MATH 261 | MATH 261: Elementary Differential Equations | MATH 261
FALL 2010 | FINAL EXAM | FALL 2010
FINAL EXAM -A2 | EXAMINATION COVER PAGE | Professor Moseley

PRINT NAME____ (______________________)
Last Name, First Name MI (What you wish to be called)

ID # ____________________________________ EXAM DATE_______________________

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

______________________________ ______________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 34 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. Page 35 contains Laplace transforms you need not memorize. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! Ask for scratch paper if you need it. You may remove the staple. Print your name on all sheets. Pages 1-34 are Fill-in-the Blank/Multiple Choice or True/False. Expect no partial credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Your thoughts should be expressed in your best mathematics on this paper. Proofread as time allows. GOOD LUCK!!

Scores

page	points	score
1	20	
2	7	
3	6	
4	8	
5	6	
6	5	
7	5	
8	9	
ST1	66	

Scores

page	points	score
9	3	
10	10	
11	7	
12	2	
13	12	
14	4	
15	9	
16	9	
ST2	56	

Scores

page	points	score
17	6	
18	5	
19	4	
20	5	
21	11	
22	6	
23	3	
24	4	
ST3	44	

Scores

page	points	score
25	9	
26	6	
27	4	
28	-	
29	5	
30	6	
31	4	
32	4	
33	6	
34	3	
35	----	
ST4	47	
ST1	66	
ST2	56	
ST3	44	
ST4	47	
Tot.	213	

PRINT NAME _________________________(_________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. You are to classify the first order ordinary differential equations given below. The classification relates to the method of solution. Recall from class (attendance is mandatory) the possible methods listed below. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is wrong. DO NOT SOLVE. Also recall the following:

a. In this context, exact means exact as given (in either of the forms discussed in class).

b. Bernoulli is not a correct method of solution if the original equation is linear.

c. Homogeneous (use the substitution v = y/x) is not a correct method of solution if it converts a separable equation into another separable equation.

1. (4 pts.) $xye^{x+y}\,dx + dy = 0$ _____ A B C D E

2.(4 pts.) $(y^2 + xy)dx + x\,dy = 0$ _____ A B C D E

3. (4 pts.) $(e^x + 2xy + x)dx + (x^2 + 2y)dy = 0$ _____ A B C D E

4. (4 pts.) $(xy + \cos(x))dx + (1 + x^2)\,dy = 0$ _____ A B C D E

5. (4 pts.) $(y^2 + x^2)dx + x^2\,dy = 0$ _____ A B C D E

Possible answers this page.
A) First order linear (y as a function of x). B) First order linear (x as a function of y).
C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).
E) Bernoulli, but not linear (y as a function of x).
AB) Bernoulli, but not linear (x as a function of y)
AC) Homogeneous, but not separable. AD) None of the above techniques works.
Total points this page = 20. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _______________________(_________________) ID No. ___________________
Last Name, First Name MI What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

Consider the first order linear ODE $y' = y + x$ (*). To solve (*), you may need to change it to a standard form.

6. (1 pts.) The correct standard form for (*) is ________________________. _____ A B C D E
A) $y' + y = x$ B) $y' + y = -x$ C) $y' - y = x$ D) $y' - y = -x$ E) $y' + 2y = x$ AB) $y' + 2y = -x$ AC) $y' - 2y = x$ AD) $y' - 2y = -x$ AE) $y' + y = 2x$ BC) $y' + y = -2x$ BD) $y' - y = 2x$ BE) $y' - y = -2x$ CD) $y' + 2y = 2x$ CE) $y' + 2y = -2x$ DE) $y' - 2y = 2x$ ABC) $y' - 2y = -2x$ ABD) None of the above

7. (2 pts.) An integrating factor for (*) is μ=_______________________. ______ A B C D E
A) x B) $-x$ C) x^2 D) $-x^2$ E) $2x$ AB) $-2x$ AC) $2x^2$ AD) $-2x^2$ AE) e^x AD) e^{-x} AE) e^{2x} BC) e^{-2x} BD) e^{x^2} BE) e^{-x^2} CD) None of the above

8. (3 pts.) In solving (*) as we did in class (attendance is mandatory), the following step occurs:

____. ______ A B C D E

A) $\frac{d(ye^x)}{dx} = xe^x$ B) $\frac{d(ye^x)}{dx} = -xe^x$ C) $\frac{d(ye^x)}{dx} = 2xe^x$ E) $\frac{d(ye^x)}{dx} = -2xe^x$ AB) $\frac{d(ye^{-x})}{dx} = xe^{-x}$ AC) $\frac{d(ye^{-x})}{dx} = -xe^{-x}$
AD) $\frac{d(ye^{-x})}{dx} = 2xe^{-x}$ AE) $\frac{d(ye^{-x})}{dx} = -2xe^{-x}$ BC) $\frac{d(ye^{2x})}{dx} = xe^{2x}$ BD) $\frac{d(ye^{2x})}{dx} = -xe^{2x}$ BE) $\frac{d(ye^{2x})}{dx} = 2xe^{2x}$
CD) $\frac{d(ye^{2x})}{dx} = -2xe^{2x}$ CE) $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$ DE) $\frac{d(ye^{-2x})}{dx} = -xe^{-2x}$ ABC) $\frac{d(ye^{-2x})}{dx} = 2xe^{-2x}$ ABD) $\frac{d(ye^{-2x})}{dx} = -2xe^{-2x}$
ABE) None of the above steps ever appears in any solution of this problem.

9. (1pt.) Let (**) be the initial value problem consisting of (*) and the initial condition $y(0) = 0$.

The number of solutions to (**) is _____________. _____A B C D E A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Possible points this page = 7. TOTAL POINTS EARNED _________

PRINT NAME ________________________(__________________) ID No. ________________

Last Name, First Name MI What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

An ODE may be considered to be a "vector" equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE, call it (*), of the form L[y] = g(x) where L is of the form L[y] = y′ + p(x)y, an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^x)}{dx} = -xe^x$, call it (**). If a problem has an infinite number of solutions, the form of the solution is not unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

10. (2 pts.) The theorem from calculus that allows you to integrate the Left Hand Side of (**)

is ____. ____ A B C D E

A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem
D) Chain Rule E) Product Rule AB) Integration by Parts AC) Partial Fractions
AD) Fundamental Theorem of Calculus AE) None off the above

11. (4 pts.) The solution (or family of solutions) to the ODE (*) may be written

as____. _______ A B C D E

A) $y = x + 1 + c\, e^x$ B) $y = -x + 1 + c\, e^x$ C) $y = x - 1 + c\, e^x$ D) $y = -x - 1 + c\, e^x$
E) $y = x + 1 + c\, e^{-x}$ AB) $y = -x + 1 + c\, e^{-x}$ AC) $y = x - 1 + c\, e^{-x}$ AD) $y = -x - 1 + ce^{-x}$
AE) $y = 2x + 2 + ce^x$ BC) $y = -2x + 2 + ce^x$ BD) $y = 2x - 2 + ce^x$ BE) $y = -2x - 2 + c\, e^x$
CD) $y = 2x + 2 + ce^{-x}$ CE) $y = -2x + 2 + ce^{-x}$ DE) $y = 2x - 2 + ce^{-x}$ ABC) $y = -2x - 2 + ce^{-x}$
ABD) None of the above solutions or families of solutions is correct.

PRINT NAME ________________________(__________________) ID No. ________________
Last Name, First Name MI What you wish to be called

Let L, p(x), g(x), (*), and (**) be as on the previous page.

12. (1 pts.) The solution set for the ODE (*) on the previous page may be written as

S = ____. _______ A B C D E

A) $\{y = x + 1 + c\,e^{x}: c \in \mathbf{R}\}$ B) $\{y = -x + 1 + c\,e^{x} : c \in \mathbf{R}\}$ C) $\{y = x - 1 + c\,e^{x}: c \in \mathbf{R}\}$
D) $\{y = -x - 1 + c\,e^{x} : c \in \mathbf{R}\}$ E) $\{y = x + 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AB) $\{y = -x + 1 + c\,e^{-x}: c \in \mathbf{R}\}$
AC) $\{y = x - 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AD) $\{y = -x - 1 + ce^{-x}: c \in \mathbf{R}\}$ AE) $\{y = 2x + 2 + e^{x} + c : c \in \mathbf{R}\}$ BC) $\{y = -2x + 2 + ce^{x}: c \in \mathbf{R}\}$ BD) $\{y = 2x - 2 + ce^{x}: c \in \mathbf{R}\}$ BE) $\{y = -2x - 2 + ce^{x}: c \in \mathbf{R}\}$
CD) $\{y = 2x + 2 + ce^{-x}: c \in \mathbf{R}\}$ CE) $\{y = -2x + 2 + ce^{-x}: c \in \mathbf{R}\}$ DE) $\{y = 2x - 2 + ce^{-x}: c \in \mathbf{R}\}$
ABC) $\{y = -2x - 2 + ce^{-x}: c \in \mathbf{R}\}$ DE) None of the above correctly describe the solution set.

13. (1 pt.) A basis for the nullspace of L is B =________________________. ______A B C D E
A) $\{1\}$ B) $\{x\}$ C) $\{1,x\}$ D) $\{e^{x}\}$ E) $\{e^{-x}\}$ AB) $\{e^{x}, e^{-x}\}$ AC) None of the above

14. (1 pt.) The general solution of L[y] = 0 is $y_c(x)$ =________________. ______A B C D E
A) c B) cx C) $c_1 + c_2x$ D) ce^{x} E) ce^{-x} AB) $c_1e^{x} + c_2e^{-x}$ AC) None of the above

15. (1 pt.)) Using the linear theory, a particular solution of L[y] = g(x) is given by

$y_p(x)$ =________________________. ______A B C D E A) 1 B) x C) x + 1
D) x − 1 E) 1 − x AB) −x −1 AC) e^{x} AD) e^{-x} AE) None of the above

16. (1 pt.) The number of solutions to (*) is ________._____A B C D E A) 0 B) 1
C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

17. (2 pts.) Let (***) be the initial value problem consisting of (*) and the initial condition y(0) = 0. The solution (or family of solutions) to (***) may be written

as____. _______ A B C D E
A) $y = x + 1 - e^{x}$ B) $y = -x + 1 - e^{x}$ C) $y = x - 1 + e^{x}$ D) $y = x + 1 - e^{-x}$
E) $y = -x + 1 - e^{-x}$ AB) $y = x + 2 - 2e^{-x}$ AC) $y = x - 1 + e^{-x}$ AD) $y = x + 1 + e^{x} - 2 + c$
AE) $y = -x + 1 + e^{x} - 2$ BC) $y = x - 1 + e^{x}$ BD) $y = x + 1 + e^{-x} - 2$ BE) $y = -x + 1 + e^{-x} - 2$
CD) $y = x - 1 + e^{-x}$ CE) $y = x + 1 - e^{-x}$
DE)None of the above solutions or families of solutions is correct.

18. (1 pt.) The number of solutions to (***) is _________._____A B C D E A) 0 B) 1
C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Possible points this page = 8. TOTAL POINTS EARNED _________

PRINT NAME ________________________(________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let $A = \begin{bmatrix} 1 & i \\ -i & 1 \end{bmatrix}$, $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, and $\vec{b} = \begin{bmatrix} 1 \\ -i \end{bmatrix}$. Also let $T(\vec{x}) = A\vec{x}$ so that $T:\mathbf{C}^2 \rightarrow \mathbf{C}^2$. Now solve Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$); that is, solve the vector equation $A\vec{x} = \vec{b}$ (i.e. $T(\vec{x}) = \vec{b}$). The form of the answer may not be unique. If a problem has an infinite number of solutions, the form of the solution is not unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

19. (3 pts.) If $\left[A \middle| \vec{b} \right]$ is reduced to $\left[U \middle| \vec{c} \right]$ using Gauss elimination we obtain

$\left[U \middle| \vec{c} \right]$ = ______________________________. _____A B C D E A) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ B) $\left[\begin{array}{cc|c} 1 & i & -1 \\ 0 & 0 & 0 \end{array}\right]$

C) $\left[\begin{array}{cc|c} 1 & -i & 1 \\ 0 & 0 & 1 \end{array}\right]$ D) $\left[\begin{array}{cc|c} 1 & -i & -1 \\ 0 & 0 & 0 \end{array}\right]$ E) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$ AB) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ AC) None of the above.

20. (3 pts.) The solution of $A\vec{x} = \vec{b}$ may be written as

$\vec{x}$ = ________________________________._____A B C D E A) No Solution B) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ E) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AB) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AC) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AD) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

BC) None of the above correctly describes the solution or collection of solutions.

PRINT NAME _________________________(_________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Let $\text{Prob}(\mathbf{C}^2; A\vec{x} = \vec{b}), A, \vec{b}, \vec{x}$, and T be as on the previous page.

21. (1 pt.) The solution set for Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$) may be written as

S =______________________.______A B C D E A) $\varnothing$ B) C) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ $\left\{\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

D) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ E) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AB) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

AC) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AD) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

BC) None of the above correctly describes the solution set for this problem

22. (1 pt.) A basis for the null space of T is B =_________________________. ______A B C D E

A) $\varnothing$ B) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ C) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ D) $\left\{\begin{bmatrix} i \\ 1 \end{bmatrix}\right\}$ E) $\left\{\begin{bmatrix} i \\ -1 \end{bmatrix}\right\}$ AB) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ AC) $\left\{\begin{bmatrix} -i \\ -1 \end{bmatrix}\right\}$

AD) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ BC) None of the above correctly describes the solution set for this problem

23. (1 pt.) The general solution of $A\vec{x} = \vec{0}$ is $\vec{x}_c$ =____________________. ______A B C D E

A) No Solution B) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ C) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} i \\ 1 \end{bmatrix}$ E) $y\begin{bmatrix} i \\ -1 \end{bmatrix}$ AB) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

AC) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AD) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AE) None of the above

24. (1 pt.) Using the linear theory, a particular solution of $A\vec{x} = \vec{b}$ is given by

$\vec{x}_p$ =__________________________. ______A B C D E A) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ B) $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} i \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} i \\ -1 \end{bmatrix}$

E) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AB) $\begin{bmatrix} -i \\ -1 \end{bmatrix}$ AC) $\begin{bmatrix} 1 \\ i \end{bmatrix}$ AD) $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ AE) $\begin{bmatrix} -1 \\ i \end{bmatrix}$ BC) $\begin{bmatrix} -1 \\ -i \end{bmatrix}$ BD) $\begin{bmatrix} i \\ i \end{bmatrix}$

BE) $A\vec{x} = \vec{b}$ has solutions but none are listed CD) $A\vec{x} = \vec{b}$ has no solutions

25. (1 pt.) The number of solutions to Prob($\mathbf{C}^2$; $A\vec{x} = \vec{b}$) is __________.______A B C D E A) 0
B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a (real or complex) vector space V to a (real or complex) vector space W. Now consider the mapping problem defined by the vector equation

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

26. (1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has a solution.

27. (1 pt.) A)True or B)False The vector equation (*) may have exactly two solutions.

28.(1 pt.) A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} = \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where $c_1, c_2, \ldots, c_n$ are arbitrary constants.

29. (1 pt.) A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

30. (1 pt.) A)True or B)False If the null space of T is $N(T) = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has a unique solution.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. _________________

Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet.

The dimension of the null space N_L of the linear operator $L[y] = y'' + y'$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^{-x}$. Hence we can deduce that $B_{N_L} = \{1, e^{-x}\}$ is a basis of N_L so that

$y_c = c_1 + c_2\, e^{-x}$ is the general solution of $y'' + y' = 0$.

Use the method of undetermined coefficients as discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

31. (3 pts.) $y'' + y' = -2$ First guess: y_p = ________________________

Second guess (if needed): y_p = ___________________________

Third guess (if needed): y_p = ____________________________

Final guess ____ . _______ A B C D E

32.(3 pts.) $y'' + y' = 3 \cos x$ First guess: y_p = ________________________

Second guess (if needed): y_p = ___________________________

Third guess (if needed): y_p = ____________________________

Final guess ____ . ______ A B C D E

33. (3 pts.) $y'' + y' = -4e^x$ First guess: y_p = ________________________

Second guess (if needed): y_p = ___________________________

Third guess (if needed): y_p = ____________________________

Final guess ____ . _____ A B C D E

Possible Answers:

A) Ae^x B) Axe^x C) Ax^2e^x D) $Axe^x + Be^x$ E) $Ax^2e^x + Bxe^x$ AB) Ae^{-x} AC) Axe^{-x}
AD) Ax^2e^{-x} AE) $Axe^{-x} + Be^{-x}$ BC) $Ax^2e^{-x} + Bxe^{-x}$ BD) $A \sin x$ BE) $A\cos x$
CD) $A x \sin x$ CE) $A x \cos x$ DE) $A \sin x + B \cos x$ ABC) $A x \sin x + B x \cos x$
ABD) A ABE) Ax ACD) $Ax + B$ ACE) $Ax^2 + Bx$ ADE) $Ax^2 + Bx + C$
BCD) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(_________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Let Prob($\mathcal{A}$((−π/2,π/2),**R**), (*)) be the problem defined by the ODE

$$y'' + y = -\tan(x) \quad I = (-\pi/2, \pi/2) \qquad (*)$$

Let L: $\mathcal{A}((-\pi/2,\pi/2),\mathbf{R})\to\mathcal{A}((-\pi/2,\pi/2),\mathbf{R})$ be defined by L[y] = y'' + y. The general solution to L[y] = 0 is $y_c = c_1 \cos(x) + c_2 \sin(x)$. To obtain a particular solution of L[y] = tan(x) we let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. You are to find y_p. Be careful!! Remember, once you make a mistake, the rest is wrong.

34. (3 pts.) Substituting $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ into (*) and making the appropriate assumption you obtained the two equations:

____ . ____A B C D E

A) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$

B) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\tan(x)$

C) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

D) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\tan(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

E) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sec(x)$

AB) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\sec(x)$

AC) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \sec(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

AD) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\sec(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

AE) None of the above.

PRINT NAME _________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let Prob(A((−π/2,π/2),**R**), (*)), (*), L and $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be as defined on the previous page.

35. (2 pts.) Solving the set of equations for $u'_1(x)$ and $u'_2(x)$ on the previous page we obtain

$u'_1(x)$ =____. ____A B C D E

36. (2 pts.) And $u'_2(x)$ = ____. ____A B C D E

37. (2 pts.) Hence we may choose $u_1(x)$ =_______________________________. ____A B C D E

38. (2 pts.) And $u_2(x)$ =____. ____A B C D E

39 (2 pts.) Hence a particular solution to (*) is

$y_p(x)$ = ____. ___A B C D E

Possible answers this page.

A)0 B)1 C)−1 D) x E) −x AB) sin x AC) −sin x AD) cos x AE) −cos x BC) sin(x) cos(x)
BD) −sin(x) cos(x) BE) $\sin^2(x)/\cos(x)$ CD) − $\sin^2(x)/\cos(x)$ CD)ln(tan(x) + sec(x))
CE) −ln(tan(x) +sec(x)) DE)[sin(x)]ln(tan(x)+sec(x)) ABC)−[sin(x)]ln(tan(x)+sec(x))
ABD)[cos(x)]ln(tan(x)+sec(x)) ABE) −[cos(x)]ln(tan(x)+sec(x)) ACD) ln(sin(x))
ACE)−ln(sin(x)) ADE) ln(cos(x)) BCD) −ln(cos(x)) BCE) (cos(x)) [ln(sin(x)) + x sin(x)]
BDE) −(cos(x)) [ln(sin(x)) + x sin(x)] CDE) (cos(x)) [ln(cos(x)) + x sin(x)
ABCD) −(cos(x)) [ln(cos(x)) + x sin(x) ABCDE)None of the above.

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. If you miss one part, it may cause you to miss other parts.

Consider $y^{IV} - 4y''' + 4y'' = 0$ (*) Also let $L: A(\mathbf{R},\mathbf{R}) \to A(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$ and answer the following questions.

40. (1 pt). The dimension of the null space of L is ____________________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 ABCDE) None of the above.

41. (1 pts). The auxiliary equation for (*) is ______________________________ . ___ A B C D E
A) $r^4 + 4r^2 + 4 = 0$ B) $r^4 - 4r^2 + 4 = 0$ C) $r^4 + 4r^3 + 4r^2 = 0$ D) $r^4 - 4r^3 + 4r^2 = 0$
E) $r^4 + 4r^3 = 0$ AB) $r^4 - 4r^3 = 0$ AC) $r^6 + 4r^3 + 4r^2 = 0$ ABCDE) None of the above.

42. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation are

r = ________________________________. ______ A B C D E A) 0,0,2,2 B)0,0,−2,−2
C)0,0,2,−2 D) 0,0,2i,−2i E)0,0,2,2i AB) r =0,0,−2,−2i ABCDE) None of the above.

43. (1 pts). A basis for the null space of L is B = ____________________________. ___ A B C D E
A)$\{1,x,e^{2x},xe^{2x}\}$ B)$\{1,x,e^{-2x},xe^{-2x}\}$ C)$\{1,x,e^{2x},e^{-2x}\}$ D)$\{1,x,\sin 2x, \cos 2x\}$ E)$\{1,x,e^{2x}, \sin 2x\}$
AB)$\{1,x, x^2, e^{-2x}, \sin 2x\}$ AC) $\{1, x, x^2, e^{-2x}\}$ AD)$\{1, x, x^2, x^3\}$ AE) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$
ABCDE) None of the above

44. (2 pt). The general solution of (*) is y(x) = _____________________________ . ___ A B C D E
A)$c_1 + c_2 e^{2x} + c_3 xe^{2x}$ B)$c_1 + c_2 e^{-2x} + c_3 xe^{-2x}$ C)$c_1 + c_2 x + c_3 e^{2x} + c_4 e^{-2x}$
D) $c_1 + c_2x + c_3 \sin 2x + c_4 \cos 2x$ E)$c_1+c_2x + c_3x^2 + c_4 e^{2x}$ AB)$c_1+c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{2x}$
AC) $c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $c_1+c_2 x + c_3 x^2 + c_4 x^3$ AE) $c_1e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$
ABCDE)None of the above

Points this page = 7. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME ________________________(________________) ID No. _________________

Last Name, First Name MI, What you wish to be called

Let (*) and L be as on the previous page.

45. (1pt.) The solution set for (*) may be written as

S = ____. ______A B C D E

A) $\{y(x) = c_1 + c_2\, e^{2x} + c_3\, xe^{2x}\!:\ c_1, c_2, c_3 \in \mathbf{R}\}$

B) $\{y(x) = c_1 + c_2\, e^{-2x} + c_3\, xe^{-2x}\!:\ c_1, c_2, c_3 \in \mathbf{R}\}$

C) $\{y(x) = c_1 + c_2\, x + c_3\, e^{2x} + c_4 x\, e^{2x}\!:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

D) $\{y(x) = c_1 + c_2\, x + c_3\, e^{-2x} + c_3 x\, e^{-2x}\!:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

E) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{2x} + c_5 x\, e^{2x}\!:\ c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$

AB) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x} + c_5\, xe^{-2x}\!: c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$

AC) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x}\!:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

AD) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x} :\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$ ABCDE) None of the above

46. (1 pt.) The number of solutions to (*) is _____________._____A B C D E A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions ABCDE) None of the above

Points this page = 2. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also circle your answers. Be careful!! Remember, once you make a mistake, the rest is wrong.

Consider the ODE $y''' + y'' = -2e^{-x} + 4\cos(x)$ (*). Let $L[y] = y''' + y''$.

47. (3 pts.) The general solution of $y''' + y'' = 0$ is

$y_c(x) =$ ____. ____ A B C D E

A)$c_1+c_2x+c_3e^x$ B)$c_1+c_2x+c_3e^{-x}$ C)$c_1+c_2e^x+c_3e^{-x}$ D)$c_1e^x+c_2\sin(x)+c_3\cos(x)$
E)$c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$ ABCDE) None of the above

48. (4 pts.) A particular solution of $y''' + y'' = 2e^{-x}$ is

$y_{p1}(x) =$ ____. ____ A B C D E

A)$2e^x$ B) $-2e^x$ C)$2e^{-x}$ D) $-2e^{-x}$ E)$2x\,e^x$ AB)$-2x\,e^x$ AC)$2x\,e^{-x}$ AD)$-2x\,e^{-x}$
ABCDE) None of the above

49. (4 pts.) A particular solution of $y''' + y'' = 4\cos(x)$ is

$y_{p2}(x) =$ ____. _____ A B C D E

A)$2\sin(x)+2\cos(x)$ B)$2\sin(x) -2\cos(x)$ C) $-2\sin(x)+2\cos(x)$ D) $-2\sin(x) -2\cos(x)$
ABCDE) None of the above

50. (1 pts.) A particular solution of (*) is

$y_p(x) =$ ____. _____ A B C D E

A)$2xe^x+2\sin(x)+2\cos(x)$ B)$2xe^x +2\sin(x) -2\cos(x)$ C) $2xe^x -2\sin(x)+2\cos(x)$
D)$2xe^x-2\sin(x)-2\cos(x)$ E) $-2xe^x+2\sin(x)+2\cos(x)$ AB) $-2xe^x +2\sin(x) -2\cos(x)$
AC) $-2xe^x -2\sin(x)+2\cos(x)$ AD) $-2xe^x-2\sin(x)-2\cos(x)$ AE)$2xe^{-x}+2\sin(x)+2\cos(x)$
BC)$2xe^{-x} +2\sin(x) -2\cos(x)$ BD)$2xe^{-x} -2\sin(x)+2\cos(x)$ CD)$2xe^{-x} -2\sin(x)-2\cos(x)$
CE) $-2xe^{-x}+2\sin(x)+2\cos(x)$ DE) $-2xe^{-x} +2\sin(x) -2\cos(x)$ ABC) $-2xe^{-x}-2\sin(x)+2\cos(x)$
ABD)$-2xe^{-x}-2\sin(x)-2\cos(x)$ ABCDE) None of the above.

PRINT NAME __________________________(__________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let (*) and L be as on the previous page.

51. (2 pts.) The general solution of (*) is

y(x) = ____. ______ A B C D E

A) $2xe^x + 2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^x$ B) $2xe^x +2\sin(x) -2\cos(x) +c_1+c_2x+c_3e^x$
C) $2xe^x -2\sin(x) +2\cos(x) +c_1+c_2x+c_3e^x$ D) $2xe^x -2\sin(x) -2\cos(x) + c_1 + c_2x + c_3e^x$
E) $-2xe^x +2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^x$ AB) $-2xe^x +2\sin(x)-2\cos(x) +c_1+c_2x+c_3e^x$
AC) $-2xe^x -2\sin(x)+2\cos(x) +c_1+c_2x+c_3e^x$ AD) $-2xe^x-2\sin(x)-2\cos(x) + c_1 + c_2x + c_3e^x$
AE) $2e^x + c_1 \sin(x)+c_2 \cos(x) + c_3 e^x$ BC) $2e^x +2\sin(x)+2\cos(x)+c_1e^x+ c_2e^x+c_3x$
BD) $2xe^{-x}+2\sin(x)+ \cos(x)+c_1+c_2x+c_3e^{-x}$ BE) $2xe^{-x} + 2\sin(2x) -\cos(2x) + c_1+ c_2x + c_3e^{-x}$
CD) $2xe^{-x} -2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^{-x}$ CE) $2xe^{-x} -2\sin(x) -2\cos(x) +c_1+c_2x+c_3e^{-x}$
DE) $-2xe^{-x} +2\sin(x) +2\cos(x) +c_1+c_2x+c_3e^{-x}$ ABC) $-2xe^{-x} +2\sin(x) -2\cos(x) + c_1 + c_2x + c_3e^{-x}$
ABD) $-2xe^{-x} -2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^{-x}$ ABE) $-2xe^{-x} -2\sin(x)-2\cos(x) +c_1+c_2x+c_3e^{-x}$
ACD) $-2e^x -2\sin(x)+2\cos(x) +c_1+c_2x+c_3e^{-x}$ ACE) $2e^x -2\sin(x) -2\cos(x) + c_1 + c_2x + c_3e^x$
ADE) $2e^x + c_1 \sin(x)+ c_2 \cos(x) + c_3 e^{-x}$ CDE) $2e^x +2\sin(x)+2\cos(x)+c_1e^x+ c_2e^{-x}+c_3x$
ABCD) $2e^{-x}+2\sin(x)+ \cos(x)+c_1xe^x+c_2e^{-x}+c_3xe^{-x}$ ABCE) $2e^x + 2\sin(2x) + \cos(2x)+c_1e^x+ c_2xe^x +c_3$
ABCDE) None of the above.

52. (1pt.) The solution set for (*) may be written as

S = ____. _______A B C D E

A) $\{y(x) = 2xe^x + 2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
B) $\{y(x) = 2xe^x +2\sin(x) -2\cos(x) +c_1+ c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
C) $\{y(x) = 2xe^x -2\sin(x) +2\cos(x) +c_1+ c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
D) $\{y(x) = 2xe^x -2\sin(x) -2\cos(x) + c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
E) $\{y(x) = -2xe^x +2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
AB) $\{y(x) = -2xe^x +2\sin(x) -2\cos(x) +c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
AC) $\{y(x) = -2xe^x -2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
AD) $\{y(x) = -2xe^x -2\sin(x)-2\cos(x) + c_1 + c_2x + c_3e^x: c_1, c_2, c_3 \in \mathbf{R}\}$
AE) $\{y(x) = 2xe^{-x} + 2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
BC) $\{y(x) = 2xe^{-x} +2\sin(x) -2\cos(x) +c_1+ c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
BD) $\{y(x) = 2xe^{-x} -2\sin(x) +2\cos(x) +c_1+ c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
BE) $\{y(x) = 2xe^{-x} -2\sin(x) -2\cos(x) + c_1 + c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
E) $\{y(x) = -2xe^{-x} +2\sin(x)+2\cos(x) +c_1 + c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
AB) $\{y(x) = -2xe^{-x} +2\sin(x) -2\cos(x) +c_1 + c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
AC) $\{y(x) = -2xe^{-x} -2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$
AD) $\{y(x) = -2xe^{-x} -2\sin(x)-2\cos(x) + c_1+c_2x+ c_3e^{-x}: c_1, c_2, c_3 \in \mathbf{R}\}$ ABCDE) None of the above

53. (1 pt.) The number of solutions to (*) is ______________.______A B C D E A) 0 B) 1
C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. .

Compute the Laplace transform of the following functions in PC[0,∞)∩Exp ⊆ **T**.

54. (3 pts.) $f(t) = 2t - 3t^2$ $\mathcal{L}(f)$ = ________________________________. ______ A B C D E

55. (3 pts.) $f(t) = -2\,e^{2t} + 3\,e^{-3t}$ $\mathcal{L}(f)$ = ____________________________. ______ A B C D E

56. (3 pts.) $f(t) = -2\sin(2t) + 3\cos(3t)$ $\mathcal{L}(f)$ = ______________________. ______ A B C D E

Possible answers this page

A) $\frac{2}{s}+\frac{3}{s^2}$ B) $\frac{2}{s}-\frac{3}{s^2}$ C) $-\frac{2}{s}+\frac{3}{s^2}$ D) $-\frac{2}{s}-\frac{3}{s^2}$ E) $\frac{2}{s^2}+\frac{6}{s^3}$ AB) $\frac{2}{s^2}-\frac{6}{s^3}$ AC) $-\frac{2}{s^2}+\frac{6}{s^3}$ AD) $-\frac{2}{s^2}-\frac{6}{s^3}$

AE) $\frac{2}{s+2}+\frac{3}{s+3}$ BC) $\frac{2}{s+2}-\frac{3}{s+3}$ BD) $-\frac{2}{s+2}+\frac{3}{s+3}$ BE) $-\frac{2}{s+2}-\frac{3}{s+3}$

CD) $\frac{2}{s-2}+\frac{3}{s+3}$ CE) $\frac{2}{s-2}-\frac{3}{s+3}$ DE) $-\frac{2}{s-2}+\frac{3}{s+3}$ ABC) $-\frac{2}{s-2}-\frac{3}{s+3}$

ABD) $\frac{4}{s^2+4}+\frac{3s}{s^2+9}$ ABE) $\frac{4}{s^2+4}-\frac{3s}{s^2+9}$ ACD) $-\frac{4}{s^2+4}+\frac{3s}{s^2+9}$ ACE) $-\frac{4}{s^2+4}-\frac{3s}{s^2+9}$

ADE) $\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ BCD) $\frac{4}{s^2-4}-\frac{3s}{s^2-9}$ BDE) $-\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ CDE) $-\frac{4}{s^2-4}-\frac{3s}{s^2-9}$

ABCD) $-\frac{2}{(s-2)^2}+\frac{3}{(s+3)^2}$ ABCE) $\frac{2}{s^2+2}+\frac{3s}{s^2+3}$ ABDE) $\frac{2s}{s^2+2}+\frac{3}{s^2+3}$

ACDE) $\mathcal{L}\{f\}$ exists but none of the above is $\mathcal{L}\{f\}$ BCDE) $\mathcal{L}\{f\}$ does not exist.

ABCDE)None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ________________________(________________) ID No. _________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions if they are in **F**:

57. (3 pts.) $F(s) = -\frac{2}{s} + \frac{3}{s+2}$ $\mathscr{L}^{-1}\{F\}$ = ________________________. _____ A B C D E

58. (3 pts.) $F(s) = \frac{-2s+4}{s^2+9}$ $\mathscr{L}^{-1}\{F\}$ = ___________________________. _____ A B C D E

59. (3 pts.) $F(s) = \frac{-2s+3}{s^2-2s+2}$ $\mathscr{L}^{-1}\{F\}$ = _________________________. ____ A B C D E

Possible answers this page

A) $2 + 3e^{2t}$ B) $2 - 3e^{2t}$ C) $-2 + 3e^{2t}$ D) $-2 - 3e^{2t}$ E) $2 + 3e^{-2t}$ AB) $2 - 3e^{-2t}$ AC) $-2 + 3e^{-2t}$
AD) $-2 - 3e^{-2t}$ AE) $2 \cos 3t + 4 \sin 3t$ BC) $2 \cos 3t - 4 \sin 3t$ AD) $-2 \cos 3t + 4 \sin 3t$
AE) $-2 \cos 3t - 4 \sin 3t$ BC) $2 \cos t + (4/3) \sin 3t$ BC) $2 \cos 3t - (4/3) \sin 3t$
AD) $-2 \cos 3t + (4/3) \sin 3t$ AE) $-2 \cos 3t - (4/3) \sin 3t$ BD) $2e^t \cos t + 5e^t \sin t$
BE) $2e^t \cos t - 5e^t \sin t$ CD) $-2e^t \cos t + 5e^t \sin t$ CE) $-2e^t \cos t - 5e^t \sin t$
BD) $2e^t \cos t + e^t \sin t$ BE) $2e^t \cos t - e^t \sin t$ CD) $-2e^t \cos t + e^t \sin t$
CE) $-2e^t \cos t - e^t \sin t$ ACDE) $\mathscr{L}^{-1}\{f\}$ exists but none of the above is $\mathscr{L}\{f\}$
BCDE) $\mathscr{L}^{-1}\{f\}$ does not exist. ABCDE) None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ________________________(________________) ID No. _________________
Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Using the procedure illustrated in class (attendance is mandatory), find the eigenvalues of

$A = \begin{bmatrix} i & 4 \\ 0 & -1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

60. (2 pts.) A polynomial p(λ) where solving p(λ) = 0 yields the eigenvalues of A can be written

as p(λ) =____. ______ A B C D E

A) (i+λ)(1+λ) B) (i+λ)(1−λ) C) (i−λ)(1+λ) D) (i−λ)(1−λ) E) (i+λ)(2+λ) AB) (i+λ)(2−λ)
AC) (i−λ)(2+λ) AD)(i−λ)(2−λ) AE) (2i+λ)(1+λ) BC)(2i+λ)(1−λ) BD)(2i−λ)(1+λ)
BE) (2i−λ)(1−λ) CD) (2i+λ)(2+λ) CE)(2i+λ)(2−λ) DE)(2i−λ)(2+λ) ABC)(2i−λ)(2−λ)
ABD) (3i−λ)(2+λ) ABE)None of the above.

61. (1 pt.) The degree of p(λ) is __________. ______ A B C D E A) 1 B) 2 C) 3 D) 4
E) 5 AB) 6 AC) 7 ABCDE) None of the above.

62. (1 pt.) Counting repeated roots, the number of eigenvalues of A

is _________. _____ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5
AC) 6 AD) 7 AE) 8 ABCDE) None of the above

63.(2 pts.) The eigenvalues of A can be written as _____________________._____ A B C D E

A) $\lambda_1 = 1, \lambda_2 = i$ B) $\lambda_1 = 1, \lambda_2 = -i$ C) $\lambda_1 = -1, \lambda_2 = i$ D) $\lambda_1 = -1, \lambda_2 = -i$ E) $\lambda_1 = 2, \lambda_2 = i$
AB) $\lambda_1 = 2, \lambda_2 = -i$ AC) $\lambda_1 = -2, \lambda_2 = i$ AD) $\lambda_1 = -2, \lambda_2 = -i$ AE) $\lambda_1 = 1, \lambda_2 = 2i$
BC) $\lambda_1 = 1, \lambda_2 = -2i$ BD) $\lambda_1 = -1, \lambda_2 = 2i$ BE) $\lambda_1 = -1, \lambda_2 = -2i$ CD) $\lambda_1 = 2, \lambda_2 = 2i$
CE) $\lambda_1 = 2, \lambda_2 = -2i$ DE) $\lambda_1 = -2, \lambda_2 = 2i$ ABC) $\lambda_1 = -2, \lambda_2 = -2i$ ABCDE) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(__________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answers. Note that $\lambda_1 = -1$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$

64.(4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with λ_1 is B = _____________________. ___A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$
AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$
BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$
ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$
ACD) $\lambda = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

65. (1pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, the eigenspace associated with λ_1 is often one dimensional. Hence conventions for selecting eigenvector(s) associated with λ_1 have been developed (by engineers). We say that <u>the</u> eigenvector(s) associated with λ_1

is (are) ____. ______A B C D E

A) $[1,1]^T, [4,4]^T$ B) $[1,1]^T$ C) $\{[1,2]^T\}$ D) $[1,2]^T, [4,8]^T$ E) $[2,1]^T$
AB) $[1,3]^T$ AC) $[1,4]^T$ AD) $[4,1]^T$ AE) $[3,1]^T$ BC) $[1,-1]^T, [4,4]^T$
BD) $[1,-1]^T$ BE) $[1,-2]^T$ CD) $[1,-2]^T, [4,8]^T$ CE) $[2,1]^T$ DE) $[1,3]^T$
ABC) $[1,-4]^T$ ABD) $[4,-1]^T$ ABE) $[3,-1]^T$
ACD) $\lambda_1 = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda_1 = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.
Consider the scalar equation $u'' - 4u' + 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du'/dt$). As was done in class (attendance is mandatory) convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y). You may think of x as the position and y as the velocity of a point particle). This system of two scalar equations can be written in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. You are to find $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; that is you are to find a, b, c, and d.

66. (1 pt.) a = ______. ____ A B C D E

67. (1 pt.) b = ______. ____ A B C D E

68. (1 pt.) c = ______. ____ A B C D E

69. (1 pt.) d = ______. ____ A B C D E

Possible answers this page.
A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9
BD) −1 BE) −2 CD) −3 CE) −4 DE) −5 ABC) −6 ABD) −7 ABE) −8 ACD) −9
ABCDE) None of the above
Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let the 2x2 matrix A have the eigenvalue table

TABLE

Eigenvalues	Eigenvectors
$r_1 = -1$	$\vec{\xi}_1 = \begin{bmatrix}1\\2\end{bmatrix}$
$r_2 = 2$	$\vec{\xi}_2 = \begin{bmatrix}2\\1\end{bmatrix}$

Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}^2)\to\mathcal{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}] = \vec{x}' - A\vec{x}$

and let the null space of L be N_L

70.(1 pt). The dimension of N_L is ________. _____ A B C D E
A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) None of the above.

72. (2 pts.) A basis for the null space of L is__________________________ . ____ A B C D E

A) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{t},\begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ B) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{-t},\begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ C) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{t},\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$ D) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{-t},\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$

E) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ AB) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ AC) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ AD) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ AE) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}\right\}$ BC) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ BD) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ BE) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ CD) None of the above

73.(2 pts.) The general solution of $\vec{x}' = A\vec{x}$ is ____________________________. ___ A B C D E

A) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ B) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ C) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$ D) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$

E) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ AB) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ AC) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ AD) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$

AE) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}$ BC) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ BD) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$

BE) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ CD)None of the above

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ____________

PRINT NAME __________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

True or False. Let f and g be real valued functions of a real variable; that is, $f:\mathbf{R} \to \mathbf{R}$ and $g:\mathbf{R} \to \mathbf{R}$. Circle True if the statement is true. Circle False if the statement is false.

73. (1 pt.) A)True B)False The function f is even if $f(-x) = f(x)\ \forall x \in \mathbf{R}$.

74. (1 pt.) A)True B)False The function f is not odd if $f(-x) = -f(x)\ \forall x \in \mathbf{R}$.

75. (1 pt.) A)True B)False If f and g are both even functions, then the product of f and g is an odd function.

76. (1 pt.) A)True B)False The function f is periodic of period T if $f(x+T) = f(x)\ \ \forall x \in \mathbf{R}$.

77. (1 pt.) A)True B)False If f is an even function, then we know that $\int_{-\ell}^{\ell} f(x)dx = 0$.

For each of the following questions write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters

Classify the following function with regard to whether they are odd or even.

78. (1 pt.) $f(x) = -x$ is ____. ______ A B C D E

79. (1 pt.) $f(x) = -3$ is ____. _______ A B C D E

80.(1 pt.) $f(x) = \cos(x)$ is ______________________________________. ______ A B C D E

81. (1 pt.) f(x) = *x* is ____. _______ A B C D E

82. (1 pt.) $f(x) = e^x$ is ____. ________A B C D E

83. (1 pt.) $f(x) = 0$ is ____. ________ A B C D E

Possible answers for questions 78-83.

A) odd, but not even B) even, but not odd C) both odd and even

D) neither odd nor even E) none of the above

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(___________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let f;**R**→**R** be in $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈*F*(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [ )ℓ,ℓ], and $f(x) = [f(x+)+f(x-)]/2$ at points of discontinuity} so that its Fourier series exists.

84. (2 pts.) The formula for the general Fourier series for f ∈ $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ given in our text is

f(x) = ____ . ______ A B C D E

A) $a_0 + \sum_{n=1}^{N} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$ B) $a_0 + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$

C) $\frac{a_0}{2} + \sum_{n=0}^{N} a_n \cos(n\pi x) + b_n \sin(n\pi x)$ D) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$

E) $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$ AB) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + b_n \cos(n\pi x)$

AC) None of the above

85. (2pts.) where for n = 0, 1, 2, ... we have a_n = __________________________. _____ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ B) $\frac{2}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ C) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(n\pi x)dx$

D) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ E) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ AB) $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\cos(x)dx$

AC) None of the above.

86. (2pts.) and for n = 1, 2, ... we have b_n = _____________________________. _____ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ B) $\frac{2}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ C) $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(n\pi x)dx$

D) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ E) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ AB) $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\sin(x)dx$

AC) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(_________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Recall from the previous page that $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈$\mathcal{F}$(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and f(x) = [f(x+) + f(x−)]/2 at points of discontinuity}. Now let $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ and $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ are inner product spaces with inner product (f,g) = $\int_{-\ell}^{\ell} f(x)g(x)dx$, that $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)} = \{1/2\}\cup\{\cos(\frac{n\pi}{\ell}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ is an orthogonal Schauder basis for $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$, and that $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)}$ is an orthogonal Hamel basis for $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$.

87. (1 pt.) Using the notation given above, an orthogonal Hamel basis for $PC^1_{ffs}(\mathbf{R},\mathbf{R};2)$

is ____.______A B C D E Hint: What is ℓ?

A) $\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$

B) $\{1/2\}\cup\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$

C) $\{1/2\}\cup\{\cos(\frac{n\pi}{3}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{3}):n\in\mathbf{N}\}$

D) $\{1/2\}\cup\{\cos(\frac{n\pi}{4}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{4}):n\in\mathbf{N}\}$

E) $\{(1/2)x\}\cup\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$

AB) None of the above.

88. (2 pts.) The Fourier series for the function f(x)∈ $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ which has period 2π and is defined on the interval [−π,π) by f(x) = 2 + 2 cos(x) + 3 sin(x) is

f(x) =____. ___A B C D E

Hint: Think Hamel basis.

A) $3 + \sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

B) $2 + \sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

C) $3 + \sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

D) $2 + \sum_{k=1}^{\infty}\frac{1}{k\pi}\sin(k\pi x)$

E) 2+ 2cos(x) + 2sin(x)

AB)2+ 2cos(x) + 3sin(x) AC)2 + 3cos(x) + 3sin(x) AD) 3+ 3cos(x)+ 3sin(x)

AE) 2+ 2cos(2πx) + 2sin(2π x) BC) 2 + 2cos(2πx) + 3sin(2πx) BD)2 + 3cos(2πx) + 3sin(2πx)

BE)3 + 3cos(2πx) + 3sin(2πx) ABCDE) None of the above.

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ and $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)}$ be as on the previous page. Now let $f(x) \in PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ be the function whose domain is **R** which has period 4 and is defined on the interval $(-2,2)$ by $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 4 & 0 < x < 2 \end{cases}$. Using the formulas on the previous page, determine the Fourier series for the function f. Begin by sketching f for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges

89. (1 pt.) To apply the formulas given on the previous page we choose ℓ = _____.___ A B C D E
A) $\ell = 1$ B) $\ell = 2$ C) $\ell = 3$ D) $\ell = 4$ E) $\ell = -1$ AB) $\ell = -2$ AC) $\ell = -3$
AD) $\ell = -4$ ABCDE) None of the above

Next write down the formulas for a Fourier series and its coefficients using this value of ℓ. After computing the a_n's and the b_n's, note what they are for n odd and n even. Then answer the questions below and on the next two pages.

90. (3 pts.) We have a_0 = _________. _____ A B C D E A) 0 B) 1 C) 2 D) 3
E) 4 AB) −1 AC) −2 AD) −3 AE) −4 ABCDE) None of the above

PRINT NAME __________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Let f(x) be as on the previous page. Continue the computation of the Fourier Series coefficients.

91. (3 pts.) For a_n with n odd (n = 1, 3, 5, ...) so that for k = 0,1,2,3,.. we have

a_{2k+1} =__________________________. ______ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

92. (3 pts.) For a_n with n even (n= 2, 4, 6, ...) so that for k = 1,2,3,... we have

a_{2k} =___________________________. _____ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) ABCDE)None of the above

93. (3 pts.) For b_n with n odd (n = 1, 3, 5, ...) so that for k = 0,1,2,3,... we have

b_{2k+1} =___________________________. ____ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(_________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Let f(x) be as on the page before the previous page. Continue the computation of the Fourier Series coefficients.

94. (3 pts.)For b_n with n even (n = 2, 4, 6, ...) so that for k = 1,2,3,...

we have b_{2k} =_____________________. _____ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) ABCDE)None of the above

95. (3 pts.) Thus the Fourier series for f(x) may be written as

f(x) = ____._____ A B C D E

A) $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ B) $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ D) $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

E) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ AB) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AC) $1+\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ AD) $1+\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AE) $1+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ BC) $1+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos\frac{(2k+1)\pi}{2}x)$

BD) $1+\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ BE) $1+\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

CD) $2+\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ CE) $2+\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

DE) $2+\sum_{k=1}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ ABC) $2+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

ABD) $2+\sum_{k=1}^{\infty}\frac{8}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ ABE) $2+\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

ABCDE) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(_________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Consider the Partial Differential Equation (PDE): $t\,u_{tt} = x\,u_{xx}$

96. (4 pts.) Using the method of separation of variables with separation constant λ (or $-\lambda$ if appropriate), one of the following sets of two Ordinary Differential Equations (ODE's) can be obtained from this PDE. Recall that the process does not yield a unique set of ODE's. (Note that we are looking for product solutions in the null space of the linear operator $L[u] = t\,u_{tt} - x\,u_{xx}$). Following the advice given in class as to how to choose the separation constant (attendance is mandatory) we may obtain the set of

ODE's ____. ____ A B C D E

A) $X'' + \lambda X = 0, \quad T'' + \lambda T = 0$

B) $X'' + \lambda X = 0, \quad T'' - \lambda T = 0$

C) $X'' + \lambda X = 0, \quad T'' + \lambda t T = 0$

D) $X'' + \lambda x X = 0, \quad T'' + \lambda T = 0$

E) $X'' + \lambda xX = 0, \quad T'' + \lambda t T = 0$

AB) $X'' + \lambda t X = 0, \quad T'' + \lambda x T = 0$

AC) $x X'' + \lambda X = 0, \quad t T'' + \lambda T = 0$

AD) $t X'' + \lambda X = 0, \quad xT'' + \lambda x T = 0$

AE) $x X'' + \lambda X = 0, \quad T'' + \lambda T = 0$

BC) $X'' + \lambda X = 0, \quad t T'' + \lambda T = 0$

BD) $X'' + \lambda x X = 0, \quad T'' + \lambda T = 0$

BE) $X'' + \lambda X = 0, \quad T'' + \lambda t T = 0$

CD) Separation of variables does not work on this PDE.

CE) Separation of variables works on this PDE, but none of the above is correct.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Recall that we have established $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ as a set where we can calculate Fourier Series. Now let $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions, $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ be the functions in $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to $[0,\ell]$, and $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$ be the subspace of $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ for which the Fourier (sine) series is finite. Recall that $B_{fss} = \{\sin(n\pi/\ell) : n \in \mathbf{N}\}$ is a Schauder basis of $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ and a Hamel basis for $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$. Consider the heat conduction in a rod problem defined by

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
BC	$u(0,t) = 0$, $u(\ell,t) = 0$,	$t > 0$
IC	$u(x,0) = u_0(x)$	$0 < x < \ell$.

Recall that $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ is the **state space** for this problem. To formulate this problem as a linear mapping problem we also need the **solution space**. Let $D = (0,\ell)\times(0,\infty)$, $\tilde{D} = [0,\ell]\times(0,\infty)$, $\bar{D} = [0,\ell]\times[0,\infty)$, $\mathcal{A}_{HC}(\bar{D},\mathbf{R},\ell) = \{u(x,t)\in\mathcal{F}(\bar{D},\mathbf{R}): u\in\mathcal{A}(D,\mathbf{R})\cap C(\tilde{D},\mathbf{R}), u(0,t) = 0, u(\ell,t) = 0$ for $t > 0, u(x,0)\in PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ and $\forall x\in(0,\ell)$ we have that $u(x,t)$ is continuous at $t = 0$ except where $u(x,0)$ is discontinuous$\}$. Thus a function $u(x,t)\in\mathcal{A}_{HC}(\bar{D},\mathbf{R},\ell)$ is analytic in D, contiuous on the boundary of $\bar{D}$ except where $u(x,0)$ is discontinuous, satisfies the boundary conditions $u(0,t) = 0$, $u(\ell,t) = 0$ for $t > 0$, and $u(x,0)\in PC^1_{fss}([0,\ell],\mathbf{R};\ell)$. We take $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ as the **solution space** for our problem. Now let $L_B:\mathcal{A}_{HC}(\bar{D},\mathbf{R};\ell)\to\mathcal{A}(D,\mathbf{R};\ell)$ be defined by $L_B[u] = u_t - \alpha^2 u_{xx}$, Thus we incorporate the boundary conditions into the domain of the operator. Now let N_{LB} be the null space of L_B and $\mathcal{A}_{NLBffss}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L: u(x,0)\in PC^1_{ffss}([0,\ell],\mathbf{R};\ell)\}$ and $\mathcal{A}_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L: u(x,0)\in PC^1_{fss}([0,\ell],\mathbf{R};\ell)\}$. Hence we see that $\mathcal{A}_{NLBffss}(\bar{D},\mathbf{R};\alpha^2,\ell) \subseteq \mathcal{A}_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell) \subseteq N_{LB} \subseteq \mathcal{A}_{HC}(\bar{D},\mathbf{R};\ell)$. Recall that $B_{HCfss} = \{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ is a Schauder basis for $\mathcal{A}_{NLBfss}(\bar{D},\mathbf{R};\ell)$ and a Hamel basis for $\mathcal{A}_{NLBffss}(\bar{D},\mathbf{R};\ell)$.

We denote the problem of finding all solutions to PDE $u_t = \alpha^2 u_{xx}$ $0 < x < 2$, $t > 0$
BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

by $\mathrm{Prob}_{HC}(\mathcal{A}_{HC}(\bar{D},\mathbf{R};\alpha^2,\ell), L_B[u] = 0; \alpha^2, \ell)$. We denote the problem of heat conduction in a rod that is given above by $\mathrm{Prob}_{HC}(\mathcal{A}_{HC}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x); \alpha^2, \ell)$

PRINT NAME ________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let $\text{Prob}_{HC}(A_{HC}(\bar{D},\mathbf{R};\alpha^2,\ell), L_B[u]=0;\alpha^2,\ell)$, $\text{Prob}_{HC}(A_{HC}(\bar{D},\mathbf{R}), L_B[u]=0, u(x,0))$, L_B, N_{LBffss}, N_{LBfss}, $u_0(x)$, all of the function spaces, and all of the basis sets, be as on the previous page.

97. (1pt.) The set (which may be thought of as a subset of $L^2([0,\ell],\mathbf{R})$) that we (attendance is mandatory) considered to be the state space for the problem of heat conduction in a rod

is____.______A B C D E

A) D B) $\bar{D}$ C) $A_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell)$ D) $A_{NLBffss}(\bar{D},\mathbf{R};\alpha^2,\ell)$ E) $A_{HC}(D,\mathbf{R};\ell)$ AB) N_{LB} AC) $\mathbf{R}$

AC) $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ AD) $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$ AE) $A(D,\mathbf{R};\ell)$ ABCDE) None of the above

98. (1pt.) A Schauder basis for the state space for heat conduction in a rod may be taken to

be____.______A B C D E A) $A_{fs,o}(\bar{D},\mathbf{R})$

B) $A(D,\mathbf{R};\ell)$ C) $A_{fss}(\bar{D},\mathbf{R};\ell)$ D) $A_{ffss}(\bar{D},\mathbf{R};\ell)$ E) N_{LB} AB) $A_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell)$ AC) $PC^1_{fs}([0,\ell],\mathbf{R};\ell)$

AD) $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$ AE) $B_{NLBfss}=\{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ BC) $B_{fss}=\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$

BD) $B_{fs}=\{1/2\}\cup\{\cos(\frac{n\pi}{\ell}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ ABCDE)None of the above

99. (1pt.) A Schauder basis for $A_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell)$, which is a subset of N_{LB},

is____.______A B C D E A) $A_{fs,o}(\bar{D},\mathbf{R})$

B) $A(D,\mathbf{R};\ell)$ C) $A_{fss}(\bar{D},\mathbf{R};\ell)$ D) $A_{ffss}(\bar{D},\mathbf{R};\ell)$ E) N_{LB} AB) $A_{NLBfss}(\bar{D},\mathbf{R};\alpha^2,\ell)$ AC) $PC^1_{fs}([0,\ell],\mathbf{R};\ell)$

AD) $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$ AE) $B_{NLBfss}=\{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ BC) $B_{fss}=\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$

BD) $B_{fs}=\{1/2\}\cup\{\cos(\frac{n\pi}{\ell}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ ABCDE)None of the above

100. (2 pts.) The "general" or formal solution of $\text{Prob}_D(A\,HC(\bar{D},\mathbf{R}),L_B(x,t)=0;\alpha^2,\ell)$, the problem of finding all solutions to PDE $u_t = \alpha^2 u_{xx}$ $0 < x < 2$, $t > 0$

BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

is given by u(x,t) =____. ______ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{\ell}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2\pi^2\ell^2 t}\sin(\frac{n\pi}{\ell}x)$ D) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(n\pi\ell x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\cos(\frac{n\pi}{\ell}x)$ AD) $\sum_{n=1}^{\infty} c_n \cos(\frac{n\pi}{\ell}x)$ ABCDE) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Let $\text{Prob}_{HC}(\mathcal{A}_{HC}(\bar{D},\mathbf{R};\alpha^2,\ell), L_B[u]=0;\alpha^2,\ell)$, $\text{Prob}_{HC}(\mathcal{A}_{HC}(\bar{D},\mathbf{R}), L_B[u]=0, u(x,0))$, L_B, N_{LBffss}, N_{LBfss}, $u_0(x)$, all of the function spaces, and all of the basis sets, be as on thepage before the previous page.

101. (2 pts.) The "general" or formal solution of

PDE $u_t = u_{xx}$ $0 < x < 2,\ t > 0$

BC $u(0,t) = 0,\ u(2,t) = 0,\ t > 0$

(we denote this problem by $\text{Prob}_D(\mathcal{A}_{HC}(\bar{D},\mathbf{R};1), L_B[u]=0;1,2)$)

is given by u(x,t) =____. _______ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{2}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(2n\pi x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$ AD) $\sum_{n=1}^{\infty} a_n\sin(\frac{n\pi}{2}x) + b_n\cos(\frac{n\pi}{2}x)$ ABCDE) None of the above.

102. (4 pts.) The solution of

	PDE	$u_t = u_{xx}$	$0 < x < 2,\ t > 0$
BVP for a PDE	BC	$u(0,t) = 0,\ u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 6\sin(3\pi x)$	$0 < x < 2$

(we denote this problem by $\text{Prob}_{HC}(\mathcal{A}_{fs,o,0}(\bar{D},\mathbf{R}), L_B[u]=0, 6\sin(3\pi x);1,2)$)

is given by u(x,t) =____. _____A B C D E

A) $\sum_{n=1}^{\infty} 6e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ B) $6e^{-\frac{\pi^2}{4}t}\sin(\pi x)$ C) $6e^{-9\pi^2 t}\sin(6\pi x)$ D) $6e^{-36\pi^2 t}\sin(6\pi x)$

E) $\sum_{n=1}^{\infty} 6e^{-36\pi^2 t}\sin(6\pi x)$ AB) $6e^{-12\pi^2 t}\sin(6\pi x)$ AC) $6e^{-9\pi^2 t}\sin(3\pi x)$ AD) $6e^{-36\pi^2 t}\sin(6\pi x)$

ABCDE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ________________________(________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Let $\text{Prob}_{HC}(A_{fs,o,0}(\bar{D},\mathbf{R};\ell), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ be the problem defined by

PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$

BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$

IC $u(x,0) = u_0(x)$ $0 < x < \ell$

103. (2 pts.) The formula for the solution of

$\text{Prob}_{HC}(A_{fs,o,0}(\bar{D},\mathbf{R};\ell), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$

is given by u(x,t) =____. _____ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{\ell}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell}x)$

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(n\pi\ell x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \cos(\frac{n\pi}{\ell}x)$ AD) $\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{\ell}x)$ AE) None of the above

104. (2 pts.) where the formula for c_n is c_n =___________________________. ____ A B C D E

A) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{\ell}{n\pi}x\right)dx$ B) $\frac{1}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ C) $\frac{2}{\ell}\int_{-\ell}^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ D) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ E) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin(n\pi\ell x)dx$ AB) $\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ AC) $\frac{1}{\ell}\int_{-\ell}^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ AD) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\cos\left(\frac{n\pi}{\ell}x\right)dx$ ABCDE) None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(___________________) ID No. _____________________

Last Name, First Name MI, What you wish to be called

Let $\text{Prob}_{HC}(A_{fs,o,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = 2\ ;\ 1, 2)$ be the problem defined by

PDE $u_t = u_{xx}$ $0 < x < 2,\ \ t > 0$

BC $u(0,t) = 0,\quad u(2,t) = 0,\quad t > 0$

IC $u(x,0) = 2$ $0 < x < 2$

105. (2 pts.) The formula for the solution of $\text{Prob}_{HC}(A_{HC}(\bar{D},\mathbf{R};\ell), L_B[u] = 0, u(x,0) = 2\ ;\ 1, 2)$ is given by $u(x,t) =$ ____. ____ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{2}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(2n\pi x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\cos(\frac{n\pi}{2}x)$ AD) $\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{2}x)$ ABCDE) None of the above.

106. (2 pts.) where the formula for c_n is $c_n =$ ________________________________. _____ A B C D E

A) $2\int_0^2 \sin\left(\frac{2}{n\pi}x\right)dx$ B) $\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ C) $2\int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$ D) $\frac{5}{2}\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

E) $3\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AB) $4\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AC) $5\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AD) $2\int_0^2 \cos\left(\frac{n\pi}{2}x\right)dx$

ABCDE) None of the above.

PRINT NAME ____________________________(__________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

Let $Prob_{HC}(\mathcal{A}_{HC}(\bar{D},\mathbf{R};\ell)$, $L_B[u] = 0$, $u(x,0) = 2$; 1, 2) be as on the previous page.

107. (2pts.) Computing c_n using the formula on the previous page, for n odd (n = 1, 3, 5, ...) we

have for k = 0,1,2,3,... that c_{2k+1} =_______________. ____ A B C D E A) 0 B) 1/[(2k+1)π] C) 2/[(2k+1)π] D) 3/[(2k+1)π] E) 4/[(2k+1)π] AB) 12/[(2k+1)π] AC) 16/[(2k+1)π] AD) 32/[(2k+1)π] AE) 64/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

108. (2 pts.)For c_n with n even (n = 2, 4, 6, ...) we have for k = 1,2,3,...that

c_{2k} =______________________. ____ A B C D E A) 0 B) 1/[(2k+1)π] C) 2/[(2k+1)π] D) 3/[(2k+1)π] E) 4/[(2k+1)π] AB) 12/[(2k+1)π] AC) 16/[(2k+1)π] AD) 32/[(2k+1)π] AE) 64/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

109. (2 pts.) Hence the solution of $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R})$, $L_B[u] = 0$, $u(x,0) = 2$; 1, 2) may be written

as u(x,t) = ____. ______ A B C D E

A) $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ B) $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ D) $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

E) $\sum_{k=0}^{\infty}\frac{16}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ AB) $\sum_{k=0}^{\infty}\frac{32}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{4}x)$

AC) $\sum_{k=0}^{\infty}\frac{64}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{4}x)$ AD) $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k+1)\pi}{2}x)$

AE) $\sum_{n=1}^{\infty}\frac{8}{(2k)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k)\pi}{2}x)$ BC) $\sum_{k=1}^{\infty}\frac{4}{k\pi}e^{-\frac{k^2\pi^2}{4}t}\sin(\frac{k\pi}{2}x)$ BD) $\sum_{k=1}^{\infty}\frac{2}{k\pi}e^{-\frac{k^2\pi^2}{2}t}\sin(\frac{k\pi}{2}x)$

ABCDE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer

110. (3 pts.) If the left end of a rod are insulated (recall that we assume that the lateral sides are also insulated so that the temperature does not vary over a cross section) and the right end is held a the constant temperature 0, then a good mathematical model of the physical heat conduction problem is given by:

	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell,\ \ t > 0$
BVP for a PDE	BC	$u_x(0,t) = 0, \quad u(\ell,t) = 0,$	$t > 0$
	IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

where $u_0(x)$ is the initial temperature distribution in the rod.

```
          ////////////////////////////////////////////////
      -→ O_________________________________O ←-
   /     ////////////////////////////////////////////////     \
insulated                                        u = 0
          *-------------------------------------*
        x = 0                                 x = ℓ
```

Applying the separation of variables process to the PDE results in the two ODE's:

1. $X'' + \lambda X = 0$
2. $T' + \alpha^2 \lambda T = 0$

where λ is the separation constant. The spacial eigenvalue problem that results from

applying the BC's given above is ___________________________________ .____ A B C D E

A) $X'' + \lambda X = 0$, $X(0) = 0,\ X(\ell) = 0$

B) $X'' + \lambda X = 0$, $X'(0) = 0,\ X(\ell) = 0$

C.) $X'' + \lambda X = 0$, $X(0) = 0,\ X'(\ell) = 0$

D) $X'' + \lambda X = 0$, $X(0) = 1,\ X(\ell) = 0$

E) $X'' + \lambda X = 0$, $X'(0) = 0,\ X'(\ell) = 0$

AB) $X'' + \lambda X = 0$, $X'(0) = 1,\ X'\ell) = 0$

AC) $X'' + \lambda X = 0$, $X(0) = 1,\ X(\ell) = 1$

AD) $X'' + \lambda X = 0$, $X'(0) = 1,\ X(\ell) = 0$

AE) $X'' + \lambda X = 0$, $X(0) = 0,\ X'(\ell) = 0$

ABCDE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(___________________) SS No. ____________________
Last Name, First Name MI, What you wish to be called

TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	$s > \|a\|$
cosh (at)	$\frac{s}{s^2 - a^2}$	$s > \|a\|$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
u(t)	$\frac{1}{s}$	$s > 0$
u(t - c)	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	$F(s - c)$	
$f(ct)$ $c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	

PRINT NAME ______________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

PARTIAL TABLE OF ANTIDERIVATIVES

1. $\int x[\sin(ax)]dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + c$

2. $\int x[\cos(ax)]dx = \frac{1}{a^2}\cos(ax) - \frac{x}{a}\sin(ax) + c$

3. $\int x^2[\sin(ax)]dx = \frac{2x}{a^2}\sin(ax) - \frac{a^2x^2 - 2}{a^3}\cos(ax) + c$

4. $\int x^2[\cos(ax)]dx = \frac{2x}{a^2}\cos(ax) - \frac{a^2x^2 - 2}{a^3}\sin(ax) + c$

5. $\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

6. $\int \cos^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

7. $\int [\sin(ax)][\cos(ax)]dx = \frac{1}{2a}\sin^2(ax) + c$

8. $\int [\sin(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

9. $\int [\cos(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

10. $\int [\sin(ax)][\cos(bx)]dx = -\frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

PRINT NAME ______________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

SETS AND SPACES FOR FOURIER SERIES AND THE HEAT CONDUCTION PROBLEM

1. $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈$\mathcal{F}$(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and f(x) =(f(x+) + f(x−))/2 at points of discontinuity}.
2. $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ is the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ and $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ are inner product spaces with inner product (f,g) = $\int_{-\ell}^{\ell} f(x)g(x)dx$. However, they are not Hilbert spaces.
3. $B_{fs} = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ is an orthogonal Schauder basis for $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$, and an orthogonal Hamel basis of $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$.
4. $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ is the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions.
5. $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ is the set of functions in $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to [0,ℓ]. This space of Fourier Sine Series is the state space for the heat conduction problem.
6. $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$ is the subspace of $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ for which the Fourier sine series is finite.
7. $B_{fss} = \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is an orthogonal Schauder basis for $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ and an orthogonal Hamel basis of $PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$.
8. Consider the heat conduction in a rod problem defined by

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
BC	$u(0,t) = 0$, $u(\ell,t) = 0$,	$t > 0$
IC	$u(x,0) = u_0(x)$	$0 < x < \ell$.

Recall that $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$ is the **state space** for this problem. To formulate this problem as a linear mapping problem we also need the Σ space. Let D = (0,ℓ)×(0,∞), $\tilde{D}$ = [0,ℓ]×(0,∞), $\bar{D}$ = [0,ℓ]×[0,∞), $\mathcal{A}_{HC}(\bar{D},\mathbf{R},\ell)$ = {u(x,t)∈$\mathcal{F}(\bar{D},\mathbf{R})$: u∈$\mathcal{A}$(D,**R**)∩C($\tilde{D}$,**R**), u(0,t) = 0, u(ℓ,t) = 0 for t > 0,u(x,0)∈ $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ and ∀x∈(0,ℓ) we have that u(x,t) is continuous at t = 0 except where u(x,0) is discontinuous}. Thus a function u(x,t)∈$\mathcal{A}_{HC}(\bar{D},\mathbf{R},\ell)$ is analytic in D, continuous on the boundary of $\bar{D}$ except where u(x,0) is discontinuous, satisfies the boundary conditions u(0,t) = 0, u(ℓ,t) = 0 for t > 0, and u(x,0)∈ $PC^1_{fss}([0,\ell],\mathbf{R};\ell)$. Thus $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ is the set where we look for solutions to the pde in D. Thus, we take $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ as the **Σ space** for our problem. These functions are “nice” at the boundary of D.
9. Now let $\mathcal{A}_{HCBC}(\bar{D},\mathbf{R})$ = {u(x,t)∈$\mathcal{A}_{HC}(\bar{D},\mathbf{R})$): u(0,t) = 0 and u(ℓ,t) = 0 for t > 0}. Then $\mathcal{A}_{HCBC}(\bar{D},\mathbf{R})$ is the set where we look for solutions of the pde in D that also satisfy the BC’s. These functions are “nice” on [0,ℓ]×{0} where t = 0. Now let L_B:$\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}) \to \mathcal{A}$(D,**R**) be defined by $L_B[u] = u_t - \alpha^2 u_{xx}$. Thus the BC’s are incorporated into the domain of the linear operator L_B. Now let $N_{L_{Bfss}}(\alpha^2,\ell)$ be the null space of L_B so that if u(x,t) ∈ $N_{L_{Bfss}}(\alpha^2,\ell)$, then it satisfies the BC’s as well as the pde. Now let $Prob_{HC}(\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2,\ell)$ be the problem defined by

PRINT NAME ______________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$
BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$.

Hence the solution set for $Prob_{HC}(\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2,\ell)$) is the null space of L_B which we have denoted by $N_{L_{Bfss}}(\alpha^2,\ell)$. These are the functions in $\mathcal{A}_{HCBC}(\bar{D},\mathbf{R})$ that satisfy the pde as well as the BC's. Also, if $u(x,t)\in N_{L_{Bfss}}(\alpha^2,\ell)$, then its restriction to $[0,\ell]\times\{0\}$ is in $PC^1_{fs,o}([0,\ell],\mathbf{R})$. Now let $N_{L_{Bffs}}(\alpha^2,\ell) = \{u(x,t)\in N_{L_{Bfs}}(\alpha^2,\ell) : u(x,0) \in PC^1_{ffss}([0,\ell],\mathbf{R};\ell)\}$ so that if $u(x,t)\in N_{L_{Bffss}}(\alpha^2,\ell)$, its restriction to $[0,\ell]\times\{0\}$ is in $PC^1_{ffss}([0,\ell],\mathbf{R})$. Now let $Prob_{HC}(\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}), L_B[u] = 0$, $u(x,0) = u_0(x)$; α^2, ℓ) be the heat conduction in a rod problem as defined in 8 above. From class, if $u_0(x)\in PC^1_{fss}([0,\ell],\mathbf{R};\ell)$, then the solution of $Prob_{HC}(\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ is in $N_{L_{Bfss}}(\alpha^2,\ell)$. If $u_0(x) \in PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$, then the solution of $Prob_{HC}(\mathcal{A}_{HCBC}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ is in $N_{L_{Bffs}(\alpha^2,\ell)}$.

10. From class, $B_{N_{L_B}}(\alpha^2,\ell) = \{e^{-(\alpha n\pi/\ell)^2 t}\sin(n\pi/\ell) : n \in \mathbf{N}\}$ is a Hamel basis for $N_{L_{Bffss}}(\alpha^2,\ell)$ and a Schauder basis for $N_{L_{Bfss}}(\alpha^2,\ell)$.