 0$, D) $r^2 - 6r + 9 = 0$, E) $r^2 + 6r + 9r = 0$, AB) None of the above.

36. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation are: A) $r = 0, 3$
B) $r = 0, 0, 3$ C) $r = 3, 3$ D) $r = -3, -3$ E) $r = 3, -3$ AB) $r = 0, -3, 3$
AC) None of the above.

37. (2 pts). A basis for the null space of the linear operator $L: A(\mathbf{R},\mathbf{R}) \to A(\mathbf{R},\mathbf{R})$ defined by
$L[y] = y'' - 6y' + 9y$ is : A) $\{1, e^{3x}\}$ B) $\{1, x, e^{3x}\}$ C) $\{e^{3x}, xe^{3x}\}$
D) $\{e^{-3x}, xe^{-3x}\}$ E) $\{e^{3x}, e^{-3x}\}$ AB) $\{1, e^{-3x}, e^{3x}\}$ AC) $\{1, x\}$ AD) $\{1, e^{-3x}\}$
AE) $\{1, xe^{-3x}\}$ BC) $\{1, e^{3x}\}$ BD) None of the above.

38. (1 pt). The general solution of $y'' - 6y' + 9y = 0$ is: A) $y(x) = c_1 + c_2 e^{3x}$
B) $y(x) = c_1 + c_2 x + c_3 e^{3x}$ C) $y)x) = c_1 e^{3x} + c_2 x e^{3x}$ D) $y(x) = c_1 e^{-3x} + c_2 x e^{-3x}$
E) $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ AB) $y(x) = c_1 + c_2 e^{-3x} + c_3 e^{3x}$ AC) $y(x) = c_1 + c_2 x$
AD) $y(x) = c_1 + c_2 e^{-3x}$ AE) $y(x) = c_1 + c_2 x e^{-3x}$ BC) $y(x) = c_1 + c_2 e^{3x}$
BD) None of the above.

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(8 pts.) You are to solve $y'' - 6y' + 18y = 0 \quad \forall\, x \in \mathbf{R}$ where $y = y(x)$ (so that $y' = \dfrac{dy}{dx}$) in steps by **circling the correct answers** to the following questions. Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

39. (1 pt) I. The order of the ODE is A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

40. (1 pt) II. The dimension of the null space for the linear operator $L[y] = y'' - 6y' + 18y$ is: A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

41. (1 pts)III. The auxiliary equation for this equation is: A) $r^2 + 6r + 18 = 0$, B) $r^4 - 6r^2 + 18 = 0$, C) $r^2 - 6r + 18 = 0$, D) $r^2 - 6r + 18r = 0$, E) $r^2 + 6r + 18r = 0$, AB) None of the above.

42. (2 pts)IV. <u>Listing repeated roots</u>, the roots of the auxiliary equation are: A) $r = 3+3i,\ 3-3i$ B) $r = 3+3i,\ 3+3i$ C) $r = 3-3i,\ 3-3i$ D) $r = -3,\ -3$ E) $r = 3,\ -3$ AB) $r = 3,\ 3$ AC) $r = -3,\ -9$ AD) $r = 3,\ -9$ AE) $r = 3,\ 9$ BC) None of the above.

43. (2 pts)V. A basis for the null space of the linear operator $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ defined by $L[y] = y'' - 6y' + 18y$ is : A) $\{e^{3x}\cos(3x),\ e^{3x}\sin(3x)\}$ B) $\{e^{3x}\cos(3x),\ x\,e^{3x}\cos(3x)\}$ C) $\{e^{3x}\sin(3x),\ x\,e^{3x}\sin(3x)\}$ D) $\{e^{-3x},\ xe^{-3x}\}$ E) $\{e^{3x},\ e^{-3x}\}$ AB) $\{e^{3x},\ x\,e^{3x}\}$ AC) $\{e^{-3x},\ e^{-9x}\}$ AD) $\{e^{3x},\ e^{-9x}\}$ AE) $\{e^{3x},\ e^{9x}\}$ BC) None of the above.

44. (1 pt)VI. The general solution of $y'' - 6y' + 18y = 0$ is:
A) $y(x) = c_1 e^{3x}\cos(3x) + c_2 e^{3x}\sin(3x)$
B) $y(x) = c_1 e^{3x}\cos(3x) + c_2\, x\, e^{3x}\cos(3x)$ C) $y(x) = c_1 e^{3x}\sin(3x) + c_2\, x\, e^{3x}\sin(3x)$
D) $y(x) = c_1 e^{-3x} + c_2\, x\, e^{-3x}$ E) $y(x) = c_1 e^{3x} + c_2 e^{-3x}$ AB) $y(x) = c_1 e^{3x} + c_2\, x\, e^{3x}$
AC) $y(x) = c_1 e^{-3x} + c_2 e^{-9x}$ AD) $y(x) = c_1 e^{3x} + c_2 e^{-9x}$ AE) $y(x) = c_1 e^{3x} + c_2 e^{9x}$
BC) None of the above.

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(8 pts.) You are to solve $y'' - 4y' + 2y = 0 \quad \forall x \in \mathbf{R}$ where $y = y(x)$ (so that $y' = \frac{dy}{dx}$) in steps by **circling the correct answers** to the following questions. Be especially careful at the beginning since once you make a mistake, the rest is wrong.

45. (1 pt) . The order of the ODE is A)1 B)2 C)3 D)4 E)5 AB)6 AC) 7
AD) None of the above.

46. (1 pt) . The dimension of the null space for the linear operator $L[y] = y'' - 4y' + 2y$ is:
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

47. (1 pts). The auxiliary equation for this equation is: A) $r^2 - 4r + 2r = 0$, B) $r^4 - 4r^2 + 2 = 0$,
C) $r^2 + 4r + 2 = 0$, D) $r^2 - 4r + 2 = 0$, E) $r^2 + 4r + 2r = 0$,
AB) None of the above.

48. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation are: A) $r = 2+2i,\ 2-2i$
B) $r = 2+2i,\ 2+2i$ C) $r = 2-2i,\ 2-2i$ D) $r = -2+\sqrt{2},\ -2-\sqrt{2}$ E) $r = 2+\sqrt{2},\ 2-\sqrt{2}$
AB) $r = -2+\sqrt{2},\ -2+\sqrt{2}$ AC) $r = -2,\ -2$ AD) $r = 2,\ -2$ AE) $r = 2,\ 2$
BC) None of the above.

49. (2 pts). A basis for the null space of the linear operator $L:A(\mathbf{R},\mathbf{R}) \to A(\mathbf{R},\mathbf{R})$ defined by
$L[y] = y'' - 4y' + 2y$ is : A) $\{\cos(\sqrt{2}\,x), \sin(\sqrt{2}\,x)\}$ B) $\{\cos(\sqrt{2}\,x), x\cos(\sqrt{2}\,x)\}$
C) $\{\sin(\sqrt{2}\,x), x\sin(\sqrt{2}\,x)\}$ D) $\{e^{(-2+\sqrt{2})x}, e^{(-2-\sqrt{2})x}\}$ E) $\{e^{(2+\sqrt{2})x}, e^{(2-\sqrt{2})x}\}$
AB) $\{e^{(-2+\sqrt{2})x}, e^{(-2+\sqrt{2})x}\}$ AC) $\{e^{-2x}, x e^{-2x}\}$ AD) $\{e^{2x}, e^{-2x}\}$
AE) $\{e^{2x}, x e^{2x}\}$ BC) None of the above.

50. (1 pt). The general solution of $y'' - 4y' + 2y = 0$ is:
A) $y(x) = c_1 \cos(\sqrt{2}\,x) + c_2 \sin(\sqrt{2}\,x)$
B) $y(x) = c_1 \cos(\sqrt{2}\,x) + c_2\, x \cos(\sqrt{2}\,x)$ C) $y(x) = c_1 \sin(\sqrt{2}\,x) + c_2\, x \sin(\sqrt{2}\,x)$
D) $y(x) = c_1 e^{(2+\sqrt{2})x} + c_2 e^{(-2-\sqrt{2})x}$ E) $y(x) = c_1 e^{(2+\sqrt{2})x} + c_2 e^{(2-\sqrt{2})x}$
AB) $y(x) = c_1 e^{2x} + c_2\, x e^{2x}$ AC) $y(x) = c_1 e^{-2x} + c_2 e^{-2x}$ AD) $y(x) = c_1 e^{2x} + c_2 e^{-2x}$
AE) $y(x) = c_1 e^{2x} + c_2 e^{2x}$ BC) None of the above.

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(8 pts.) You are to solve $2y'' - 5y' + 3y = 0 \quad \forall\, x \in \mathbf{R}$ where $y = y(x)$ (so that $y' = \dfrac{dy}{dx}$) in steps by **circling the correct answers** to the following questions. Be careful, especially at the beginning since once you make a mistake, the rest is wrong.

51. (1 pt) I. The order of the ODE is A)1 B)2 C)3 D)4 E)5 AB)6 AC)7
AD) None of the above.

52. (1 pt) II. The dimension of the null space for the linear operator $L[y] = 2y'' - 5y' + 3y$ is:
A)1 B)2 C)3 D)4 E)5 AB)6 AC)7 AD) None of the above.

53. (1 pts)III. The auxiliary equation for this equation is: A) $2r^2 - 5r + 3 = 0$,
B) $2r^4 - 5r^2 + 3 = 0$, C) $2r^2 + 5r + 3 = 0$, D) $2r^2 - 5r + 3r = 0$, E) $2r^2 + 5r + 3r = 0$,
AB) None of the above.

54. (2 pts)IV. <u>Listing repeated roots</u>, the roots of the auxiliary equation are: A) $r = 1+i,\ 1-i$
B) $r = 2 + i,\ 2 - i$ C) $r = 3/2,\ 1$ D) $r = -3/2,\ -1$ E) $r = 3/2,\ -1$
AB) $r = -3/2,\ 1$ AC) $r = -2,\ -2$ AD) $r = 2,\ -2$ AE) $r = 2,\ 2$
BC) None of the above.

55. (2 pts)V. A basis for the null space of the linear operator $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ defined by
$L[y] = 2y'' - 5y' + 3y$ is : A) $\{e^x \cos(x),\ e^x \sin(x)\}$ B) $\{e^{2x} \cos(x),\ e^{2x} \cos(x)\}$
C) $\{ e^{\frac{3}{2}x},\ e^x \}$ D) $\{ e^{-\frac{3}{2}x},\ e^{-x} \}$ E) $\{ e^{\frac{3}{2}x},\ e^{-x} \}$ AB) $\{ e^{-\frac{3}{2}x},\ e^x \}$
AC) $\{ e^{-2x},\ x e^{-2x} \}$ AD) $\{ e^{2x},\ e^{-2x} \}$ AE) $\{ e^{2x},\ x e^{2x} \}$ BC None of the above.

56. (1 pt)VI. The general solution of $2y'' - 5y' + 3y = 0$ is: A) $y(x) = c_1 e^x \cos(x) + c_2 e^x \sin(x)$
B) $y(x) = c_1 e^{2x} \cos(x) + c_2 e^{2x} \cos(x)$ C) $y(x) = c_1 e^{\frac{3}{2}x} + c_2 e^x$ D) $y(x) = c_1 e^{-\frac{3}{2}x} + c_2 e^{-x}$
E) $y(x) = c_1 e^{\frac{3}{2}x} + c_2 e^{-x}$ AB) $y(x) = c_1 e^{-\frac{3}{2}x} + c_2 e^x$ AC) $y(x) = c_1 e^{-2x} + c_2 e^{-2x}$
AD) $y(x) = c_1 e^{2x} + c_2 e^{-2x}$ AE) $y(x) = c_1 e^{2x} + c_2 e^{2x}$ BC) None of the above.

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Write in the correct letter(s) for the posible answers given below.

DEFINITION. An operator T:V→W where V and W are vector spaces over the same field **K** is

linear if for all α,β∈**K** and $\vec{v}_1, \vec{v}_2 \in V$, we have 57. (2 pts.) ____________

THEOREM. The operator L: $\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ defined by L[y] = y'' + 2y′ + 6y.is a linear operator.
Proof. By the above definition, to show that the operator L is a linear operator, we must show that if

c_1 and c_2 are constants in **R** and $\varphi_1(x)$ and $\varphi_2(x)$ are functions in $\mathcal{A}(\mathbf{R},\mathbf{R})$, then 58.(2 pts.) ________
Since this is an identity, we can use the standard format for proving identities.

STATEMENT	REASON
59. (2 pts) _______ $= [c_1\varphi_1(x)+c_2\varphi_2(x)]''+2[c_1\varphi_1(x)+c_2\varphi_2(x)]'+6[c_1\varphi_1(x)+c_2\varphi_2(x)]$	60. (2 pts.) _______
= 61. (2 pts) _________	Calculus theorems
= 62 (1 pt) ___________	Definition of L.

Since we have shown the appropriate identity, we have that L is a linaer operator.
QED

Possible answers.
A. $T(\vec{v}_1 + \vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$, B. $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$, C. $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = T(\vec{v}_1) + T(\vec{v}_2)$,
D. $T(\alpha\vec{v}_1) = T(\vec{v}_1)$, E. $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = \alpha T(\vec{v}_1) + \beta T(\vec{v}_2)$, AB. $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = T(\alpha\vec{v}_1) + T(\beta\vec{v}_2)$, AC. $L[\varphi_1(x)+\varphi_2(x)] = L[\varphi_1(x)] + L[\varphi_2(x)]$, AD. $L[c_1\varphi_1(x)] = c_1 L[\varphi_1(x)]$,
AE. $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[\varphi_1(x)]+L[\varphi_2(x)]$, BC . $L[c_1\varphi_1(x)] = L[\varphi_1(x)]$,
BD. $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$, BE. $L[c_1\varphi_1(x)+c_2\varphi_2(x)] = L[c_1\varphi_1(x)]+L[c_2\varphi_2(x)]$,
CD. $L[c_1\varphi_1(x)] = c_1 L[\varphi_1(x)]$, CE $L[\varphi_1(x)]+L[\varphi_2(x)]$, DE. $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$,
ABC. $c_1 L[\varphi_1(x)]+c_2L[\varphi_2(x)]$, ABD $[c_1\varphi_1(x)+c_2\varphi_2(x)]''+[c_1\varphi_1(x) + c_2\varphi_2(x)]'+[c_1\varphi_1(x) + c_2\varphi_2(x)$
ABE. $c_1[\varphi_1''(x)+2\varphi_1'(x)+6\varphi_1(x)]+c_2[\varphi_2''(x)+2\varphi_2'(x)+6\varphi_2(x)]$ ACD. $c_1[\varphi_1''(x)+2\varphi_1'(x)+6\varphi_1(x)]$
ACE. $c_2[\varphi_2''(x)+2\varphi_2'(x)+6\varphi_2(x)]$ ADE. Definition of L, BCD .Theorems from Calculus,
BCE. Definition of T, BDE. Definition of $\mathcal{A}(\mathbf{R},\mathbf{R})$, CDE. None of the above.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE ________