

PRINT NAME \_\_\_\_\_ ( \_\_\_\_\_ )  
 Last Name, First Name MI (What you wish to be called)

ID # \_\_\_\_\_ EXAM DATE Friday, Dec.17, 2010, 8am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

\_\_\_\_\_  
 SIGNATURE

\_\_\_\_\_  
 DATE

INSTRUCTIONS: Besides this cover page, there are 34 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES.** If a page is missing, you will receive a grade of zero for that page. Page 35 contains Laplace transforms you need not memorize. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS!** Ask for scratch paper if you need it. You may remove the staple. Print your name on all sheets. Pages 1-34 are Fill-in-the-Blank/Multiple Choice or True/False. Expect no partial credit on these pages. For each Fill-in-the-Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Your thoughts should be expressed in your best mathematics on this paper. Proofread as time allows. **GOOD LUCK!!**

Scores  
 page points score

25	9	
26	6	
27	4	
28	2	
29	3	
30	6	
31	4	
32	4	
33	6	
34	3	
35	----	
ST4	47	
ST1	66	
ST2	56	
ST3	44	
ST4	47	
Tot.	213	

Scores  
 page points score

1	20	
2	7	
3	6	
4	8	
5	6	
6	5	
7	5	
8	9	
ST1	66	

Scores  
 page points score

9	3	
10	10	
11	7	
12	2	
13	12	
14	4	
15	9	
16	9	
ST2	56	

Scores  
 page points score

17	6	
18	5	
19	4	
20	5	
21	11	
22	6	
23	3	
24	4	
ST3	44	

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. You are to classify the first order ordinary differential equations given below. The classification relates to the method of solution. Recall from class (attendance is mandatory) the possible methods listed below. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is wrong. DO NOT SOLVE. Also recall the following:

- In this context, exact means exact as given (in either of the forms discussed in class).
- Bernoulli is not a correct method of solution if the original equation is linear.
- Homogeneous (use the substitution  $v = y/x$ ) is not a correct method of solution if it converts a separable equation into another separable equation.

1.(4 pts.)  $(y^2 + xy)dx + x dy = 0$  \_\_\_\_\_ A B C D E

2. (4 pts.)  $(e^x + 2xy + x)dx + (x^2 + 2y)dy = 0$  \_\_\_\_\_ A B C D E

3. (4 pts.)  $(xy + \cos(x))dx + (1 + x^2) dy = 0$  \_\_\_\_\_ A B C D E

4. (4 pts.)  $(y^2 + x^2)dx + x^2 dy = 0$  \_\_\_\_\_ A B C D E

5. (4 pts.)  $xye^{x+y} dx + dy = 0$  \_\_\_\_\_ A B C D E

Possible answers this page.

- A) First order linear (y as a function of x). B) First order linear (x as a function of y).  
 C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).  
 E) Bernoulli, but not linear (y as a function of x).  
 AB) Bernoulli, but not linear (x as a function of y)  
 AC) Homogeneous, but not separable. AD) None of the above techniques works.

Total points this page = 20. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

Consider the first order linear ODE  $y' = y + 2x$  (\*). To solve (\*), you may need to change it to a standard form.

6. ( 1 pts.) The correct standard form for (\*) is \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $y' + y = x$  B)  $y' + y = -x$  C)  $y' - y = x$  D)  $y' - y = -x$  E)  $y' + y = 2x$  AB)  $y' + y = -2x$   
 AC)  $y' - y = 2x$  AD)  $y' - y = -2x$  AE)  $y' + y = 3x$  BC)  $y' + y = -3x$  BD)  $y' - y = 3x$   
 BE)  $y' - y = -3x$  CD)  $y' + y = 4x$  CE)  $y' + y = -4x$  DE)  $y' - y = 4x$  ABC)  $y' - y = -4x$   
 ABD) None of the above

7. ( 2 pts.) An integrating factor for (\*) is  $\mu =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $x$  B)  $-x$  C)  $x^2$  D)  $-x^2$  E)  $e^x$  AB)  $e^{-x}$  AC)  $e^{2x}$  AD)  $e^{-2x}$  AE)  $e^{3x}$  AD)  $e^{-3x}$   
 AE)  $e^{4x}$  BC)  $e^{-4x}$  BD)  $e^{x^2}$  BE)  $e^{-x^2}$  CD) None of the above

8. ( 3 pts.) In solving (\*) as we did in class (attendance is mandatory), the following step occurs:

- \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $\frac{d(ye^x)}{dx} = xe^x$  B)  $\frac{d(ye^{-x})}{dx} = xe^{-x}$  C)  $\frac{d(ye^x)}{dx} = 2xe^x$  E)  $\frac{d(ye^{-x})}{dx} = 2xe^{-x}$  AB)  $\frac{d(ye^x)}{dx} = 3xe^x$  AC)  $\frac{d(ye^{-x})}{dx} = 3xe^{-x}$   
 AD)  $\frac{d(ye^x)}{dx} = 4xe^x$  AE)  $\frac{d(ye^{-x})}{dx} = 4xe^{-x}$  BC)  $\frac{d(ye^{-x})}{dx} = xe^{-x}$  BD)  $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$  BE)  $\frac{d(ye^{3x})}{dx} = -xe^{3x}$   
 CD)  $\frac{d(ye^{-4x})}{dx} = xe^{-4x}$  CE)  $\frac{d(ye^{-2x})}{dx} = -xe^{-2x}$  DE)  $\frac{d(ye^{-2x})}{dx} = -2xe^{-2x}$  ABC)  $\frac{d(ye^{-2x})}{dx} = 2xe^{-2x}$  ABD)  $\frac{d(ye^{-2x})}{dx} = -3xe^{-2x}$

ABE) None of the above steps ever appears in any solution of this problem.

9. (1pt.) Let (\*\*) be the initial value problem consisting of (\*) and the initial condition  $y(0) = 0$ .

The number of solutions to (\*\*) is \_\_\_\_\_. \_\_\_\_\_ A B C D E A) 0 B) 1  
 C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

An ODE may be considered to be a “vector” equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function’s domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE, call it (\*), of the form  $L[y] = g(x)$  where  $L$  is of the form  $L[y] = y' + p(x)y$ , an integrating factor and the product rule were used to reach the following step:  $\frac{d(ye^x)}{dx} = 2xe^x$ , call it (\*\*). If a problem has an infinite number of solutions, the form of the solution is not unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

10. (2 pts.) The theorem from calculus that allows you to integrate the Left Hand Side of (\*\*)

is \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A) Intermediate Value Theorem    B) Mean Value Theorem    C) Rolle's Theorem    D) Chain Rule  
 E) Product Rule    AB) Fundamental Theorem of Calculus  
 AC) Integration by Parts    AD) Partial Fractions    AE) None off the above

11. (4 pts.) The solution (or family of solutions) to the ODE (\*) may be written

as \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $y = x + 1 + c e^x$     B)  $y = x + 1 + c e^{-x}$     C)  $y = 2x + 2 + c e^x$     D)  $y = 2x + 2 + c e^{-x}$   
 E)  $y = 3x + 3 + c e^x$     AB)  $y = 3x + 3 + c e^{-x}$     AC)  $y = 4 + 4 + c e^x$     AD)  $y = 4x + 4 + c e^{-x}$   
 AE)  $y = x - 1 + c e^x$     BC)  $y = 2x - 2 + c e^x$     BD)  $y = 3x - 3 + c e^x$     BE)  $y = 4x - 4 + c e^x$   
 CD)  $y = x - 1 + c e^{-x}$     CE)  $y = 2x - 2 + c e^{-x}$     DE)  $y = 3x - 3 + c e^{-x}$     ABC)  $y = 4x - 4 + c e^{-x}$   
 ABCDE) None of the above solutions or families of solutions is correct.

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Let  $L$ ,  $p(x)$ ,  $g(x)$ ,  $(*)$ , and  $(**)$  be as on the previous page.

12. (1 pts.) The solution set for the ODE  $(*)$  on the previous page may be written as

- $S =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E
- A)  $\{y = x + 1 + c e^x : c \in \mathbf{R}\}$  B)  $\{y = x + 1 + c e^{-x} : c \in \mathbf{R}\}$  C)  $\{y = 2x + 2 + c e^x : c \in \mathbf{R}\}$   
 D)  $\{y = 2x + 1 + c e^{-x} : c \in \mathbf{R}\}$  E)  $\{y = 3x + 3 + c e^x : c \in \mathbf{R}\}$  AB)  $\{y = 3x + 3 + c e^{-x} : c \in \mathbf{R}\}$   
 AC)  $\{y = 4x + 4 + c e^x : c \in \mathbf{R}\}$  AD)  $\{y = 4x + 4 + c e^{-x} : c \in \mathbf{R}\}$  AE)  $\{y = x - 1 + c e^x : c \in \mathbf{R}\}$   
 BC)  $\{y = 2x - 2 + c e^x : c \in \mathbf{R}\}$  BD)  $\{y = 3x - 3 + c e^x : c \in \mathbf{R}\}$  BE)  $\{y = 4x - 4 + c e^x : c \in \mathbf{R}\}$   
 CD)  $\{y = x - 1 + c e^{-x} : c \in \mathbf{R}\}$  CE)  $\{y = 2x - 2 + c e^{-x} : c \in \mathbf{R}\}$  DE)  $\{y = 3x - 3 + c e^{-x} : c \in \mathbf{R}\}$  ABC)  $\{y = 4x - 4 + c e^{-x} : c \in \mathbf{R}\}$  DE) None of the above correctly describe the solution set.

13. (1 pt.) A basis for the nullspace of  $L$  is  $B =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $\{1\}$  B)  $\{x\}$  C)  $\{1, x\}$  D)  $\{e^x\}$  E)  $\{e^{-x}\}$  AB)  $\{e^x, e^{-x}\}$  AC) None of the above

14. (1 pt.) The general solution of  $L[y] = 0$  is  $y_c(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $c$  B)  $cx$  C)  $c_1 + c_2x$  D)  $ce^x$  E)  $ce^{-x}$  AB)  $c_1e^x + c_2e^{-x}$  AC) None of the above

15. (1 pt.) Using the linear theory, a particular solution of  $L[y] = g(x)$  is given by

- $y_p(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E A)  $x - 1$  B)  $2x - 2$  C)  $3x - 3$  D)  $4x - 4$   
 E)  $x + 1$  AB)  $2x + 2$  AC)  $3x + 3$  AD)  $4x - 4$  AE) None of the above

16. (1 pt.) The number of solutions to  $(*)$  is \_\_\_\_\_ . \_\_\_\_\_ A B C D E A) 0 B) 1

- C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

17. (2 pts.) Let  $(***)$  be the initial value problem consisting of  $(*)$  and the initial condition

$y(0) = 0$ . The solution (or family of solutions) to  $(***)$  may be written

as \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $y = x + 1 + e^x$  B)  $y = 2x + 2 + 2e^x$  C)  $y = 3x + 3 + 3e^x$  D)  $y = 4x + 4 + 4e^x$   
 E)  $y = x - 1 + e^{-x}$  AB)  $y = 2x - 2 + 2e^{-x}$  AC)  $y = 3x - 3 + 3e^{-x}$  AD)  $y = 4x - 4 + 4e^{-x}$   
 AE)  $y = x - 1 + ce^x$  BC)  $y = 2x - 2 + ce^x$  BD)  $y = 3 - 3 + ce^{-x}$  BE)  $y = 4x - 4 + ce^{-x}$   
 CD)  $y = x - 1 + ce^{-2x}$  CE)  $y = x + 1 - ce^{-2x}$

ABCDE) None of the above solutions or families of solutions is correct.

18. (1 pt.) The number of solutions to  $(***)$  is \_\_\_\_\_ . \_\_\_\_\_ A B C D E A) 0 B) 1

- C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions ABCDE) None of the above

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let  $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$ ,  $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ , and  $\vec{b} = \begin{bmatrix} -2 \\ 2i \end{bmatrix}$ . Also let  $T(\vec{x}) = A\vec{x}$  so that  $T: \mathbb{C}^2 \rightarrow \mathbb{C}^2$ . Now solve

$\text{Prob}(\mathbb{C}^2, A\vec{x} = \vec{b})$ ; that is, solve the vector equation  $A\vec{x} = \vec{b}$  (i.e.  $T(\vec{x}) = \vec{b}$ ). The form of the answer may not be unique. If a problem has an infinite number of solutions, the form of the solution is not unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

19. (3 pts.) If  $\begin{bmatrix} A & \vec{b} \end{bmatrix}$  is reduced to  $\begin{bmatrix} U & \vec{c} \end{bmatrix}$  using Gauss elimination we obtain

- $\begin{bmatrix} U & \vec{c} \end{bmatrix} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E    A)  $\begin{bmatrix} 1 & i & | & -1 \\ 0 & 0 & | & 0 \end{bmatrix}$     B)  $\begin{bmatrix} 1 & i & | & -2 \\ 0 & 0 & | & 0 \end{bmatrix}$   
 C)  $\begin{bmatrix} 1 & i & | & -3 \\ 0 & 0 & | & 0 \end{bmatrix}$     D)  $\begin{bmatrix} 1 & i & | & -4 \\ 0 & 0 & | & 0 \end{bmatrix}$     E)  $\begin{bmatrix} 1 & i & | & -1 \\ 0 & 0 & | & 1 \end{bmatrix}$     AB)  $\begin{bmatrix} 0 & 0 & | & 0 \\ 0 & 0 & | & 0 \end{bmatrix}$     ABCDE) None of the above.

20. (3 pts.) The solution of  $A\vec{x} = \vec{b}$  may be written as

- $\vec{x} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E    A) No Solution    B)  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$   
 C)  $\begin{bmatrix} -i \\ 1 \end{bmatrix}$     D)  $y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     E)  $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     AB)  $\begin{bmatrix} -2 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     AC)  $\begin{bmatrix} -3 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     AD)  $\begin{bmatrix} -4 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix}$   
 ABCDE) None of the above correctly describes the solution or collection of solutions.

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Let  $\text{Prob}(\mathbf{C}^2; \mathbf{A}\vec{x} = \vec{b})$ ,  $\mathbf{A}$ ,  $\vec{b}$ ,  $\vec{x}$ , and  $\mathbf{T}$  be as on the previous page.

21. (1 pt.) The solution set for  $\text{Prob}(\mathbf{C}^2, \mathbf{A}\vec{x} = \vec{b})$  may be written as

S = \_\_\_\_\_ ABCD E    A)  $\emptyset$     B)  $\left\{ \begin{bmatrix} -i \\ 1 \end{bmatrix} \right\} \left\{ \vec{x} = y \begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C} \right\}$

D)  $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$  E)  $\left\{ \vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C} \right\}$     AB)  $\left\{ \vec{x} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C} \right\}$

AC)  $\left\{ \vec{x} = \begin{bmatrix} -3 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C} \right\}$     AD)  $\left\{ \vec{x} = \begin{bmatrix} -4 \\ 0 \end{bmatrix} + y \begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C} \right\}$

ABCDE) None of the above correctly describes the solution set for this problem

22. (1 pt.) A basis for the null space of  $\mathbf{T}$  is  $\mathbf{B} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\emptyset$     B)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix} \right\}$     C)  $\left\{ \begin{bmatrix} 0 \\ 1 \end{bmatrix} \right\}$     D)  $\left\{ \begin{bmatrix} i \\ 1 \end{bmatrix} \right\}$     E)  $\left\{ \begin{bmatrix} i \\ -1 \end{bmatrix} \right\}$     AB)  $\left\{ \begin{bmatrix} -i \\ 1 \end{bmatrix} \right\}$     AC)  $\left\{ \begin{bmatrix} -i \\ -1 \end{bmatrix} \right\}$

AD)  $\left\{ \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix} \right\}$     ABCDE) None of the above correctly describes the solution set for this problem

23. (1 pt.) The general solution of  $\mathbf{A}\vec{x} = \vec{0}$  is  $\vec{x}_c =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A) No Solution    B)  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$     C)  $\begin{bmatrix} -i \\ 1 \end{bmatrix}$     D)  $y \begin{bmatrix} i \\ 1 \end{bmatrix}$     E)  $y \begin{bmatrix} i \\ -1 \end{bmatrix}$     AB)  $y \begin{bmatrix} -i \\ 1 \end{bmatrix}$

AC)  $y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     AD)  $y \begin{bmatrix} -i \\ 1 \end{bmatrix}$     ABCDE) None of the above

24. (1 pt.) Using the linear theory, a particular solution of  $\mathbf{A}\vec{x} = \vec{b}$  is given by

$\vec{x}_p =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E    A)  $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$     B)  $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$     C)  $\begin{bmatrix} i \\ 1 \end{bmatrix}$     D)  $\begin{bmatrix} i \\ -1 \end{bmatrix}$     E)

$\begin{bmatrix} -i \\ 1 \end{bmatrix}$     AB)  $\begin{bmatrix} -i \\ -1 \end{bmatrix}$     AC)  $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$     AD)  $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$     AE)  $\begin{bmatrix} -3 \\ 0 \end{bmatrix}$     BC)  $\begin{bmatrix} -4 \\ 0 \end{bmatrix}$     BD)  $\begin{bmatrix} i \\ i \end{bmatrix}$

BE)  $\mathbf{A}\vec{x} = \vec{b}$  has solutions but none are listed    CD)  $\mathbf{A}\vec{x} = \vec{b}$  has no solutions

25. (1 pt.) The number of solutions to  $\text{Prob}(\mathbf{C}^2; \mathbf{A}\vec{x} = \vec{b})$  is \_\_\_\_\_ . \_\_\_\_\_ A B C D E    A) 0

B) 1    C) 2    D) 3    E) 4    AB) 5    AC) Infinite number of solutions    ABCDE) None of the above

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True or false. Solution of Abstract Linear Equations (having either  $\mathbf{R}$  or  $\mathbf{C}$  as the field of scalars). Assume  $T: V \rightarrow W$  is a linear operator from a (real or complex) vector space  $V$  to a (real or complex) vector space  $W$ . Now consider the mapping problem defined by the vector equation

$$T(\vec{x}) = \vec{b}. \quad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

26. (1 pt.) A)True or B)False If  $\vec{b} = \vec{0}$ , then (\*) always has a solution.
27. (1 pt.) A)True or B)False The vector equation (\*) may have exactly two solutions.
- 28.(1 pt.) A)True or B)False If the null space of  $T$  has a basis  $\mathbf{B} = \{\vec{x}_1, \vec{x}_2, \dots, \vec{x}_n\}$  and  $\vec{b} = \vec{0}$ , then the general solution of (\*) is given by  $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \dots + c_n\vec{x}_n$  where  $c_1, c_2, \dots, c_n$  are arbitrary constants.
29. (1 pt.) A)True or B)False Either (\*) has no solutions, exactly one solution, or an infinite number of solutions.
30. (1 pt.) A)True or B)False If the null space of  $T$  is  $N_T = \{\vec{0}\}$  and  $\vec{b}$  is in the range space of  $T$ , then (\*) has a unique solution.



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Answer questions using the instructions on the Exam Cover Sheet.

The dimension of the null space  $N_L$  of the linear operator  $L[y] = y'' + y'$  that maps  $\mathcal{A}(\mathbf{R}, \mathbf{R})$  to  $\mathcal{A}(\mathbf{R}, \mathbf{R})$  is 2. Assuming a solution of the homogeneous equation  $L[y] = 0$  of the form  $y = e^{rx}$  leads to the two linearly independent solutions  $y_1 = 1$  and  $y_2 = e^{-x}$ . Hence we can deduce that

$B_{N_L} = \{1, e^{-x}\}$  is a basis of  $N_L$  so that

$$y_c = c_1 + c_2 e^{-x} \quad \text{is the general solution of} \quad y'' + y' = 0.$$

Use the method of undetermined coefficients as discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution  $y_p$  of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

31. (3 pts.)  $y'' + y' = 2$       First guess:  $y_p =$  \_\_\_\_\_  
 Second guess (if needed):  $y_p =$  \_\_\_\_\_  
 Third guess (if needed):  $y_p =$  \_\_\_\_\_

Final guess \_\_\_\_\_ . \_\_\_\_\_ A B C D E

32.(3 pts.)  $y'' + y' = 3 \sin x$       First guess:  $y_p =$  \_\_\_\_\_  
 Second guess (if needed):  $y_p =$  \_\_\_\_\_  
 Third guess (if needed):  $y_p =$  \_\_\_\_\_

Final guess \_\_\_\_\_ . \_\_\_\_\_ A B C D E

33. (3 pts.)  $y'' + y' = 4 e^x$       First guess:  $y_p =$  \_\_\_\_\_  
 Second guess (if needed):  $y_p =$  \_\_\_\_\_  
 Third guess (if needed):  $y_p =$  \_\_\_\_\_

Final guess \_\_\_\_\_ . \_\_\_\_\_ A B C D E

Possible Answers:

- A)  $Ae^x$    B)  $Axe^x$    C)  $Ax^2e^x$    D)  $Axe^x + Be^x$    E)  $Ax^2e^x + Bxe^x$    AB)  $Ae^{-x}$    AC)  $Axe^{-x}$
- AD)  $Ax^2e^{-x}$    AE)  $Axe^{-x} + Be^{-x}$    BC)  $Ax^2e^{-x} + Bxe^{-x}$    BD)  $A \sin x$    BE)  $A \cos x$
- CD)  $A x \sin x$    CE)  $A x \cos x$    DE)  $A \sin x + B \cos x$    ABC)  $A x \sin x + B x \cos x$
- ABD)  $A$    ABE)  $Ax$    ACD)  $Ax + B$    ACE)  $Ax^2 + Bx$    ADE)  $Ax^2 + Bx + C$
- BCD) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Let  $\text{Prob}(\mathcal{A}((-\pi/2, \pi/2), \mathbf{R}), (*))$  be the problem defined by the ODE

$$y'' + y = -\tan(x) \quad I = (-\pi/2, \pi/2) \quad (*)$$

Let  $L: \mathcal{A}((-\pi/2, \pi/2), \mathbf{R}) \rightarrow \mathcal{A}((-\pi/2, \pi/2), \mathbf{R})$  be defined by  $L[y] = y'' + y$ . The general solution to  $L[y] = 0$  is  $y_c = c_1 \cos(x) + c_2 \sin(x)$ . To obtain a particular solution of  $L[y] = \tan(x)$  we let  $y_p(x) = u_1(x) \cos(x) + u_2(x) \sin(x)$ . You are to find  $y_p$ . Be careful!! Remember, once you make a mistake, the rest is wrong.

34. (3 pts.) Substituting  $y_p(x) = u_1(x) \cos(x) + u_2(x) \sin(x)$  into (\*) and making the appropriate assumption you obtained the two equations:

\_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0, \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = \tan(x)$   
 B)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0, \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = -\tan(x)$   
 C)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = \tan(x), \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$   
 D)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = -\tan(x), \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$   
 E)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0, \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = \sec(x)$   
 AB)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0, \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = -\sec(x)$   
 AC)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = \sec(x), \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$   
 AD)  $u'_1(x) \cos(x) + u'_2(x) \sin(x) = -\sec(x), \quad -u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$   
 ABCDE) None of the above.

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Let  $\text{Prob}(\mathcal{A}(-\pi/2, \pi/2), \mathbf{R}, (*)), (*), L$  and  $y_p(x) = u_1(x) \cos(x) + u_2(x) \sin(x)$  be as defined on the previous page.

35. (2 pts.) Solving the set of equations for  $u'_1(x)$  and  $u'_2(x)$  on the previous page we obtain

$u'_1(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

36. (2 pts.) And  $u'_2(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

37. (2 pts.) Hence we may choose  $u_1(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

38. (2 pts.) And  $u_2(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

39 (2 pts.) Hence a particular solution to (\*) is

$y_p(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

Possible answers this page.

- A) 0 B) 1 C) -1 D) x E) -x AB)  $\sin x$  AC)  $-\sin x$  AD)  $\cos x$  AE)  $-\cos x$  BC)  $\tan(x)$
- BD)  $-\tan(x)$  BE)  $\sin^2(x)/\cos(x)$  CD)  $-\sin^2(x)/\cos(x)$  CD)  $\ln(\cos(x))$  CE)  $-\ln(\cos(x))$
- DE)  $[\cos(x)]\ln(\tan(x)+\sec(x))$  ABC)  $-\ln(\cos(x))$  ABC)  $-\ln(\cos(x))$
- ABD)  $-\ln(\tan(x)+\sec(x))+\sin x$  ABE)  $\ln(\tan(x)+\sec(x)) - \sin x$  ACD)  $\ln(\sin(x))$
- ACE)  $-\ln(\sin(x))$  ADE)  $\ln(\cos(x))$  BCD)  $-\ln(\cos(x))$  ACD)  $(\cos(x)) [\ln(\sin(x)) + x \sin(x)]$
- ACE)  $-(\cos(x)) [\ln(\sin(x)) + x \sin(x)]$  ADE)  $(\cos x) \ln(\cos(x)) + x \sin(x)$
- BCD)  $-(\cos(x)) \ln(\cos(x) - x \sin(x))$  ABCDE) None of the above.

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.  
 Also, circle your answer. Be careful. If you miss one part, it may cause you to miss other parts.

Consider  $y^{IV} + 4y''' + 4y'' = 0$  (\*) Also let  $L: \mathcal{A}(\mathbf{R}, \mathbf{R}) \rightarrow \mathcal{A}(\mathbf{R}, \mathbf{R})$  be defined by  
 $L[y] = y^{IV} + 4y''' + 4y''$  and answer the following questions.

40. (1 pt). The dimension of the null space of L is \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 ABCDE) None of the above.

41. (1 pts). The auxiliary equation for (\*) is \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $r^4 + 2r^3 + r^2 = 0$  B)  $r^4 + 4r^3 + 4r^2 = 0$  C)  $r^4 + 6r^3 + 9r^2 = 0$  D)  $r^4 + 8r^3 + 16r^2 = 0$   
 E)  $r^4 - 2r^3 + r^2 = 0$  AB)  $r^4 - 4r^3 + 4r^2 = 0$  AC)  $r^4 - 6r^3 + 9r^2 = 0$  AD)  $r^4 - 8r^3 + 16r^2 = 0$   
 ABCDE) None of the above.

42. (2 pts). Listing repeated roots, the roots of the auxiliary equation are

$r =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E A) 0,0,1,1 B) 0,0,-1,-1  
 C) 0,0,2,2 D) 0,0,-2,-2 E) 0,0,3,3 AB)  $r = 0,0,-3,-3$  AC) 0,0,4,4 AD) 0,0,-4,-4  
 ABCDE) None of the above.

43. (1 pts). A basis for the null space of L is B = \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $\{1, x, e^x, xe^x\}$  B)  $\{1, x, e^{-x}, xe^{-x}\}$  C)  $\{1, x, e^{2x}, e^{2x}\}$  D)  $\{1, x, e^{-2x}, xe^{-2x}\}$  E)  $\{1, x, e^{3x}, e^{3x}\}$   
 AB)  $\{1, x, e^{-3x}, e^{-3x}\}$  AC)  $\{1, x, e^{4x}, e^{4x}\}$  AD)  $\{1, x, e^{-4x}, e^{-4x}\}$  AE)  $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$   
 ABCDE) None of the above

44. (2 pt). The general solution of (\*) is  $y(x) =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $c_1 + c_2 e^x + c_3 x e^x$  B)  $c_1 + c_2 e^{-x} + c_3 x e^{-x}$  C)  $c_1 + c_2 x + c_3 e^{2x} + c_4 e^{2x}$   
 D)  $c_1 + c_2 x + c_3 e^{-2x} + c_4 e^{-2x}$  E)  $c_1 + c_2 x + c_3 e^{3x} + c_4 x e^{3x}$  AB)  $c_1 + c_2 x + c_3 e^{-3x} + c_4 x e^{-3x}$   
 AC)  $c_1 + c_2 x + c_3 e^{4x} + c_4 x e^{4x}$  AD)  $c_1 + c_2 x + c_3 e^{-4x} + c_4 x e^{-4x}$  AE)  $c_1 e^{2x} + c_2 x e^{2x} + c_3 e^{-2x} + c_4 x e^{-2x}$   
 ABCDE) None of the above

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
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Let (\*) and L be as on the previous page.

45. (1pt.) The solution set for (\*) may be written as

S = \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\{y(x) = c_1 + c_2 x, c_3 e^x + c_4 x e^x : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

B)  $\{y(x) = c_1 + c_2 x + c_3 e^{-x} + c_4 x e^{-x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

C)  $\{y(x) = c_1 + c_2 x + c_3 e^{2x} + c_4 x e^{2x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

D)  $\{y(x) = c_1 + c_2 x + c_3 e^{-2x} + c_4 x e^{-2x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

E)  $\{y(x) = c_1 + c_2 x + c_3 e^{3x} + c_4 x e^{3x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

AB)  $\{y(x) = c_1 + c_2 x + c_3 e^{-3x} + c_4 x e^{-3x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

AC)  $\{y(x) = c_1 + c_2 x + c_3 e^{4x} + c_4 x e^{4x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

AD)  $\{y(x) = c_1 + c_2 x + c_3 e^{-4x} + c_4 x e^{-4x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$       ABCDE) None of the above

46. (1 pt.) The number of solutions to (\*) is \_\_\_\_\_. \_\_\_\_\_ A B C D E    A) 0 B) 1  
 C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
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Answer questions using the instructions on the Exam Cover Sheet. Also circle your answers. Be careful!!

Remember, once you make a mistake, the rest is wrong.

Consider the ODE  $y''' + y'' = 2e^{-x} + 4 \sin(x)$  (\*). Let  $L[y] = y''' + y''$ .

47. (3 pts.) The general solution of  $y''' + y'' = 0$  is

$y_c(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $c_1 + c_2x + c_3e^x$  B)  $c_1 + c_2x + c_3e^{-x}$  C)  $c_1 + c_2e^x + c_3e^{-x}$  D)  $c_1e^x + c_2 \sin(x) + c_3 \cos(x)$

E)  $c_1e^{-x} + c_2 \sin(x) + c_3 \cos(x)$  ABCDE) None of the above

48. (4 pts.) A particular solution of  $y''' + y'' = 2e^{-x}$  is

$y_{p1}(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $xe^{-x}$  B)  $-xe^{-x}$  C)  $2xe^{-x}$  D)  $-2xe^{-x}$  E)  $3xe^{-x}$  AB)  $-3xe^{-x}$  AC)  $4xe^{-x}$  AD)  $-4xe^{-x}$

ABCDE) None of the above

49. (4 pts.) A particular solution of  $y''' + y'' = 4 \sin(x)$  is

$y_{p2}(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\cos(x) - \sin(x)$  B)  $2\cos(x) - 2\sin(x)$  C)  $3\cos(x) - 3\sin(x)$  D)  $4\cos(x) - 4\sin(x)$

ABCDE) None of the above

50. (1 pts.) A particular solution of (\*) is

$y_p(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $xe^x + \cos(x) - \sin(x)$  B)  $xe^{-x} + \cos(x) - \sin(x)$  C)  $-xe^{-x} + \cos(x) - \sin(x)$

D)  $2xe^x + 2\cos(x) - 2\sin(x)$  E)  $2xe^{-x} + 2\cos(x) - 2\sin(x)$  AB)  $-2xe^{-x} + 2\cos(x) - 2\sin(x)$

AC)  $3xe^x + 3\cos(x) - 3\sin(x)$  AD)  $3xe^{-x} + 2\cos(x) - 3\sin(x)$  AE)  $-3xe^{-x} + 3\cos(x) - 3\sin(x)$

BC)  $4xe^x + 4\cos(x) - 4\sin(x)$  BD)  $4xe^{-x} + 4\cos(x) - 4\sin(x)$  CD)  $-4xe^{-x} + 4\cos(x) - 4\cos(x)$

CE)  $5xe^x + 5\cos(x) - 5\sin(x)$  DE)  $5xe^{-x} + 5\cos(x) - 5\cos(x)$  ABC)  $-5xe^{-x} + 5\cos(x) - 5\sin(x)$

ABD)  $-2xe^{-x} - 2\sin(x) - 2\cos(x)$  ABCDE) None of the above.

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Let (\*) and L be as on the previous page.

51. (2 pts.) The general solution of (\*) is

$y(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $xe^x + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^x$  B)  $xe^x + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^{-x}$   
 C)  $xe^{-x} + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^x$  D)  $xe^{-x} + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^{-x}$   
 E)  $xe^x + \cos(x) - \sin(x) + c_1 + c_2x + c_3xe^x$  AB)  $xe^{-x} + \cos(x) - \sin(x) + c_1 + c_2x + c_3xe^{-x}$   
 AC)  $2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^x$  AD)  $2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 AE)  $2xe^{-x} + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^x$  BC)  $2xe^{-x} + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 BD)  $2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3xe^x$  BE)  $2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3xe^{-x}$   
 CD)  $3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^x$  CE)  $3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 DE)  $3xe^{-x} + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^x$  ABC)  $3xe^{-x} + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 ABD)  $3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3xe^x$  ABE)  $3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3xe^{-x}$   
 ACD)  $4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^x$  ACE)  $4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 ADE)  $4xe^{-x} + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^x$  CDE)  $4xe^{-x} + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^{-x}$   
 ABCD)  $4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3xe^x$  ABCE)  $4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3xe^{-x}$   
 ABCDE) None of the above.

52. (1pt.) The solution set for (\*) may be written as

S = \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $\{y(x) = xe^x + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 B)  $\{y(x) = xe^x + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 C)  $\{y(x) = xe^{-x} + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 D)  $\{y(x) = xe^{-x} + \cos(x) - \sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 E)  $\{y(x) = 2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 AB)  $\{y(x) = 2xe^x + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 AC)  $\{y(x) = 2xe^{-x} + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 AD)  $\{y(x) = 2xe^{-x} + 2\cos(x) - 2\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 AE)  $\{y(x) = 3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 BC)  $\{y(x) = 3xe^x + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 BD)  $\{y(x) = 3xe^{-x} + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 BE)  $\{y(x) = 3xe^{-x} + 3\cos(x) - 3\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 CD)  $\{y(x) = 4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 CE)  $\{y(x) = 4xe^x + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$   
 DE)  $\{y(x) = 4xe^{-x} + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$   
 ABC)  $\{y(x) = 4xe^{-x} + 4\cos(x) - 4\sin(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$  ABCDE) None of the above

53. (1 pt.) The number of solutions to (\*) is \_\_\_\_\_ . \_\_\_\_\_ A B C D E A) 0 B) 1

C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Answer questions using the instructions on the Exam Cover Sheet. .

Compute the Laplace transform of the following functions in  $PC[0, \infty) \cap \text{Exp} \subseteq \mathbf{T}$ .

54. (3 pts.)  $f(t) = 3t - 4t^2$   $\mathcal{L}(f) =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

55. (3 pts.)  $f(t) = -3e^{2t} - 4e^{-3t}$   $\mathcal{L}(f) =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

56. (3 pts.)  $f(t) = -3\sin(2t) + 4\cos(3t)$   $\mathcal{L}(f) =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

Possible answers this page

A)  $\frac{3}{s} + \frac{4}{s^2}$  B)  $\frac{3}{s} + \frac{-4}{s^2}$  C)  $\frac{-3}{s} + \frac{4}{s^2}$  D)  $\frac{-3}{s} + \frac{-4}{s^2}$  E)  $\frac{3}{s^2} + \frac{8}{s^3}$  AB)  $\frac{3}{s^2} + \frac{-8}{s^3}$  AC)  $\frac{-3}{s^2} + \frac{8}{s^3}$  AD)  $\frac{-3}{s^2} + \frac{-8}{s^3}$

AE)  $\frac{3}{s+2} + \frac{4}{s+3}$  BC)  $\frac{3}{s+2} + \frac{-4}{s+3}$  BD)  $\frac{-3}{s+2} + \frac{4}{s+3}$  BE)  $\frac{-3}{s+2} + \frac{-4}{s+3}$

CD)  $\frac{3}{s-2} + \frac{4}{s+3}$  CE)  $\frac{3}{s-2} + \frac{-4}{s+3}$  DE)  $\frac{-3}{s-2} + \frac{4}{s+3}$  ABC)  $\frac{-3}{s-2} + \frac{-4}{s+3}$

ABD)  $\frac{6}{s^2+4} + \frac{4s}{s^2+9}$  ABE)  $\frac{6}{s^2+4} + \frac{-4s}{s^2+9}$  ACD)  $\frac{-6}{s^2+4} + \frac{4s}{s^2+9}$  ACE)  $\frac{-6}{s^2+4} + \frac{-4s}{s^2+9}$  ADE)  $\frac{6}{s^2-4} + \frac{4s}{s^2-9}$

BCD)  $\frac{6}{s^2-4} + \frac{-4s}{s^2-9}$  BDE)  $\frac{-6}{s^2-4} + \frac{4s}{s^2-9}$  CDE)  $\frac{-6}{s^2-4} + \frac{-4s}{s^2-9}$

ABCD)  $\frac{3}{(s-2)^2} + \frac{4}{(s+3)^2}$  ABCE)  $\frac{3}{s^2+2} + \frac{4s}{s^2+3}$  ABDE)  $\frac{3s}{s^2+4} + \frac{6}{s^2+9}$

ACDE)  $\mathcal{L}\{f\}$  exists but none of the above is  $\mathcal{L}\{f\}$  BCDE)  $\mathcal{L}\{f\}$  does not exist.

ABCDE) None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_



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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions if they are in **F**:

57. (3 pts.)  $F(s) = -\frac{3}{s} - \frac{4}{s+2}$   $\mathcal{L}^{-1}\{F\} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

58. (3 pts.)  $F(s) = \frac{2s-6}{s^2+9}$   $\mathcal{L}^{-1}\{F\} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

59. (3 pts.)  $F(s) = \frac{2s-4}{s^2-2s+2}$   $\mathcal{L}^{-1}\{F\} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

Possible answers this page

A)  $3 + 4e^{2t}$  B)  $3 - 4e^{2t}$  C)  $-3 + 4e^{2t}$  D)  $-3 - 4e^{2t}$  E)  $3 + 4e^{-2t}$  AB)  $3 - 4e^{-2t}$  AC)  $-3 + 4e^{-2t}$

AD)  $-3 - 4e^{-2t}$  AE)  $2 \cos 3t + \sin 3t$  BC)  $2 \cos 3t - \sin 3t$  BD)  $2 \cos 3t + 2 \sin 3t$

BE)  $2 \cos 3t - 2 \sin 3t$  CD)  $2 \cos t + 3 \sin 3t$  CE)  $2 \cos 3t - 3 \sin 3t$

DE)  $2 \cos 3t + 4 \sin 3t$  ABC)  $2 \cos 3t - 4 \sin 3t$  ABD)  $2e^t \cos t + e^t \sin t$

ABE)  $2e^t \cos t - e^t \sin t$  ACD)  $2e^t \cos t + 2e^t \sin t$  ACE)  $2e^t \cos t - 2e^t \sin t$

ADE)  $2e^t \cos t + 3e^t \sin t$  BCD)  $2e^t \cos t - 3e^t \sin t$  BCE)  $2e^t \cos t + 4e^t \sin t$

BDE)  $2e^t \cos t - 4e^t \sin t$  CDE)  $\mathcal{L}^{-1}\{f\}$  exists but none of the above is  $\mathcal{L}\{f\}$

ABCD)  $\mathcal{L}^{-1}\{f\}$  does not exist. ABCDE) None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
 Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Using the procedure illustrated in class (attendance is mandatory), find the eigenvalues of

$$A = \begin{bmatrix} i & 4 \\ 0 & 2 \end{bmatrix} \in \mathbb{C}^{2 \times 2}.$$

60. (2 pts.) A polynomial  $p(\lambda)$  where solving  $p(\lambda) = 0$  yields the eigenvalues of A can be written

as  $p(\lambda) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $(i+\lambda)(1+\lambda)$  B)  $(i+\lambda)(1-\lambda)$  C)  $(i-\lambda)(1+\lambda)$  D)  $(i-\lambda)(1-\lambda)$  E)  $(i-\lambda)(2-\lambda)$  AB)  $(i+\lambda)(2-\lambda)$   
 AC)  $(i-\lambda)(2+\lambda)$  AD)  $(i-\lambda)(3-\lambda)$  AE)  $(i+\lambda)(4+\lambda)$  BC)  $(i+\lambda)(4-\lambda)$  BD)  $(i-\lambda)(4+\lambda)$   
 BE)  $(i-\lambda)(4-\lambda)$  CD)  $(2i+\lambda)(2+\lambda)$  CE)  $(2i+\lambda)(2-\lambda)$  DE)  $(2i-\lambda)(2+\lambda)$  ABC)  $(2i-\lambda)(2-\lambda)$   
 ABD)  $(3i-\lambda)(2+\lambda)$  ABE) None of the above.

61. (1 pt.) The degree of  $p(\lambda)$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E      A) 1   B) 2   C) 3  
 D) 4   E) 5   AB) 6   AC) 7   AD) None of the above.

62. (1pt.) Counting repeated roots, the number of eigenvalues of A

is \_\_\_\_\_. \_\_\_\_\_ A B C D E      A) 0   B) 1   C) 2   D) 3   E) 4   AB) 5  
 AC) 6   AD) 7   AE) 8      BC) None of the above

63. (2 pts.) The eigenvalues of A can be written as \_\_\_\_\_. \_\_\_\_\_ A B C D E

- A)  $\lambda_1 = 1, \lambda_2 = i$  B)  $\lambda_1 = 1, \lambda_2 = -i$  C)  $\lambda_1 = -1, \lambda_2 = i$  D)  $\lambda_1 = -1, \lambda_2 = -i$  E)  $\lambda_1 = 2, \lambda_2 = i$   
 AB)  $\lambda_1 = 2, \lambda_2 = -i$  AC)  $\lambda_1 = -2, \lambda_2 = i$  AD)  $\lambda_1 = -2, \lambda_2 = -i$  AE)  $\lambda_1 = 3, \lambda_2 = i$   
 BC)  $\lambda_1 = 3, \lambda_2 = -i$  BD)  $\lambda_1 = 4, \lambda_2 = i$  BE)  $\lambda_1 = 4, \lambda_2 = -i$  CD)  $\lambda_1 = 2, \lambda_2 = 2i$   
 CE)  $\lambda_1 = 2, \lambda_2 = -2i$  DE)  $\lambda_1 = -2, \lambda_2 = 2i$  ABC)  $\lambda_1 = -2, \lambda_2 = -2i$  ABD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answers.

Note that  $\lambda_1 = 2$  is an eigenvalue of the matrix  $A = \begin{bmatrix} 3 & -2 \\ -6 & 14 \end{bmatrix}$

64.(4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with  $\lambda_1$  is B = \_\_\_\_\_. \_\_\_\_\_ A B C D E

- A)  $\{[1,1]^T, [4,4]^T\}$  B)  $\{[1,1]^T\}$  C)  $\{[1,2]^T\}$  D)  $\{[1,2]^T, [4,8]^T\}$  E)  $\{[2,1]^T\}$   
 AB)  $\{[1,3]^T\}$  AC)  $\{[1,4]^T\}$  AD)  $\{[4,1]^T\}$  AE)  $\{[3,1]^T\}$  BC)  $\{[1,-1]^T, [4,4]^T\}$   
 BD)  $\{[1,-1]^T\}$  BE)  $\{[1,-2]^T\}$  CD)  $\{[1,-2]^T, [4,8]^T\}$  CE)  $\{[2,1]^T\}$  DE)  $\{[1,3]^T\}$   
 ABC)  $\{[1,-4]^T\}$  ABD)  $\{[4,-1]^T\}$  ABE)  $\{[3,-1]^T\}$

ACD)  $\lambda = 2$  is not an eigenvalue of the matrix A

ACE)  $\lambda = -1$  is not an eigenvalue of the matrix A

ADE)  $\lambda = 3$  is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

65. (1pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, the eigenspace associated with  $\lambda_1$  is often one dimensional. Hence conventions for selecting eigenvector(s) associated with  $\lambda_1$  have been developed (by engineers). We say that the eigenvector(s) associated with  $\lambda_1$

is (are) \_\_\_\_\_. \_\_\_\_\_ A B C D E

- A)  $[1,1]^T, [4,4]^T$  B)  $[1,1]^T$  C)  $[1,2]^T$  D)  $[1,2]^T, [4,8]^T$  E)  $[2,1]^T$   
 AB)  $[1,3]^T$  AC)  $[1,4]^T$  AD)  $[4,1]^T$  AE)  $[3,1]^T$  BC)  $[1,-1]^T, [4,4]^T$   
 BD)  $[1,-1]^T$  BE)  $[1,-2]^T$  CD)  $[1,-2]^T, [4,8]^T$  CE)  $[2,1]^T$  DE)  $[1,3]^T$   
 ABC)  $[1,-4]^T$  ABD)  $[4,-1]^T$  ABE)  $[3,-1]^T$

ACD)  $\lambda_1 = 2$  is not an eigenvalue of the matrix A

ACE)  $\lambda_1 = -1$  is not an eigenvalue of the matrix A

ADE)  $\lambda = 3$  is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer. Consider the scalar equation  $u'' + 4u' - 2u = 0$  where  $u = u(t)$  (i.e. the dependent variable  $u$  is a function of the independent variable  $t$  so that  $u' = du/dt$  and  $u'' = du'/dt$ ). As was done in class (attendance is mandatory) convert this to a system of two first order equations by letting  $u = x$  and  $u' = y$  (i.e. obtain two first order scalar equations in  $x$  and  $y$ ). You may think of  $x$  as the position and  $y$  as the velocity of a point particle). This system of two scalar

equations can be written in the vector form  $\vec{x}' = A\vec{x}$  where  $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$  and  $A$  is a  $2 \times 2$  matrix. You are to find  $A$

$= \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ ; that is you are to find  $a$ ,  $b$ ,  $c$ , and  $d$ .

66. (1 pt.)  $a =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

67. (1 pt.)  $b =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

68. (1 pt.)  $c =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

69. (1 pt.)  $d =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E

Possible answers this page.

A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9  
 BD) -1 BE) -2 CD) -3 CE) -4 DE) -5 ABC) -6 ABD) -7 ABE) -8 ACD) -9  
 ABCDE) None of the above

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

TABLE

Let the 2x2 matrix A have the eigenvalue table

	Eigenvalues	Eigenvectors
Let $L: \mathcal{A}(\mathbf{R}, \mathbf{R}^2) \rightarrow \mathcal{A}(\mathbf{R}, \mathbf{R}^2)$ be defined by $L[\bar{x}] = \bar{x}' - A\bar{x}$	$r_1 = 2$	$\bar{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
and let the null space of L be $N_L$	$r_2 = 1$	$\bar{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

70. (1 pt). The dimension of  $N_L$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A) 0    B) 1    C) 2    D) 3    E) 4    AB) 5    AC) 6    AD) None of the above.
71. (2 pts.) A basis for the null space of L may be written

- as \_\_\_\_\_ . \_\_\_\_\_ A B C D E
- A)  $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} \right\}$     B)  $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} \right\}$     C)  $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} \right\}$     D)  $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} \right\}$
- E)  $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$     AB)  $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$     AC)  $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$     AD)  $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$     AE)
- $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t} \right\}$     BC)  $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$     BD)  $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$     BE)  $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$     CD) None of the above

72. (2 pts.) The general solution of  $\bar{x}' = A\bar{x}$  may be written
- as \_\_\_\_\_ . \_\_\_\_\_ A B C D E
- A)  $\bar{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$     B)  $\bar{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$     C)  $\bar{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$     D)  $\bar{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$
- E)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$     AB)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$     AC)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^t + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$     AD)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$
- AE)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t}$     BC)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$     BD)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$
- BE)  $\bar{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$     CD) None of the above

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
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True or False. Let  $f$  and  $g$  be real valued functions of a real variable; that is,  $f: \mathbf{R} \rightarrow \mathbf{R}$  and  $g: \mathbf{R} \rightarrow \mathbf{R}$ . Circle True if the statement is true. Circle False if the statement is false.

73. (1 pt.) A) True B) False The function  $f$  is even if  $f(-x) = -f(x) \forall x \in \mathbf{R}$ .

74. (1 pt.) A) True B) False The function  $f$  is even if  $f(-x) = f(x) \forall x \in \mathbf{R}$ .

75. (1 pt.) A) True B) False If  $f$  and  $g$  are both odd functions, then the product of  $f$  and  $g$  is an even function.

76. (1 pt.) A) True B) False The function  $f$  is periodic of period  $T$  if  $f(x+T) = f(x) \forall x \in \mathbf{R}$ .

77. (1 pt.) A) True B) False If  $f$  is an even function, then we know that  $\int_{-l}^l f(x) dx = 0$ .

For each of the following questions write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters

Classify the following function with regard to whether they are odd or even.

78. (1 pt.)  $f(x) = 5$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

79. (1 pt.)  $f(x) = x$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

80. (1 pt.)  $f(x) = \cos(x)$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

81. (1 pt.)  $f(x) = x^2$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

82. (1 pt.)  $f(x) = e^x$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

83. (1 pt.)  $f(x) = 0$  is \_\_\_\_\_. \_\_\_\_\_ A B C D E

Possible answers for questions 78-83.

A) odd, but not even      B) even, but not odd      C) both odd and even  
 D) neither odd nor even      ABCDE) none of the above

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

PRINT NAME KEY \_\_\_\_\_ ( \_\_\_\_\_ ) ID No. \_\_\_\_\_

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let  $f: \mathbf{R} \rightarrow \mathbf{R}$  be in  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell) = \{f \in \mathcal{F}(\mathbf{R}, \mathbf{R}) : f \text{ is periodic of period } 2\ell, f \text{ and } f' \text{ are piecewise continuous on } ]\ell, \ell], \text{ and } f(x) = \frac{f(x+) + f(x-)}{2} \text{ at points of discontinuity}\}$  so that its Fourier series exists.

84. (2 pts.) The formula for the general Fourier series for  $f \in PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  given in our text is

$f(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $a_0 + \sum_{n=1}^N a_n \cos\left(\frac{n\pi}{\ell}x\right) + b_n \sin\left(\frac{n\pi}{\ell}x\right)$  B)  $\sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{\ell}x\right) + b_n \sin\left(\frac{n\pi}{\ell}x\right)$

C)  $a_0 + \sum_{n=1}^{\infty} a_n \cos\left(\frac{n\pi}{\ell}x\right) + b_n \sin\left(\frac{n\pi}{\ell}x\right)$  D)  $\frac{a_0}{2} + \sum_{n=0}^N a_n \cos(n\pi x) + b_n \sin(n\pi x)$

E)  $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos\left(\frac{n\pi}{\ell}x\right) + b_n \sin\left(\frac{n\pi}{\ell}x\right)$  AB)  $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + b_n \cos(n\pi x)$

ABCDE) None of the above

85. (2pts.) where for  $n = 0, 1, 2, \dots$  we have  $a_n =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\frac{2}{\ell} \int_{-\ell}^{\ell} f(x) \cos\left(\frac{n\pi}{\ell}x\right) dx$  B)  $\frac{2}{\ell} \int_0^{\ell} f(x) \cos\left(\frac{n\pi}{\ell}x\right) dx$  C)  $\frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \cos\left(\frac{n\pi}{\ell}x\right) dx$

D)  $\frac{1}{\ell} \int_0^{\ell} f(x) \cos\left(\frac{n\pi}{\ell}x\right) dx$  E)  $\frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \cos(n\pi x) dx$  AB)  $\frac{\ell}{2} \int_{-\ell}^{\ell} f(x) \cos(x) dx$

ABCDE) None of the above.

86. (2pts.) and for  $n = 1, 2, \dots$  we have  $b_n =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\frac{2}{\ell} \int_{-\ell}^{\ell} f(x) \sin\left(\frac{n\pi}{\ell}x\right) dx$  B)  $\frac{2}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{n\pi}{\ell}x\right) dx$  C)  $\frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \sin\left(\frac{n\pi}{\ell}x\right) dx$

D)  $\frac{1}{\ell} \int_0^{\ell} f(x) \sin\left(\frac{n\pi}{\ell}x\right) dx$  E)  $b_n = \frac{1}{\ell} \int_{-\ell}^{\ell} f(x) \sin(n\pi x) dx$  AB)  $\frac{\ell}{2} \int_{-\ell}^{\ell} f(x) \sin(x) dx$

ABCDE) None of the above.

PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_  
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Recall from the previous page that  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell) = \{f \in \mathcal{F}(\mathbf{R}, \mathbf{R}) : f \text{ is periodic of period } 2\ell, f \text{ and } f' \text{ are piecewise continuous on } ]\ell, \ell], \text{ and } f(x) = (f(x+) + f(x-))/2 \text{ at points of discontinuity}\}$ . Now let  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  be the subspace of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  and  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  are inner product spaces with inner product  $(f, g) = \int_{-\ell}^{\ell} f(x)g(x)dx$ , that  $B_{fs}(\ell) = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$  is an orthogonal Schauder basis for  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$ , and an orthogonal Hamel basis of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$ .

87. (1 pt.) Using the notation given above, an orthogonal Hamel basis for  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; 2)$

is \_\_\_\_\_ . \_\_\_\_\_ A B C D E

Hint: What is  $\ell$ ?

- A)  $\{1/2\} \cup \{\cos(n\pi) : n \in \mathbf{N}\} \cup \{\sin(n\pi) : n \in \mathbf{N}\}$
  - B)  $\{1/2\} \cup \{\cos(\frac{n\pi}{2}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{2}) : n \in \mathbf{N}\}$
  - C)  $\{1/2\} \cup \{\cos(\frac{n\pi}{3}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{3}) : n \in \mathbf{N}\}$
  - D)  $\{1/2\} \cup \{\cos(\frac{n\pi}{4}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{4}) : n \in \mathbf{N}\}$
  - E)  $\{(1/2)x\} \cup \{\cos(\frac{n\pi}{2}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{2}) : n \in \mathbf{N}\}$
- ABCDE) None of the above.

88. (2 pts.) The Fourier series for the function  $f(x) \in PC_{fs}^1(\mathbf{R}, \mathbf{R}; \pi)$  which has period  $2\pi$  and is defined on the interval  $[-\pi, \pi]$  by  $f(x) = 2 + 2 \cos(x) + 2 \sin(x)$  is

$f(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

Hint: Think Hamel basis.

- A)  $3 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$
  - B)  $2 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$
  - C)  $3 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$
  - D)  $2 + \sum_{k=1}^{\infty} \frac{1}{k\pi} \sin(k\pi x)$
  - E)  $2 + 2\cos(x) + \sin(x)$
- AB)  $2 + 2\cos(x) + 2\sin(x)$  AC)  $2 + 2\cos(x) + 3\sin(x)$  AD)  $2 + 2\cos(x) + 4\sin(x)$   
 AE)  $2 + 2\cos(2\pi x) + 2\sin(2\pi x)$  BC)  $2 + 2\cos(2\pi x) + 3\sin(2\pi x)$  BD)  $2 + 3\cos(2\pi x) + 3\sin(2\pi x)$   
 BE)  $3 + 3\cos(2\pi x) + 3\sin(2\pi x)$  ABCDE) None of the above.



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Let  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  and  $B_{fs}(\ell)$  be as on the previous page. Now let  $f(x) \in PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  be the function whose domain is  $\mathbf{R}$  which has period 4 and is defined on the interval  $(-2, 2)$  by  $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 4 & 0 < x < 2 \end{cases}$ . Using the formulas on the previous page, determine the Fourier series for the function  $f$ . Begin by sketching  $f$  for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges

89. (1 pt.) To apply the formulas given on the previous page we choose  $\ell =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E  
 A)  $\ell = 1$  B)  $\ell = 2$  C)  $\ell = 3$  D)  $\ell = 4$  E)  $\ell = -1$  AB)  $\ell = -2$  AC)  $\ell = -3$   
 AD)  $\ell = -4$  ABCDE) None of the above

Next write down the formulas for a Fourier series and its coefficients using this value of  $\ell$ . After computing the  $a_n$ 's and the  $b_n$ 's, note what they are for  $n$  odd and  $n$  even. Then answer the questions below and on the next two pages.

90. (3 pts.) We have  $a_0 =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E A) 0 B) 1 C) 2 D) 3  
 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 ABCDE) None of the above

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Let  $f(x)$  be as on the previous page. Continue the computation of the Fourier Series coefficients.

91. (3 pts.) For  $a_n$  with  $n$  odd ( $n = 1, 3, 5, \dots$ ) so that for  $k = 0, 1, 2, 3, \dots$  we have

$a_{2k+1} =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E      A) 0    B)  $1/(2k+1)$   
 C)  $2/(2k+1)$     D)  $3/(2k+1)$     E)  $1/[(2k+1)\pi]$     AB)  $2/[(2k+1)\pi]$     AC)  $3/[(2k+1)\pi]$   
 AD)  $4/[(2k+1)\pi]$     AE)  $8/[(2k+1)\pi]$     BC)  $-1/(2k+1)$     BD)  $-2/(2k+1)$     BE)  $-3/(2k+1)$   
 CD)  $-4/(2k+1)$     CE)  $-1/[(2k+1)\pi]$     DE)  $-2/[(2k+1)\pi]$     ABC)  $-3/[(2k+1)\pi]$   
 ABD)  $-4/[(2k+1)\pi]$     ABE)  $-8/[(2k+1)\pi]$     ABCDE) None of the above

92. (3 pts.) For  $a_n$  with  $n$  even ( $n = 2, 4, 6, \dots$ ) so that for  $k = 1, 2, 3, \dots$  we have

$a_{2k} =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E      A) 0    B)  $1/(2k)$     C)  $1/k$   
 D)  $3/(2k)$     E)  $1/(2k\pi)$     AB)  $1/(k\pi)$     AC)  $3/(2k\pi)$     AD)  $2/(k\pi)$     AE)  $4/(k\pi)$   
 BC)  $-1/(2k)$     BD)  $-1/k$     BE)  $-3/(2k)$     CD)  $-2/k$     CE)  $-1/(2k\pi)$     DE)  $-1/(k\pi)$   
 ABC)  $-3/(2k\pi)$     ABD)  $-2/(2k\pi)$     ABE)  $-4/(k\pi)$     ABCDE) None of the above

93. (3 pts.) For  $b_n$  with  $n$  odd ( $n = 1, 3, 5, \dots$ ) so that for  $k = 0, 1, 2, 3, \dots$  we have

$b_{2k+1} =$  \_\_\_\_\_. \_\_\_\_\_ A B C D E      A) 0    B)  $1/(2k+1)$     C)  $2/(2k+1)$     D)  
 $3/(2k+1)$     E)  $4/[(2k+1)\pi]$     AB)  $8/[(2k+1)\pi]$     AC)  $(12)/[(2k+1)\pi]$     AD)  $(16)/[(2k+1)\pi]$   
 AE)  $8/[(2k+1)\pi]$     BC)  $-4/(2k+1)$     BD)  $-8/(2k+1)$     BE)  $-(12)/(2k+1)$     CD)  $-(16)/(2k+1)$   
 CE)  $-1/[(2k+1)\pi]$     DE)  $-2/[(2k+1)\pi]$     ABC)  $-3/[(2k+1)\pi]$     ABD)  $-4/[(2k+1)\pi]$   
 ABE)  $-8/[(2k+1)\pi]$     ABCDE) None of the above

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Let  $f(x)$  be as on the page before the previous page. Continue the computation of the Fourier Series coefficients.

94. (3 pts.) For  $b_n$  with  $n$  even ( $n = 2, 4, 6, \dots$ ) so that for  $k = 1, 2, 3, \dots$

we have  $b_{2k} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E    A) 0    B)  $1/(2k)$     C)  $1/k$   
 D)  $3/(2k)$     E)  $1/(2k\pi)$     AB)  $1/(k\pi)$     AC)  $3/(2k\pi)$     AD)  $2/(k\pi)$     AE)  $4/(k\pi)$   
 BC)  $-1/(2k)$     BD)  $-1/k$     BE)  $-3/(2k)$     CD)  $-2/k$     CE)  $-1/(2k\pi)$     DE)  $-1/(k\pi)$   
 ABC)  $-3/(2k\pi)$     ABD)  $-2/(2k\pi)$     ABE)  $-4/(k\pi)$     ABCDE) None of the above

95. (3 pts.) Thus the Fourier series for  $f(x)$  may be written as

$f(x) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- |  |  |
|--|--|
| A) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$        | B) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$        |
| C) $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$        | D) $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$        |
| E) $\sum_{k=0}^{\infty} \frac{12}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$       | AB) $\sum_{k=0}^{\infty} \frac{12}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$      |
| AC) $\sum_{k=0}^{\infty} \frac{16}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$      | AD) $\sum_{k=0}^{\infty} \frac{16}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$      |
| AE) $1 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$   | BC) $1 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$   |
| BD) $2 + \sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$   | BE) $2 + \sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$   |
| CD) $3 + \sum_{k=0}^{\infty} \frac{12}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$  | CE) $3 + \sum_{k=0}^{\infty} \frac{12}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$  |
| DE) $4 + \sum_{k=0}^{\infty} \frac{16}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$  | ABC) $4 + \sum_{k=0}^{\infty} \frac{16}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$ |
| ABD) $5 + \sum_{k=0}^{\infty} \frac{20}{(2k+1)\pi} \sin\left(\frac{(2k+1)\pi}{2} x\right)$ | ABE) $5 + \sum_{k=0}^{\infty} \frac{20}{(2k+1)\pi} \cos\left(\frac{(2k+1)\pi}{2} x\right)$ |
- ABCDE) None of the above

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Consider the Partial Differential Equation (PDE):  $x u_{tt} = 2 t u_{xx}$

96. (4 pts.) Using the method of separation of variables with separation constant  $\lambda$  (or  $-\lambda$  if appropriate), one of the following sets of two Ordinary Differential Equations (ODE's) can be obtained from this PDE. Recall that the process does not yield a unique set of ODE's. (Note that we are looking for product solutions in the null space of the linear operator  $L[u] = x u_{tt} - t u_{xx}$ ). Following the advice given in class as to how to choose the separation constant (attendance is mandatory) we may obtain the set of

ODE's \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- |  |  |
|--|--|
| A) $X'' + \lambda x X = 0, \quad T'' + \lambda t T = 0$    | B) $X'' + \lambda X = 0, \quad T'' - \lambda T = 0$        |
| C) $X'' + \lambda x X = 0, \quad T'' + 2 \lambda t T = 0$  | D) $X'' + \lambda x X = 0, \quad T'' + \lambda T = 0$      |
| E) $X'' + \lambda x X = 0, \quad T'' + 3 \lambda t T = 0$  | AB) $X'' + \lambda t X = 0, \quad T'' + \lambda x T = 0$   |
| AC) $X'' + \lambda x X = 0, \quad T'' + 4 \lambda t T = 0$ | AD) $t X'' + \lambda X = 0, \quad x T'' + \lambda x T = 0$ |
| AE) $x X'' + \lambda X = 0, \quad T'' + \lambda T = 0$     | BC) $X'' + \lambda X = 0, \quad t T'' + \lambda T = 0$     |
| BD) $X'' + \lambda x X = 0, \quad T'' + \lambda T = 0$     | BE) $X'' + \lambda X = 0, \quad T'' + \lambda t T = 0$     |
- CD) Separation of variables does not work on this PDE.  
 CE) Separation of variables works on this PDE, but none of the above is correct.  
 ABCDE) None of the above

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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Recall that we have established  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  as a function space where we can calculate Fourier Series. Now let  $PC_{fs,o}^1(\mathbf{R}, \mathbf{R}; \ell)$  be the subspace of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  containing only odd functions,  $PC_{fs}^1([0, \ell], \mathbf{R})$  be the functions in  $PC_{fs,o}^1(\mathbf{R}, \mathbf{R}; \ell)$  with their domains restricted to  $[0, \ell]$ , and  $PC_{fss}^1([0, \ell], \mathbf{R})$  be the subspace of  $PC_{fs}^1([0, \ell], \mathbf{R})$  for which the Fourier (sine) series is finite. From class  $B_{fss}(\ell) = \{\sin(n\pi / \ell) : n \in \mathbf{N}\}$  is a Hamel basis of  $PC_{fss}^1([0, \ell], \mathbf{R})$  and a Schauder basis of  $PC_{fss}^1([0, \ell], \mathbf{R})$ . We view  $PC_{fss}^1([0, \ell], \mathbf{R})$  as the state space for the problem of heat conduction in a rod. It contains the functions that are the temperatures in the rod. The problem of heat conduction in a rod is:

$$\begin{array}{lll} \text{PDE} & u_t = \alpha^2 u_{xx} & 0 < x < \ell, \quad t > 0 \\ \text{BC} & u(0,t) = 0, \quad u(\ell,t) = 0, & t > 0 \\ \text{IC} & u(x,0) = u_0(x) & 0 < x < \ell \end{array}$$

To formulate the problem of heat conduction in a rod as a linear mapping problem we first let  $D = (0, \ell) \times (0, \infty)$  and  $\bar{D} = [0, \ell] \times [0, \infty)$ . Next we let  $\mathcal{A}_{HC}(\bar{D}, \mathbf{R})$  be the set of functions in  $\mathcal{F}(\bar{D}, \mathbf{R})$  whose restriction to  $D$  is analytic, whose restriction to  $[0, \ell] \times (0, \infty)$  is continuous, and whose restriction to  $[0, \ell] \times \{0\}$  is in  $PC_{fss}^1([0, \ell], \mathbf{R})$  with the additional condition that the function be continuous at all points in  $[0, \ell] \times \{0\}$  where its restriction to  $[0, \ell] \times \{0\}$  is continuous. Thus

$\mathcal{A}_{HC}(\bar{D}, \mathbf{R})$  is the set where we look for solutions to the pde in  $D$ . (These functions are “nice” at the boundary of  $D$ .) Now let  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}) = \{u(x,t) \in \mathcal{A}_{HC}(\bar{D}, \mathbf{R}) : u(0,t) = 0 \text{ and } u(\ell,t) = 0 \text{ for } t > 0\}$ . Then  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R})$  is the set where we look for solutions of the pde in  $D$  that also satisfy the BC’s. (These functions are “nice” on  $[0, \ell] \times \{0\}$  where  $t = 0$ .)

Now let  $L_B : \mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}) \rightarrow \mathcal{A}(D, \mathbf{R})$  be defined by  $L_B[u] = u_t - \alpha^2 u_{xx}$ . Thus the BC’s are incorporated into the domain of the linear operator  $L_B$ . Now let  $N_{L_B}(\alpha^2, \ell)$  be the null space of  $L_B$  so that if  $u(x,t) \in N_{L_B}(\alpha^2, \ell)$ , then it satisfies the BC’s as well as the pde. Now let

$$\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$$

$$\begin{array}{lll} \text{PDE} & u_t = \alpha^2 u_{xx} & 0 < x < \ell, \quad t > 0 \\ \text{BC} & u(0,t) = 0, \quad u(\ell,t) = 0, & t > 0. \end{array}$$

Hence the solution set for  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$  is the null space of  $L_B$  which we have denoted by  $N_{L_B}(\alpha^2, \ell)$ . These are the functions in  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R})$  that satisfy the pde as well as the BC’s. Also, if

$u(x,t) \in N_{L_B}(\alpha^2, \ell)$ , then its restriction to  $[0, \ell] \times \{0\}$  is in  $PC_{fs,o}^1([0, \ell], \mathbf{R})$ . Now let  $N_{L_{Bfs}}(\alpha^2, \ell) = \{u(x,t) \in N_{L_B}(\alpha^2, \ell) : u(x,0) \in PC_{fss}^1([0, \ell], \mathbf{R}; \ell)\}$  so that if  $u(x,t) \in N_{L_{Bfs}}(\alpha^2, \ell)$ , its restriction to  $[0, \ell] \times \{0\}$  is in  $PC_{fss}^1([0, \ell], \mathbf{R})$ . Now let  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  be the heat conduction in a rod problem as defined above. From class, if  $u_0(x) \in PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$ , then the solution of  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  is in  $N_{L_{Bfs}}(\alpha^2, \ell)$ . If  $u_0(x) \in PC_{fs,o}^1([0, \ell], \mathbf{R}; \ell)$ , then the solution of  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  is in  $N_{L_{Bfs}}(\alpha^2, \ell)$ . From class,  $B_{N_{L_B}}(\alpha^2, \ell) = \{e^{-(\alpha n \pi / \ell)^2 t} \sin(n\pi / \ell) : n \in \mathbf{N}\}$  is a Hamel basis for  $N_{L_{Bfs}}(\alpha^2, \ell)$  and a Schauder basis for  $N_{L_{Bfs}}(\alpha^2, \ell)$ .

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Let  $L_B$ ,  $N_{L_{Bis}(\alpha^2, \ell)}$ ,  $N_{L_B(\alpha^2, \ell)}$ ,  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$ ,  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x, 0) = u_0(x); \alpha^2, \ell)$ , all of the function spaces, and all of the basis sets be as on the previous page.

97. (1pt.) The set (which may be thought of as a subset of  $L^2([0, \ell], \mathbf{R})$ ) that we (attendance is mandatory) considered to be the state space for heat conduction in a rod

is \_\_\_\_\_ A B C D E

98. (1pt.) A Schauder basis for the state space for heat conduction in a rod may be taken to

be \_\_\_\_\_ A B C D E

99. (1pt.) A Schauder basis for  $N_{L_{Bis}(\alpha^2, \ell)}$ , the null space of  $L_B$ ,

is \_\_\_\_\_ A B C D E

Possible answers for 97-99

A)  $\mathbf{R}$  B)  $D$  C)  $\bar{D}$  D)  $\mathcal{A}(D, \mathbf{R})$  E)  $\mathcal{A}_{HC}(\bar{D}, \mathbf{R})$  AB)  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R})$  AC)  $PC_{\text{fiss}}^1([0, \ell], \mathbf{R}; \ell)$

AD)  $PC_{\text{fiss}}^1([0, \ell], \mathbf{R}; \ell)$  AE)  $N_{L_{Bis}(\alpha^2, \ell)}$  BC)  $N_B(\alpha^2, \ell)$  BD)  $B_{\text{is}}(\ell) = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}x) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}x) : n \in \mathbf{N}\}$

BC)  $B_{\text{iss}}(\ell) = \{\sin(\frac{n\pi}{\ell}x) : n \in \mathbf{N}\}$  BD)  $B_{N_{L_B}}(\alpha^2, \ell) = \{e^{-\frac{(\alpha n \pi)^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x) : n \in \mathbf{N}\}$

ABCDE) None of the above

100. (2 pts.) The "general" or formal solution of the problem of heat conduction in a rod,  $\text{Prob}_{HC}(\mathcal{A}_{HC,BC}(\bar{D}, \mathbf{R}), L_B(x, t) = 0; \alpha^2, \ell)$ , is given by

$u(x, t) =$  \_\_\_\_\_ A B C D E

A)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{\ell}{n\pi}x)$  B)  $\sum_{n=1}^N c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$  C)  $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell}x)$

D)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$  E)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(n\pi \ell x)$  AB)  $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$

AC)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \cos(\frac{n\pi}{\ell}x)$  AD)  $\sum_{n=1}^{\infty} c_n \cos(\frac{n\pi}{\ell}x)$  ABCDE) None of the above.

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Let  $L_B, N_{L_{BS}(\alpha^2, \ell)}, N_{L_{BS}(\alpha^2, \ell)}, \text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell), \text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}),$

$L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$ , all of the function spaces, and all of the basis sets be as on the page before the previous page.

101. (2 pts.) The "general" or formal solution of PDE  $u_t = u_{xx} \quad 0 < x < 2, \quad t > 0$   
 BC  $u(0,t) = 0, \quad u(2,t) = 0, \quad t > 0$

(we denote this problem by  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; 1, 2)$ )

is given by  $u(x,t) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4} t} \sin(\frac{2}{n\pi} x)$     B)  $\sum_{n=1}^N c_n e^{-\frac{n^2 \pi^2}{4} t} \sin(\frac{n\pi}{2} x)$     C)  $\sum_{n=1}^{\infty} c_n e^{-4n^2 \pi^2 t} \sin(\frac{n\pi}{2} x)$

D)  $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4} t} \sin(\frac{n\pi}{2} x)$     E)  $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4} t} \sin(2n\pi x)$     AB)  $\sum_{n=1}^{\infty} c_n e^{\frac{n^2 \pi^2}{4} t} \sin(\frac{n\pi}{2} x)$

AC)  $\sum_{n=1}^{\infty} c_n e^{-n^2 \pi^2 t} \sin(\frac{n\pi}{2} x)$     AD)  $\sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2} x) + b_n \cos(\frac{n\pi}{2} x)$     ABCDE) None of the above.

102. (4 pts.) The solution of

BVP for a PDE PDE  $u_t = u_{xx} \quad 0 < x < 2, \quad t > 0$   
 BC  $u(0,t) = 0, \quad u(2,t) = 0, \quad t > 0$   
 IC  $u(x,0) = a \sin(6\pi x) \quad 0 < x < 2$

(we denote this problem by  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0, 6 \sin(6\pi x); 1, 2)$ )

is given by  $u(x,t) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\sum_{n=1}^{\infty} e^{-\frac{36 \pi^2}{4} t} \sin(\frac{n\pi}{2} x)$     B)  $e^{-36 \pi^2 t} \sin(6\pi x)$     C)  $2 e^{-36 \pi^2 t} \sin(6\pi x)$     D)  $3 e^{-36 \pi^2 t} \sin(6\pi x)$

E)  $\sum_{n=1}^{\infty} 4 e^{-36 \pi^2 t} \sin(6\pi x)$     AB)  $6 e^{-36 \pi^2 t} \sin(6\pi x)$     AC)  $6 e^{-12 \pi^2 t} \sin(3\pi x)$     AD)  $6 e^{-144 \pi^2 t} \sin(6\pi x)$

ABCDE) None of the above.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Let  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  be the previously defined problem:

PDE  $u_t = \alpha^2 u_{xx} \quad 0 < x < \ell, \quad t > 0$

BC  $u(0,t) = 0, \quad u(\ell,t) = 0, \quad t > 0$

IC  $u(x,0) = u_0(x) \quad 0 < x < \ell$

103. ( 2 pts.) The formula for the solution of  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$

is given by  $u(x,t) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin\left(\frac{\ell}{n\pi} x\right)$     B)  $\sum_{n=1}^N c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin\left(\frac{n\pi}{\ell} x\right)$     C)  $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin\left(\frac{n\pi}{\ell} x\right)$

D)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin\left(\frac{n\pi}{\ell} x\right)$     E)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(n\pi \ell x)$     AB)  $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin\left(\frac{n\pi}{\ell} x\right)$

AC)  $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \cos\left(\frac{n\pi}{\ell} x\right)$     AD)  $\sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{\ell} x\right)$     AE) None of the above

104. (2 pts.) where the formula for  $c_n$  is  $c_n =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

A)  $\frac{2}{\ell} \int_0^{\ell} u_0(x) \sin\left(\frac{\ell}{n\pi} x\right) dx$     B)  $\frac{1}{\ell} \int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$     C)  $\frac{2}{\ell} \int_{-\ell}^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$     D)  $\frac{2}{\ell} \int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$

E)  $\frac{2}{\ell} \int_0^{\ell} u_0(x) \sin(n\pi \ell x) dx$     AB)  $\int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$     AC)  $\frac{1}{\ell} \int_{-\ell}^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$     AD)  $\frac{2}{\ell} \int_0^{\ell} u_0(x) \cos\left(\frac{n\pi}{\ell} x\right) dx$

ABCDE) None of the above.



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Let  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 1 ; 1, 2)$  be the problem defined by

$$\text{PDE} \quad u_t = u_{xx} \quad 0 < x < 2, \quad t > 0$$

$$\text{BC} \quad u(0,t) = 0, \quad u(2,t) = 0, \quad t > 0$$

$$\text{IC} \quad u(x,0) = 1 \quad 0 < x < 2$$

105. (2 pts.) The formula for the solution of  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 1 ; 1, 2)$  is given by

$u(x,t) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

$$\text{A) } \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin\left(\frac{2}{n\pi}x\right) \quad \text{B) } \sum_{n=1}^N c_n e^{-\frac{n^2\pi^2}{4}t} \sin\left(\frac{n\pi}{2}x\right) \quad \text{C) } \sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2t} \sin\left(\frac{n\pi}{2}x\right)$$

$$\text{D) } \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin\left(\frac{n\pi}{2}x\right) \quad \text{E) } \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(2n\pi x) \quad \text{AB) } \sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t} \sin\left(\frac{n\pi}{2}x\right)$$

$$\text{AC) } \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \cos\left(\frac{n\pi}{2}x\right) \quad \text{AD) } \sum_{n=1}^{\infty} c_n \sin\left(\frac{n\pi}{2}x\right) \quad \text{ABCDE) None of the above.}$$

106. (2 pts.) where the formula for  $c_n$  is  $c_n =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

$$\text{A) } \int_0^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{B) } \int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{C) } 2 \int_0^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{D) } 2 \int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right) dx$$

$$\text{E) } 3 \int_0^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{AB) } 3 \int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{AC) } 4 \int_0^2 \sin\left(\frac{n\pi}{2}x\right) dx \quad \text{AD) } 4 \int_{-2}^2 \cos\left(\frac{n\pi}{2}x\right) dx$$

ABCDE) None of the above.

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Let  $\text{Prob}_{\text{HC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 1; 1, 2)$  be as on the previous page.

107. (2pts.) Computing  $c_n$  using the formula on the previous page, for  $n$  odd ( $n = 1, 3, 5, \dots$ ) we

- have for  $k = 0, 1, 2, 3, \dots$  that  $c_{2k+1} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E
- A) 0 B)  $1/[(2k+1)\pi]$  C)  $2/[(2k+1)\pi]$  D)  $4/[(2k+1)\pi]$  E)  $8/[(2k+1)\pi]$  AB)  $12/[(2k+1)\pi]$   
 AC)  $16/[(2k+1)\pi]$  AD)  $32/[(2k+1)\pi]$  AE)  $64/[(2k+1)\pi]$  BC)  $-1/(2k+1)$  BD)  $-2/(2k+1)$   
 BE)  $-3/(2k+1)$  CD)  $-4/(2k+1)$  CE)  $-1/[(2k+1)\pi]$  DE)  $-2/[(2k+1)\pi]$  ABC)  $-3/[(2k+1)\pi]$   
 ABD)  $-4/[(2k+1)\pi]$  ABE)  $-8/[(2k+1)\pi]$  ABCDE) None of the above

108. (2 pts.) For  $c_n$  with  $n$  even ( $n = 2, 4, 6, \dots$ ) we have for  $k = 1, 2, 3, \dots$  that

- $c_{2k} =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E
- A) 0 B)  $1/[(2k+1)\pi]$  C)  $2/[(2k+1)\pi]$  D)  $3/[(2k+1)\pi]$  E)  $4/[(2k+1)\pi]$  AB)  $12/[(2k+1)\pi]$  AC)  $16/[(2k+1)\pi]$   
 AD)  $32/[(2k+1)\pi]$  AE)  $64/[(2k+1)\pi]$  BC)  $-1/(2k+1)$  BD)  $-2/(2k+1)$  BE)  $-3/(2k+1)$   
 CD)  $-4/(2k+1)$  CE)  $-1/[(2k+1)\pi]$  DE)  $-2/[(2k+1)\pi]$  ABC)  $-3/[(2k+1)\pi]$   
 ABD)  $-4/[(2k+1)\pi]$  ABE)  $-8/[(2k+1)\pi]$  ABCDE) None of the above

109. (2 pts.) Hence the solution of  $\text{Prob}_D(\bar{A}_{\text{HC},0}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 1; 1, 2)$  may be written

as  $u(x,t) =$  \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- A)  $\sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{2} x\right)$  B)  $\sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{2} x\right)$   
 C)  $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{2} x\right)$  D)  $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{2} x\right)$   
 E)  $\sum_{k=0}^{\infty} \frac{12}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{2} x\right)$  AB)  $\sum_{k=0}^{\infty} \frac{16}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{4} x\right)$   
 AC)  $\sum_{k=0}^{\infty} \frac{32}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \sin\left(\frac{(2k+1)\pi}{4} x\right)$  AD)  $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \cos\left(\frac{(2k+1)\pi}{2} x\right)$   
 AE)  $\sum_{n=1}^{\infty} \frac{8}{(2k)\pi} e^{-\frac{(2k+1)^2 \pi^2}{4} t} \cos\left(\frac{(2k)\pi}{2} x\right)$  BC)  $\sum_{k=1}^{\infty} \frac{4}{k\pi} e^{-\frac{k^2 \pi^2}{4} t} \sin\left(\frac{k\pi}{2} x\right)$  BD)  $\sum_{k=1}^{\infty} \frac{2}{k\pi} e^{-\frac{k^2 \pi^2}{2} t} \sin\left(\frac{k\pi}{2} x\right)$

ABCDE) None of the above.

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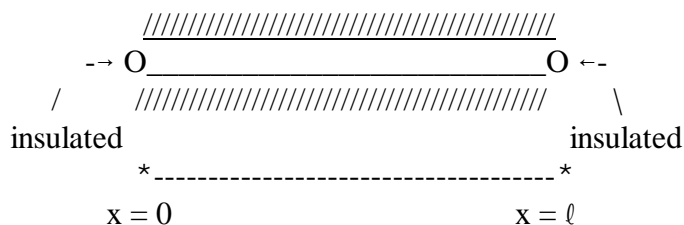
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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer

110. ( 3 pts.) If both ends of a rod are insulated (recall that we assume that the lateral sides are also insulated so that the temperature does not vary over a cross section), then a good mathematical model of the physical heat conduction problem is given by:

	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell, \quad t > 0$
BVP for a PDE	BC	$u_x(0,t) = 0, \quad u_x(\ell,t) = 0,$	$t > 0$
	IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

where  $u_0(x)$  is the initial temperature distribution in the rod.



Applying the separation of variables process to the PDE results in the two ODE's:

1.  $X'' + \lambda X = 0$
2.  $T' + \alpha^2 \lambda T = 0$

where  $\lambda$  is the separation constant. The spacial eigenvalue problem that results from

applying the BC given above is \_\_\_\_\_ . \_\_\_\_\_ A B C D E

- |  |   |  |
|--|---|--|
| A) $X'' + \lambda X = 0$<br>$X(0) = 0, X(\ell) = 0$  | B) $X'' + \lambda X = 0$<br>$X'(0) = 0, X(\ell) = 0$  | C.) $X'' + \lambda X = 0$<br>$X(0) = 0, X'(\ell) = 0$  |
| D) $X'' + \lambda X = 0$<br>$X(0) = 1, X(\ell) = 0$  | E) $X'' + \lambda X = 0$<br>$X'(0) = 0, X'(\ell) = 0$ | AB) $X'' + \lambda X = 0$<br>$X'(0) = 1, X'(\ell) = 0$ |
| AC) $X'' + \lambda X = 0$<br>$X(0) = 1, X(\ell) = 1$ | AD) $X'' + \lambda X = 0$<br>$X'(0) = 1, X(\ell) = 0$ | AE) $X'' + \lambda X = 0$<br>$X(0) = 0, X'(\ell) = 0$  |
- BC) None of the above.

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TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$ )))))))))	$F(s) = \mathcal{L}\{f(t)\}$ )))))))))	Domain $F(s)$ )))))))))
$t^n \quad n = \text{positive integer}$	$\frac{n!}{s^{n+1}}$	$s > 0$
$\sinh(at)$	$\frac{1}{s^2 - a^2}$	$s >  a $
$\cosh(at)$	$\frac{s}{s^2 - a^2}$	$s >  a $
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at} \quad n = \text{positive integer}$	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u(t)$	$\frac{1}{s}$	$s > 0$
$u(t - c)$	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	$F(s - c)$	
$f(ct) \quad c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t - c)$	$e^{-cs}$	

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## PARTIAL TABLE OF ANTIDERIVATIVES

$$1. \int x[\sin(ax)]dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + c$$

$$2. \int x[\cos(ax)]dx = \frac{1}{a^2}\cos(ax) - \frac{x}{a}\sin(ax) + c$$

$$3. \int x^2[\sin(ax)]dx = \frac{2x}{a^2}\sin(ax) - \frac{a^2x^2 - 2}{a^3}\cos(ax) + c$$

$$4. \int x^2[\cos(ax)]dx = \frac{2x}{a^2}\cos(ax) - \frac{a^2x^2 - 2}{a^3}\sin(ax) + c$$

$$5. \int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$$

$$6. \int \cos^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$$

$$7. \int [\sin(ax)][\cos(ax)]dx = \frac{1}{2a}\sin^2(ax) + c$$

$$8. \int [\sin(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$$

$$9. \int [\cos(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$$

$$10. \int [\sin(ax)][\cos(bx)]dx = -\frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$$

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### SETS AND SPACES FOR FOURIER SERIES AND THE HEAT CONDUCTION PROBLEM

- $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell) = \{f \in \mathcal{A}(\mathbf{R}, \mathbf{R}) : f \text{ is periodic of period } 2\ell, f \text{ and } f' \text{ are piecewise continuous on } ]\ell, \ell], \text{ and } f(x) = (f(x^-) + f(x^+))/2 \text{ at points of discontinuity}\}$ .
- $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  is the subspace of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  and  $PC_{fss}^1(\mathbf{R}, \mathbf{R}; \ell)$  are inner product spaces with inner product  $(f, g) = \int_{-\ell}^{\ell} f(x)g(x)dx$ . However, they are not Hilbert spaces.
- $B_{fs} = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$  is an orthogonal Schauder basis for  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$ , and an orthogonal Hamel basis of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$ .
- $PC_{fs,o}^1(\mathbf{R}, \mathbf{R}; \ell)$  is the subspace of  $PC_{fs}^1(\mathbf{R}, \mathbf{R}; \ell)$  containing only odd functions.
- $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$  is the set of functions in  $PC_{fs,o}^1(\mathbf{R}, \mathbf{R}; \ell)$  with their domains restricted to  $[0, \ell]$ . This space of Fourier Sine Series is the state space for the heat conduction problem.
- $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$  is the subspace of  $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$  for which the Fourier sine series is finite.
- $B_{fss} = \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$  is an orthogonal Schauder basis for  $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$  and an orthogonal Hamel basis of  $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$ .
- Consider the heat conduction in a rod problem defined by

$$\begin{array}{lll} \text{PDE} & u_t = \alpha^2 u_{xx} & 0 < x < \ell, \quad t > 0 \\ \text{BC} & u(0, t) = 0, \quad u(\ell, t) = 0, & t > 0 \\ \text{IC} & u(x, 0) = u_0(x) & 0 < x < \ell. \end{array}$$

Recall that  $PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$  is the **state space** for this problem. To formulate this problem as a linear mapping problem we also need the  $\Sigma$  space. Let  $D = (0, \ell) \times (0, \infty)$ ,  $\bar{D} = [0, \ell] \times (0, \infty)$ ,  $\bar{D} = [0, \ell] \times [0, \infty)$ ,  $\mathcal{A}_{HC}(\bar{D}, \mathbf{R}, \ell) = \{u(x, t) \in \mathcal{F}(\bar{D}, \mathbf{R}) : u \in \mathcal{A}(D, \mathbf{R}) \cap C(\bar{D}, \mathbf{R}), u(0, t) = 0, u(\ell, t) = 0 \text{ for } t > 0, u(x, 0) \in PC_{fss}^1([0, \ell], \mathbf{R}; \ell) \text{ and } \forall x \in (0, \ell) \text{ we have that } u(x, t) \text{ is continuous at } t = 0 \text{ except where } u(x, 0) \text{ is discontinuous}\}$ . Thus a function  $u(x, t) \in \mathcal{A}_{HC}(\bar{D}, \mathbf{R}, \ell)$  is analytic in  $D$ , continuous on the boundary of  $\bar{D}$  except where  $u(x, 0)$  is discontinuous, satisfies the boundary conditions  $u(0, t) = 0, u(\ell, t) = 0$  for  $t > 0$ , and  $u(x, 0) \in PC_{fss}^1([0, \ell], \mathbf{R}; \ell)$ . Thus  $\mathcal{A}_{HC}(\bar{D}, \mathbf{R})$  is the set where we look for solutions to the pde in  $D$ . Thus, we take  $\mathcal{A}_{HC}(\bar{D}, \mathbf{R})$  as the  **$\Sigma$  space** for our problem. These functions are “nice” at the boundary of  $D$ .

- Now let  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}) = \{u(x, t) \in \mathcal{A}_{HC}(\bar{D}, \mathbf{R}) : u(0, t) = 0 \text{ and } u(\ell, t) = 0 \text{ for } t > 0\}$ . Then  $\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R})$  is the set where we look for solutions of the pde in  $D$  that also satisfy the BC's. These functions are “nice” on  $[0, \ell] \times \{0\}$  where  $t = 0$ . Now let  $L_B : \mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}) \rightarrow \mathcal{A}(D, \mathbf{R})$  be defined by  $L_B[u] = u_t - \alpha^2 u_{xx}$ . Thus the BC's are incorporated into the domain of the linear operator  $L_B$ . Now let  $N_{L_B}(\alpha^2, \ell)$  be the null space of  $L_B$  so that if  $u(x, t) \in N_{L_B}(\alpha^2, \ell)$ , then it satisfies the BC's as well as the pde. Now let  $\text{Prob}_{HC}(\mathcal{A}_{HCBC}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$  be the problem defined by

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$$\begin{array}{ll} \text{PDE} & u_t = \alpha^2 u_{xx} \quad 0 < x < \ell, \quad t > 0 \\ \text{BC} & u(0,t) = 0, \quad u(\ell,t) = 0, \quad t > 0. \end{array}$$

Hence the solution set for  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$  is the null space of  $L_B$  which we have denoted by  $N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ . These are the functions in  $\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R})$  that satisfy the pde as well as the BC's. Also, if  $u(x,t) \in N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ , then its restriction to  $[0, \ell] \times \{0\}$  is in

$PC_{\text{fss},o}^1([0, \ell], \mathbf{R})$ . Now let  $N_{L_{\text{Bfss}}}(\alpha^2, \ell) = \{u(x,t) \in N_{L_{\text{Bfss}}}(\alpha^2, \ell) : u(x,0) \in PC_{\text{fss}}^1([0, \ell], \mathbf{R}; \ell)\}$  so that if  $u(x,t) \in N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ , its restriction to  $[0, \ell] \times \{0\}$  is in  $PC_{\text{fss}}^1([0, \ell], \mathbf{R})$ . Now let

$\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  be the heat conduction in a rod problem as defined in 8 above. From class, if  $u_0(x) \in PC_{\text{fss}}^1([0, \ell], \mathbf{R}; \ell)$ , then the solution of

$\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  is in  $N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ . If  $u_0(x) \in$

$PC_{\text{fss},o}^1([0, \ell], \mathbf{R}; \ell)$ , then the solution of  $\text{Prob}_{\text{HC}}(\mathcal{A}_{\text{HCBC}}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$  is in  $N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ .

10. From class,  $B_{N_{L_B}}(\alpha^2, \ell) = \{e^{-(\alpha n \pi / \ell)^2 t} \sin(n\pi / \ell) : n \in \mathbf{N}\}$  is a Hamel basis for  $N_{L_{\text{Bfss}}}(\alpha^2, \ell)$  and a Schauder basis for  $N_{L_{\text{Bfss}}}(\alpha^2, \ell)$ .