MATH 261
FALL 2008
FINAL EXAM

MATH 261: Elementary Differential Equations
FINAL EXAM
EXAMINATION COVER PAGE

MATH 261
FALL 2008
Professor Moseley

PRINT NAME____ (_________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Wednesday, Dec.10, 2008, 8am-10am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

__________________________________ _________________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 34 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. Page 35 contains Laplace transforms you need not memorize. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! Ask for scratch paper if you need it. You may remove the staple. Print your name on all sheets. Pages 1-34 are Fill-in-the Blank/Multiple Choice or True/False. Expect no partial credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Your thoughts should be expressed in your best mathematics on this paper. Proofread as time allows. GOOD LUCK!!

Scores

page	points	score
1	20	
2	7	
3	6	
4	8	
5	6	
6	5	
7	5	
8	9	
ST1	66	

Scores

page	points	score
9	3	
10	10	
11	7	
12	2	
13	12	
14	4	
15	9	
16	9	
ST2	56	

Scores

page	points	score
17	6	
18	5	
19	4	
20	5	
21	11	
22	6	
23	3	
24	4	
ST3	44	

Scores

page	points	score
25	9	
26	6	
27	4	
28	2	
29	3	
30	6	
31	4	
32	4	
33	6	
34	3	
35	----	
ST4	47	
ST1	66	
ST2	56	
ST3	44	
ST4	47	
Tot.	213	

PRINT NAME __________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

You are to classify the first order ordinary differential equations given below. The classification relates to the method of solution. Recall from class (attendance is mandatory) the possible methods listed below. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is wrong. DO NOT SOLVE. Also recall the following:

a. In this context, exact means exact as given (in either of the forms discussed in class).

b. Bernoulli is not a correct method of solution if the original equation is linear.

c. Homogeneous (use the substitution $v = y/x$) is not a correct method of solution if it converts a separable equation into another separable equation.

1. (4 pts.) $xye^{x+y}\,dx + dy = 0$ _____ A B C D E

2.(4 pts.) $(y^2 + xy)dx + x\,dy = 0$ _____ A B C D E

3. (4 pts.) $(e^x + 2xy + x)dx + (x^2 + 2y)dy = 0$ _____ A B C D E

4. (4 pts.) $(xy + \cos(x))dx + (1 + x^2)\,dy = 0$ _____ A B C D E

5. (4 pts.) $(y^2 + x^2)dx + x^2\,dy = 0$ _____ A B C D E

Possible answers this page.

A) First order linear (y as a function of x). B) First order linear (x as a function of y).

C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).

E) Bernoulli, but not linear (y as a function of x).

AB) Bernoulli, but not linear (x as a function of y)

AC) Homogeneous, but not separable. AD) None of the above techniques works.

Total points this page = 20. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(__________________) ID No. ___________________
Last Name, First Name MI What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

Consider the first order linear ODE $y' = y + x$ (*). To solve (*), you may need to change it to a standard form.

6. (1 pts.) The correct standard form for (*) is ________________________. _____ A B C D E
A) $y' + y = x$ B) $y' + y = -x$ C) $y' - y = x$ D) $y' - y = -x$ E) $y' + 2y = x$ AB) $y' + 2y = -x$ AC) $y' - 2y = x$ AD) $y' - 2y = -x$ AE) $y' + y = 2x$ BC) $y' + y = -2x$ BD) $y' - y = 2x$ BE) $y' - y = -2x$ CD) $y' + 2y = 2x$ CE) $y' + 2y = -2x$ DE) $y' - 2y = 2x$ ABC) $y' - 2y = -2x$ ABD) None of the above

7. (2 pts.) An integrating factor for (*) is μ=________________________. ______ A B C D E
A) x B) $-x$ C) x^2 D) $-x^2$ E) $2x$ AB) $-2x$ AC) $2x^2$ AD) $-2x^2$ AE) e^x AD) e^{-x} AE) e^{2x} BC) e^{-2x} BD) e^{x^2} BE) e^{-x^2} CD) None of the above

8. (3 pts.) In solving (*) as we did in class (attendance is mandatory), the following step occurs:

____. ______ A B C D E
A) $\frac{d(ye^x)}{dx} = xe^x$ B) $\frac{d(ye^x)}{dx} = -xe^x$ C) $\frac{d(ye^x)}{dx} = 2xe^x$ E) $\frac{d(ye^x)}{dx} = -2xe^x$ AB) $\frac{d(ye^{-x})}{dx} = xe^{-x}$ AC) $\frac{d(ye^{-x})}{dx} = -xe^{-x}$
AD) $\frac{d(ye^{-x})}{dx} = 2xe^{-x}$ AE) $\frac{d(ye^{-x})}{dx} = -2xe^{-x}$ BC) $\frac{d(ye^{2x})}{dx} = xe^{2x}$ BD) $\frac{d(ye^{2x})}{dx} = -xe^{2x}$ BE) $\frac{d(ye^{2x})}{dx} = 2xe^{2x}$
CD) $\frac{d(ye^{2x})}{dx} = -2xe^{2x}$ CE) $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$ DE) $\frac{d(ye^{-2x})}{dx} = -xe^{-2x}$ ABC) $\frac{d(ye^{-2x})}{dx} = 2xe^{-2x}$ ABD) $\frac{d(ye^{-2x})}{dx} = -2xe^{-2x}$
ABE) None of the above steps ever appears in any solution of this problem.

9. (1pt.) Let (**) be the initial value problem consisting of (*) and the initial condition $y(0) = 0$.
The number of solutions to (**) is _____________. _____A B C D E A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Possible points this page = 7. TOTAL POINTS EARNED _________

PRINT NAME ________________________(___________________) ID No. _________________
Last Name, First Name MI What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer. Be careful. If you miss one part, it may cause you to miss other parts.

An ODE may be considered to be a "vector" equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE, call it (*), of the form L[y] = g(x) where L is of the form L[y] = y′ + p(x)y, an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^x)}{dx} = xe^x$, call it (**).

10. (2 pts.) The theorem from calculus that allows you to integrate the Left Hand Side of (**)

is ____. ____ A B C D E

A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem D) Chain Rule E) Product Rule AB) Fundamental Theorem of Calculus AC) Integration by Parts AD) Partial Fractions AE) None off the above

11. (4 pts.) The solution (or family of solutions) to the ODE (*) may be written

as____. _______ A B C D E

A) $y = x + 1 + c\,e^x$ B) $y = -x + 1 + c\,e^x$ C) $y = x - 1 + c\,e^x$ D) $y = -x - 1 + c\,e^x$ E) $y = x + 1 + c\,e^{-x}$ AB) $y = -x + 1 + c\,e^{-x}$ AC) $y = x - 1 + c\,e^{-x}$ AD) $y = -x - 1 + ce^{-x}$ AE) $y = 2x + 2 + ce^x$ BC) $y = -2x + 2 + ce^x$ BD) $y = 2x - 2 + ce^x$ BE) $y = -2x - 2 + c\,e^x$ CD) $y = 2x + 2 + ce^{-x}$ CE) $y = -2x + 2 + ce^{-x}$ DE) $y = 2x - 2 + ce^{-x}$ ABC) $y = -2x - 2 + ce^{-x}$ ABD) None of the above solutions or families of solutions is correct.

PRINT NAME ________________________(__________________) ID No. ________________
Last Name, First Name MI What you wish to be called

Let L, p(x), g(x), (*), and (**) be as on the previous page.

12. (1 pts.) The solution set for the ODE (*) on the previous page may be written as

S = ____. _______ A B C D E

A) $\{y = x + 1 + c\,e^x : c \in \mathbf{R}\}$ B) $\{y = -x + 1 + c\,e^x : c \in \mathbf{R}\}$ C) $\{y = x - 1 + c\,e^x : c \in \mathbf{R}\}$
D) $\{y = -x - 1 + c\,e^x : c \in \mathbf{R}\}$ E) $\{y = x + 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AB) $\{y = -x + 1 + c\,e^{-x} : c \in \mathbf{R}\}$
AC) $\{y = x - 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AD) $\{y = -x - 1 + ce^{-x} : c \in \mathbf{R}\}$ AE) $\{y = 2x + 2 + e^x + c : c \in \mathbf{R}\}$ BC) $\{y = -2x + 2 + ce^x : c \in \mathbf{R}\}$ BD) $\{y = 2x - 2 + ce^x : c \in \mathbf{R}\}$ BE) $\{y = -2x - 2 + ce^x : c \in \mathbf{R}\}$
CD) $\{y = 2x + 2 + ce^{-x} : c \in \mathbf{R}\}$ CE) $\{y = -2x + 2 + ce^{-x} : c \in \mathbf{R}\}$ DE) $\{y = 2x - 2 + ce^{-x} : c \in \mathbf{R}\}$ ABC) $\{y = -2x - 2 + ce^{-x} : c \in \mathbf{R}\}$ DE) None of the above correctly describe the solution set.

13. (1 pt.) A basis for the nullspace of L is B =________________________. ______A B C D E
A) {1} B) {x} C) {1,x} D) $\{e^x\}$ E) $\{e^{-x}\}$ AB) $\{e^x, e^{-x}\}$ AC) None of the above

14. (1 pt.) The general solution of L[y] = 0 is $y_c(x)$ =________________. ______A B C D E
A) c B) cx C) $c_1 + c_2x$ D) ce^x E) ce^{-x} AB) $c_1e^x + c_2e^{-x}$ AC) None of the above

15. (1 pt.) A particular solution of L[y] = g(x) is given by

$y_p(x)$ =________________________. ______A B C D E A) 1 B) x C) x + 1
D) x − 1 E) 1 − x AB) −x −1 AC) e^x AD) e^{-x} AE) None of the above

16. (1 pt.) The number of solutions to (*) is ________._____A B C D E A) 0 B) 1
C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

17. (2 pts.) Let (***) be the initial value problem consisting of (*) and the initial condition y(0) = 0. The solution (or family of solutions) to (***) may be written

as____. _______ A B C D E
A) $y = x + 1 - e^x$ B) $y = -x + 1 - e^x$ C) $y = x - 1 + e^x$ D) $y = x + 1 - e^{-x}$
E) $y = -x + 1 - e^{-x}$ AB) $y = x + 2 - 2e^{-x}$ AC) $y = x - 1 + e^{-x}$ AD) $y = x + 1 + e^x - 2 + c$
AE) $y = -x + 1 + e^x - 2$ BC) $y = x - 1 + e^x$ BD) $y = x + 1 + e^{-x} - 2$ BE) $y = -x + 1 + e^{-x} - 2$
CD) $y = x - 1 + e^{-x}$ CE) $y = x + 1 - e^{-x}$
DE) None of the above solutions or families of solutions is correct.

18. (1 pt.) The number of solutions to (***) is _________._____A B C D E A) 0 B) 1
C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Possible points this page = 8. TOTAL POINTS EARNED _________

PRINT NAME _________________________(________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let $A=\begin{bmatrix}1 & -i\\ -i & -1\end{bmatrix}$, $\vec{x}=\begin{bmatrix}x\\ y\end{bmatrix}$, and $\vec{b}=\begin{bmatrix}-1\\ i\end{bmatrix}$. Also let $T(\vec{x}) = A\vec{x}$ so that $T:\mathbf{C}^2\to\mathbf{C}^2$. Now solve Prob($\mathbf{C}^2$, $A\vec{x}=\vec{b}$); that is, solve the vector equation $A\vec{x}=\vec{b}$ (i.e. $T(\vec{x})=\vec{b}$). The form of the answer may not be unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory).

19. (3 pts.) If $\left[A\middle|\vec{b}\right]$ is reduced to $\left[U\middle|\vec{c}\right]$ using Gauss elimination we obtain

$\left[U\middle|\vec{c}\right]$ =_____________________________. _____A B C D E A) $\left[\begin{array}{cc|c}1 & i & 1\\ 0 & 0 & 0\end{array}\right]$ B) $\left[\begin{array}{cc|c}1 & i & -1\\ 0 & 0 & 0\end{array}\right]$

C) $\left[\begin{array}{cc|c}1 & -i & 1\\ 0 & 0 & 1\end{array}\right]$ D) $\left[\begin{array}{cc|c}1 & -i & -1\\ 0 & 0 & 0\end{array}\right]$ E) $\left[\begin{array}{cc|c}1 & i & 1\\ 0 & 0 & 1\end{array}\right]$ AB) $\left[\begin{array}{cc|c}0 & 0 & 0\\ 0 & 0 & 0\end{array}\right]$ AC)None of the above.

20. (3 pts.) The solution of $A\vec{x}=\vec{b}$ may be written as

$\vec{x}$= _______________________________._____A B C D E A) No Solution B) $\begin{bmatrix}0\\ 1\end{bmatrix}$

C) $\begin{bmatrix}-i\\ 1\end{bmatrix}$ D) $y\begin{bmatrix}-i\\ 1\end{bmatrix}$ E) $\begin{bmatrix}1\\ 0\end{bmatrix}+y\begin{bmatrix}i\\ 1\end{bmatrix}$ AB) $\begin{bmatrix}1\\ 0\end{bmatrix}+y\begin{bmatrix}-i\\ 1\end{bmatrix}$ AC) $\begin{bmatrix}-1\\ 0\end{bmatrix}+y\begin{bmatrix}i\\ 1\end{bmatrix}$ AD) $\begin{bmatrix}-1\\ 0\end{bmatrix}+y\begin{bmatrix}-i\\ 1\end{bmatrix}$ BC) None of the above correctly describes the solution or collection of solutions.

PRINT NAME __________________________(__________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let $\text{Prob}(\mathbf{C}^2; A\vec{x} = \vec{b}), A, \vec{b}, \vec{x}, \text{ and } T$ be as on the previous page.

21. (1 pt.) The solution set for Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$) may be written as

S =______________________.______A B C D E A) $\varnothing$ B) C) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ $\left\{\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

D) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ E) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AB) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

AC) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ AD) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

BC) None of the above correctly describes the solution set for this problem

22. (1 pt.) A basis for the null space of T is B =__________________________. ______A B C D E

A) $\varnothing$ B) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ C) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ D) $\left\{\begin{bmatrix} i \\ 1 \end{bmatrix}\right\}$ E) $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ AB) $\left\{\begin{bmatrix} i \\ -1 \end{bmatrix}\right\}$ AC) $\left\{\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right\}$ AD) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ BC)

None of the above correctly describes the solution set for this problem

23. (1 pt.) The general solution of $A\vec{x} = \vec{0}$ is $\vec{x}_c$ =______________________. ______A B C D E

A) No Solution B) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ C) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ E) $y\begin{bmatrix} -i \\ -1 \end{bmatrix}$ AB) $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$

AC) $y\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ AD) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AE) None of the above

24. (1 pt.) A particular solution of $A\vec{x} = \vec{b}$ is given by

$\vec{x}_p$ =__________________________. ______A B C D E A) $A\vec{x} = \vec{b}$ has no solutions B) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ C) $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $\begin{bmatrix} i \\ -1 \end{bmatrix}$ E) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ AB) $\begin{bmatrix} i \\ 1 \end{bmatrix}$ AC) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ AD) $\begin{bmatrix} i \\ -1 \end{bmatrix}$

AE) None of the above is a solution of $A\vec{x} = \vec{b}$

25. (1 pt.) The number of solutions to Prob($\mathbf{C}^2$; $A\vec{x} = \vec{b}$) is __________._____A B C D E A) 0
B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume $T: V \to W$ is a linear operator from a (real or complex) vector space V to a (real or complex) vector space W. Now consider the mapping problem defined by the vector equation

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

26. (1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has a solution.

27. (1 pt.) A)True or B)False The vector equation (*) may have exactly two solutions.

28.(1 pt.) A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} = \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where $c_1, c_2, \ldots, c_n$ are arbitrary constants.

29. (1 pt.) A)True or B)False Either (*) has no solutions or exactly one solution (no other possibilities).

30. (1 pt.) A)True or B)False If the null space of T is $N(T) = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has a unique solution.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(__________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

The dimension of the null space N_L of the linear operator $L[y] = y'' - y'$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that $B_{N_L} = \{1, e^{x}\}$ is a basis of N_L so that

$y_c = c_1 + c_2\ e^{x}$ is the general solution of $y'' - y' = 0$.

Use the method of undetermined coefficients as discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

31. (3 pts.) $y'' - y' = -2x$ First guess: y_p = ___________________________

Second guess (if needed): y_p = ______________________________

Third guess (if needed): y_p = ________________________________

Final guess ____ . ________ A B C D E

32.(3 pts.) $y'' - y' = 3\sin x$ First guess: y_p = ___________________________

Second guess (if needed): y_p = ______________________________

Third guess (if needed): y_p = ________________________________

Final guess ____ . _______ A B C D E

33. (3 pts.) $y'' - y' = -4e^{x}$ First guess: y_p = ___________________________

Second guess (if needed): y_p = ______________________________

Third guess (if needed): y_p = ________________________________

Final guess ____ . _____ A B C D E

Possible Answers:

A) Ae^{x} B) Axe^{x} C) Ax^2e^{x} D) $Axe^{x} + Be^{x}$ E) $Ax^2e^{x} + Bxe^{x}$ AB) Ae^{-x} AC) Axe^{-x}
AD) Ax^2e^{-x} AE) $Axe^{-x} + Be^{-x}$ BC) $Ax^2e^{-x} + Bxe^{-x}$ BD) $A\sin x$ BE) $A\cos x$
CD) $A\,x\sin x$ CE) $A\,x\cos x$ DE) $A\sin x + B\cos x$ ABC) $A\,x\sin x + B\,x\cos x$
ABD) A ABE) Ax ACD) $Ax + B$ ACE) $Ax^2 + Bx$ ADE) $Ax^2 + Bx + C$
BCD) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let Prob($\mathcal{A}((-\pi/2,\pi/2),\mathbf{R})$, (*)) be the problem defined by the ODE

$$y'' + y = \sec(x) \quad I = (-\pi/2, \pi/2) \qquad (*)$$

Let L: $\mathcal{A}((-\pi/2,\pi/2),\mathbf{R})\to\mathcal{A}((-\pi/2,\pi/2),\mathbf{R})$ be defined by L[y] = y'' + y. The general solution to L[y] = 0 is $y_c = c_1\cos(x) + c_2\sin(x)$. To obtain a particular solution of L[y] = tan(x) we let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. You are to find y_p. Be careful!! Remember, once you make a mistake, the rest is wrong.

34. (3 pts.) Substituting $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ into (*) and making the appropriate assumption you obtained the two equations:

____ . ____A B C D E

A) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$
B) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\tan(x)$
C) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
D) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\tan(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
E) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sec(x)$
AB) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\sec(x)$
AC) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \sec(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
AD) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\sec(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
AE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Let Prob($\mathcal{A}$((−π/2,π/2),**R**), (*)), (*), L and $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be as defined on the previous page.

35. (2 pts.) Solving the set of equations for $u'_1(x)$ and $u'_2(x)$ on the previous page we obtain

$u'_1(x)$ =____. ____A B C D E

36. (2 pts.) And $u'_2(x)$ = ____. ____A B C D E

37. (2 pts.) Hence we may choose $u_1(x)$ =_________________________________. ____A B C D E

38. (2 pts.) And $u_2(x)$ =____. ____A B C D E

39 (2 pts.) Hence a particular solution to (*) is

$y_p(x)$ = ____. ___A B C D E

Possible answers this page.
A) 0 B) 1 C) x D) x^2 E) sin x AB) −sin x AC) cos x AD) −cos x AE) sin(x) cos(x)
BC) 2sin(x) cos(x) BD) $-\sin^2(x)/\cos(x)$ BE) −ln(tan(x) +sec(x)) CD)−ln(tan(x) + sec(x)) +sin(x) CE) −[cos(x)] ln(tan(x) +sec(x)) DE)−[sin(x)]ln(tan(x)+sec(x)) ABC)−[tan(x)]ln(tan(x)+sec(x)) ABD)−[sin(x) cos(x)] ln(tan(x) +sec(x) ABE) ln(cos(x)) ACD)ln(sin(x)) ACE)−ln(cos(x)) ADE)−ln(sin(x)) BCD) (cos(x))ln(cos(x)) + x sin(x) ACD) −(cos(x)) ln(sin(x)) + x sin(x)
ACE)−(cos(x)) ln(cos(x)) + x sin(x) ADE)−(cos(x)) ln(sin(x)) + x sin(x) BCD) None of the above.
Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. If you miss one part, it may cause you to miss other parts.

Consider $y^{IV} + 4y''' + 4y'' = 0$ (*) Also let L:$\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} + 4y''' + 4y''$ and answer the following questions.

40. (1 pt). The dimension of the null space of L is ____________________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

41. (1 pts). The auxiliary equation for (*) is ______________________________ . ____ A B C D E
A) $r^4 - 4r^2 + 4 = 0$ B) $r^4 + 4r^3 + 4r^2 = 0$ C) $r^4 - 4r^3 + 4r^2 = 0$ D) $r^5 + 4r^4 + 4r^3 = 0$
E) $r^5 - 4r^4 + 4r^3 = 0$ AB) $r^3 + 4r^2 + 4r = 0$ AC) $r^3 - 4r^2 + 4r = 0$ AD) None of the above.

42. (2 pts). Listing repeated roots, the roots of the auxiliary equation are

r = ______________________________. ______ A B C D E A) 0, 2, 2 B) 0, −2, −2
C) 0, 0, 2, 2 D) 0, 0, −2, −2 E) 0, 0, 0, 2, 2 AB) r = 0, 0, 0, −2, −2 AC) None of the above.

43. (1 pts). A basis for the null space of L is B = ___________________________. ___ A B C D E
A) $\{1, e^{2x}, xe^{2x}\}$ B) $\{1, e^{-2x}, xe^{-2x}\}$ C) $\{1, x, e^{2x}, xe^{2x}\}$ D) $\{1, x, e^{-2x}, xe^{-2x}\}$
E) $\{1, x, x^2, e^{2x}, xe^{2x}\}$ AB) $\{1, x, x^2, e^{-2x}, xe^{-2x}\}$ AC) $\{1, x, x^2, e^{-2x}\}$
AD) $\{1, x, x^2, x^3\}$ AE) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ BC) None of the above

44. (2 pt). The general solution of (*) is y(x) = ____________________________ . ___ A B C D E
A) $c_1 + c_2 e^{2x} + c_3 xe^{2x}$ B) $c_1 + c_2 e^{-2x} + c_3 xe^{-2x}$ C) $c_1 + c_2 x + c_3 e^{2x} + c_4 e^{2x}$
D) $c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$ E) $c_1 + c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{2x}$ AB) $c_1 + c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{2x}$
AC) $c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $c_1 + c_2 x + c_3 x^2 + c_4 x^3$ AE) $c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$ BC) None of the above

Points this page = 7. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Let (*) and L be as on the previous page.

45. (1pt.) The solution set for (*) may be written as

S = ____. ______A B C D E

A) $\{y(x) = c_1 + c_2\, e^{2x} + c_3\, xe^{2x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
B) $\{y(x) = c_1 + c_2\, e^{-2x} + c_3\, xe^{-2x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
C) $\{y(x) = c_1 + c_2\, x + c_3\, e^{2x} + c_4 x\, e^{2x}:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$
D) $\{y(x) = c_1 + c_2\, x + c_3\, e^{-2x} + c_3 x\, e^{-2x}:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$
E) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{2x} + c_5 x\, e^{2x}:\ c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$
AB) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x} + c_5\, xe^{-2x}: c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$
AC) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x}:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$
AD) $\{y(x) = c_1 + c_2\, x + c_3\, x^2 + c_4\, e^{-2x}:\ c_1, c_2, c_3, c_4 \in \mathbf{R}\}$ AE) None of the above

46. (1 pt.) The number of solutions to (*) is _____________._____A B C D E A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

Points this page = 2. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME ________________________(________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Consider the ODE $y''' + y'' = 4e^x + 4\cos(x)$ (*). Let $L[y] = y''' + y''$.

47. (3 pts.) The general solution of $y''' + y'' = 0$ is

$y_c(x) =$ ____. ____ A B C D E

48. (4 pts.) A particular solution of $y''' + y'' = 4e^x$ is

$y_{p1}(x) =$ ____. ____ A B C D E

49. (4 pts.) A particular solution of $y''' + y'' = 4\cos(x)$ is

$y_{p2}(x) =$ ____. _____ A B C D E

50. (1 pts.) A particular solution of (*) is

$y_p(x) =$ ____. _____ A B C D E

Possible answers this page

A) $c_1 + c_2x + c_3e^x$ B) $c_1 + c_2x + c_3e^{-x}$ C) $c_1 + c_2e^x + c_3e^{-x}$ D) $c_1e^x + c_2\sin(x) + c_3\cos(x)$
E) $c_1 + c_2\sin(x) + c_3\cos(x)$ AB) $c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$ AC)) $2e^x$ AD) $-2e^x$ AE)) $2x\,e^x$
BC) $2x\,e^x + e^x$ BD) $2x\,e^x + e^x$ BE) $2\sin(x)$ CD) $2\cos(x)$ CE) $2\sin(x)+2\cos(x)$ DE) $2\sin(x)-2\cos(x)$
ABC) $-2\sin(x)+2\cos(x)$ ABD) $-2\sin(x)-2\cos(x)$ ABE) $2e^x+2\sin(x)+2\cos(x)$
ACD) $2e^x+2\sin(x)-2\cos(x)$ ACE) $2e^x-2\sin(x)+2\cos(x)$ ADE) $2e^x-2\sin(x)-2\cos(x)$
BCD) $3x-2\sin(x)-10\cos(2x)$ BCE) $-2e^x+2\sin(x)+2\cos(x)$ BDE) $-2e^x+2\sin(x)-2\cos(x)$
CDE) $-2e^x-2\sin(x)+2\cos(x)$ ABCD) $-2e^x-2\sin(x)-2\cos(x)$ ABCE) None of the above.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let (*) and L be as on the previous page.

51. (2 pts.) The general solution of (*) is

y(x) = ____. _____ A B C D E

A) $2e^x + 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}$ B) $2e^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ C) $2e^x - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ D) $2e^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x$ E) $-2e^x + 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}$ AB) $-2e^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ AC) $-2e^x - 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}$ AD) $2e^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x$ AE) $2e^x + c_1\sin(x) + c_2\cos(x) + c_3e^{-x}$ BC) $2e^x + 2\sin(x)+2\cos(x) + c_1e^x + c_2e^{-x} + c_3x$ BD) $2e^{-x} + 2\sin(x) + \cos(x) + c_1xe^x + c_2e^{-x} + c_3xe^{-x}$ BE) $e^x + 2\sin(2x) + \cos(2x) + c_1e^x + c_2xe^x + c_3$ CD) None of the above.

52. (1pt.) The solution set for (*) may be written as

S = ____. ______A B C D E

A) $\{y(x) = 2e^x + 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
B) $\{y(x) = 2e^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
C) $\{y(x) = 2e^x - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
D) $\{y(x) = 2e^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x:\ c_1, c_2, c_3 \in \mathbf{R}\}$
E) $\{y(x) = -2e^x + 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
AB) $\{y(x) = -2e^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
AC) $\{y(x) = -2e^x - 2\sin(x)+2\cos(x) + c_1 + c_2x + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
AD) $\{y(x) = 2e^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x:\ c_1, c_2, c_3 \in \mathbf{R}\}$
AE) $\{y(x) = 2e^x + c_1\sin(x) + c_2\cos(x) + c_3e^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
BC) $\{y(x) = 2e^x + 2\sin(x)+2\cos(x) + c_1e^x + c_2e^{-x} + c_3x:\ c_1, c_2, c_3 \in \mathbf{R}\}$
BD) $\{y(x) = 2e^{-x} + 2\sin(x) + \cos(x) + c_1xe^x + c_2e^{-x} + c_3xe^{-x}:\ c_1, c_2, c_3 \in \mathbf{R}\}$
BE) $\{y(x) = e^x + 2\sin(2x) + \cos(2x) + c_1e^x + c_2xe^x + c_3:\ c_1, c_2, c_3 \in \mathbf{R}\}$ CD) None of the above

53. (1 pt.) The number of solutions to (*) is _____________._____A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) Infinite number of solutions AD) None of the above

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the Laplace transform of the following functions in PC[0,∞)∩Exp ⊆ **T**.

54. (3 pts.) $f(t) = 2t + 3t^2$ $\mathscr{L}(f)$ = ___________________________________. _______ A B C D E

55. (3 pts.) $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ $\mathscr{L}(f)$ = ________________________________. _______ A B C D E

56 (3 pts.) $f(t) = 2\sin(2t) + 3\cos(3t)$ $\mathscr{L}(f)$ = __________________________. _______ A B C D E

Possible answers this page

A) $\frac{2}{s}+\frac{3}{s^2}$ B) $\frac{2}{s}-\frac{3}{s^2}$ C) $\frac{2}{s^2}+\frac{3}{s^3}$ D) $\frac{2}{s^2}-\frac{3}{s^3}$ E) $\frac{2}{s^2}+\frac{6}{s^3}$ AB) $\frac{2}{s^2}-\frac{6}{s^3}$

AC) $\frac{2}{s+2}+\frac{3}{s+3}$ AD) $\frac{2}{s-2}+\frac{3}{s+3}$ AE) $\frac{2}{s+2}+\frac{3}{s-3}$ BC) $\frac{2}{s-2}+\frac{3}{s-3}$

BD) $\frac{2}{(s-2)^2}+\frac{3}{(s+3)^2}$ BE) $\frac{2}{s^2+2}+\frac{3s}{s^2+3}$ CD) $\frac{2s}{s^2+2}+\frac{3}{s^2+3}$ CE) $\frac{4}{s^2+4}+\frac{3s}{s^2+9}$

DE) $\frac{2s}{s^2+4}+\frac{9}{s^2+9}$ ABC) $\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ ABD) $\frac{2s}{s^2-4}+\frac{9}{s^2-9}$

ABE) $\mathscr{L}\{f\}$ exists but none of the above is $\mathscr{L}\{f\}$ ACD) $\mathscr{L}\{f\}$ does not exist.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ____________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions in **F**:

57. (3 pts.)) F(s) = $\frac{2}{s}\frac{3}{s-2}$ $\mathcal{L}^{-1}\{F\}$ = ______________________________. _____ A B C D E

58. (3 pts.) F(s) = $\frac{2s-4}{s^2+9}$ $\mathcal{L}^{-1}\{F\}$ = _________________________________. _____ A B C D E

59. (3 pts.) F(s) = $\frac{2s+3}{s^2-2s+2}$ $\mathcal{L}^{-1}\{F\}$ = _______________________________. ____ A B C D E

A) $2 + 3e^{2t}$ B) $2 + 3e^{-2t}$ C) $2 - 3e^{2t}$ D) $2 - 3e^{-2t}$ E) $2 + e^{-2t}$ AB) 2 cos 3t + 4 sin 3t
AC) 2 cos 3t − 4 sin 3t AD) 2 cos 3t + (4/3) sin 3t AE) 3 cos 3t − (4/3) sin 3t
BC) 2 cos t + 3 sin t BD) $2e^t \cos t + 5e^t \sin t$ BE) $2e^t \cos t - 5e^t \sin t$
CD) $2e^t \cos t + e^t \sin t$ CE) $2e^t \cos t - e^t \sin t$ DE) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Using the procedure illustrated in class (attendance is mandatory) find the eigenvalues of $A = \begin{bmatrix} i & -4 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

60. (2 pts.) A polynomial p(λ) where solving p(λ) = 0 yields the eigenvalues of A can be written

as p(λ) =____. ______ A B C D E
A) (i+λ)(1+λ) B) (i+λ)(1−λ) C) (i−λ)(1+λ) D) (i−λ)(1−λ) E) (i+λ)(2+λ) AB) (i+λ)(2−λ)
AC) (i−λ)(2+λ) AD)(i−λ)(2−λ) AE) (2i+λ)(1+λ) BC)(2i+λ)(1−λ) BD)(2i−λ)(1+λ)
BE) (2i−λ)(1−λ) CD) (2i+λ)(2+λ) CE)(2i+λ)(2−λ) DE)(2i−λ)(2+λ) ABC)(2i−λ)(2−λ)
ABD) (3i−λ)(2+λ) ABE)None of the above.

p(λ) =

61. (1 pt.) The degree of p(λ) is __________. ______ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

62. (1 pt.) Counting repeated roots, the number of eigenvalues of A

is _________. _____ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5
AC) 6 AD) 7 AE) 8 BC) None of the above

63. (2 pts.) The eigenvalues of A can be written as ______________________._____ A B C D E
A) $\lambda_1 = 1, \lambda_2 = i$ B) $\lambda_1 = 1, \lambda_2 = -i$ C) $\lambda_1 = -1, \lambda_2 = i$ D) $\lambda_1 = -1, \lambda_2 = -i$ E) $\lambda_1 = 2, \lambda_2 = i$
AB) $\lambda_1 = 2, \lambda_2 = -i$ AC) $\lambda_1 = -2, \lambda_2 = i$ AD) $\lambda_1 = -2, \lambda_2 = -i$ AE) $\lambda_1 = 1, \lambda_2 = 2i$
BC) $\lambda_1 = 1, \lambda_2 = -2i$ BD) $\lambda_1 = -1, \lambda_2 = 2i$ BE) $\lambda_1 = -1, \lambda_2 = -2i$ CD) $\lambda_1 = 2, \lambda_2 = 2i$
CE) $\lambda_1 = 2, \lambda_2 = -2i$ DE) $\lambda_1 = -2, \lambda_2 = 2i$ ABC) $\lambda_1 = -2, \lambda_2 = -2i$ ABD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Note that $\lambda_1 = -1$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$

64. (4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with λ_1 is B = _____________________. ___A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$
AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$
BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$
ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$
ACD) $\lambda = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda = -1$ is not an eigenvalue of the matrix A
ADE)None of the above is correct.

65. (1pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, since the eigenspace associated with λ_1 is one dimensional and we have developed conventions for selecting a basis for the eigenspace associated with λ_1, we say that <u>the</u> eigenvector associated

with λ_1 is ____. ______A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$
AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$
BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$
ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$
ACD) $\lambda_1 = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda_1 = -1$ is not an eigenvalue of the matrix A ADE)None of the above is correct.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Consider the scalar equation $u'' - 4u' - 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du'/dt$). As was done in class (attendance is mandatory) convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y). You may think of x as the position and y as the velocity of a point particle). This system of two scalar equations can be written in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. You are to find

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; that is you are to find a, b, c, and d.

66. (1 pt.) a = ______. ____ A B C D E

67. (1 pt.) b = ______. ____ A B C D E

68. (1 pt.) c = ______. ____ A B C D E

69. (1 pt.) d = ______. ____ A B C D E

Possible answers this page.
A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9
BD) −1 BE) −2 CD) −3 CE) −4 DE) −5 ABC) −6 ABD) −7 ABE) −8 ACD) −9
ACE) None of the above

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let the 2x2 matrix A have the eigenvalue table

TABLE

Eigenvalues	Eigenvectors
$r_1 = -1$	$\vec{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
$r_2 = -2$	$\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}^2)\to\mathcal{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}] = \vec{x}' - A\vec{x}$

and let the null space of L be N_L

70. (1 pt). The dimension of N_L is _________. ______ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

71. (2 pts.) A basis for the null space of L is_____________________________ . ____ A B C D E

A) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{t},\begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ B) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{-t},\begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ C) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{t},\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$ D) $B=\left\{\begin{bmatrix}2\\1\end{bmatrix}e^{-t},\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$

E) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ AB) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ AC) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ AD) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ AE) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}\right\}$ BC) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ BD) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$

BE) $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ CD) None of the above

72. (2 pts.) The general solution of $\vec{x}' = A\vec{x}$ is _______________________________. ____ A B C D E

A) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ B) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ C) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$ D) $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$

E) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ AB) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ AC) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ AD) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$

AE) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}$ BC) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ BD) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$

BE) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ CD)None of the above

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _____________

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

True or False. Let f and g be real valued functions of a real variable; that is, f:**R** → **R** and g:**R** → **R**. Circle True if the statement is true. Circle False if the statement is false.

73. (1 pt.) A)True B)False The function f is even if $f(-x) = f(x)\ \forall\ x \in \mathbf{R}$.

74. (1 pt.) A)True B)False The function f is even if $f(-x) = -f(x)\ \forall\ x \in \mathbf{R}$.

75. (1 pt.) A)True B)False If f and g are both odd functions, then the product of f and g is an even function.

76. (1 pt.) A)True B)False The function f is periodic of period T if $f(x + T) = f(x)\ \ \forall\ x \in \mathbf{R}$.

77. (1 pt.) A)True B)False If f is an even function, then we know that $\int_{-\ell}^{\ell} f(x)\,dx = 0$.

For each of the following questions write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters

Classify the following function with regard to whether they are odd or even.

78. (1 pt.) $f(x) = -x$ is ____. ______ A B C D E

79. (1 pt.) $f(x) = -3$ is ____. _______ A B C D E

80.(1 pt.) $f(x) = \sin(x)$ is ____. ______ A B C D E

81. (1 pt.) f(x) = *-x* is ____. _______ A B C D E

82. (1 pt.) $f(x) = -e^{x}$ is ____. ________A B C D E

83. (1 pt.) $f(x) = 0$ is ____. ________ A B C D E

Possible answers for questions 78-83.
A) odd, but not even B) even, but not odd C) both odd and even
D) neither odd nor even E) none of the above

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions Also circle your answer.

Let f;**R**→**R** be in $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈F(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [ )ℓ,ℓ], and $f(x) = \frac{f(x+)+f(x-)}{2}$ at points of discontinuity} so that its Fourier series exists.

84. (2 pts.) The formula for the general Fourier series for $f \in PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ given in our text

is f(x) = ____ . ______ A B C D E

A) $a_0 + \sum_{n=1}^{N} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$ B) $a_0 + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$

C) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$ D) $\frac{a_0}{2} + \sum_{n=0}^{N} a_n \cos(n\pi x) + b_n \sin(n\pi x)$

E) $\frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$ AB) $\frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + b_n \cos(n\pi x)$

AC) None of the above

85. (2pts.) where for n = 0, 1, 2, ... we have a_n = ________________________. _____ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ B) $\frac{2}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ C) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$

D) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ E) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(n\pi x)dx$ AB) $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\cos(x)dx$

AC) None of the above.

86. (2pts.) and for n = 1, 2, ... we have b_n = __________________________. _____ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ B) $\frac{2}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ C) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$

D) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ E) $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(n\pi x)dx$ AB) $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\sin(x)dx$

AC) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Recall from the previous page that $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈F(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [ )ℓ,ℓ], and f(x) = (f(x+) + f(x))/2 at points of discontinuity}. Now let $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ and $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ are inner product spaces with inner product (f,g) = $\int_{-\ell}^{\ell} f(x)g(x)dx$, that $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)} = \{1/2\}\cup\{\cos(\frac{n\pi}{\ell}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ is an orthogonal Schauder basis for $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$, and that $B_{PC^1_{ffs}(\mathbf{R},\mathbf{R},\ell)}$ is an orthogonal Hamel basis of $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$.

87. (1 pt.) Using the notation given above, an orthogonal Hamel basis for $PC^1_{ffs}(\mathbf{R},\mathbf{R};2)$

is ____. ___A B C D E Hint: What is ℓ?

A) $\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$
B) $\{1/2\}\cup\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$
C) $\{1/2\}\cup\{\cos(\frac{n\pi}{4}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{4}):n\in\mathbf{N}\}$
D) $\{1/2\}\cup\{\cos(\frac{n\pi}{3}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{3}):n\in\mathbf{N}\}$
E) $\{(1/2)x\}\cup\{\cos(\frac{n\pi}{2}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{2}):n\in\mathbf{N}\}$
AB) None of the above.

88. (2 pts.) The Fourier series for the function f(x)∈ $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ which has period 4 and is defined on the interval [-2,2) by f(x) = 3 + 2 cos(2πx) + 2 sin(3π x) is

f(x) =____. ___A B C D E
Hint: Think Hamel basis.

A) $3 + \sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$
B) $2 + \sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$
C) $3 + \sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$
D) $2 + \sum_{k=1}^{\infty}\frac{1}{k\pi}\sin(k\pi x)$
E) 3+ 3cos(2πx) + 2sin(3πx)
AB) 3+ 2cos(2πx) + 3sin(3πx)
AC) 3 + 2cos(πx) + 2sin(3πx)
AD) 3+ 2cos(2πx)+ 2 sin(πx)
AE) 2+ 2cos(2πx) + 2sin(3π x)
BC) 2 + 2cos(πx) + 2sin(3π x)
BD) 3 + 2cos(2πx) + 2sin(3πx)
BE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ and $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)}$ be as on the previous page. Now let $f(x) \in PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ be the function whose domain is **R** which has period 4 and is defined on the interval (−2,2) by $f(x) = \begin{cases} 0 & -2 < x < 0 \\ 2 & 0 < x < 2 \end{cases}$. Using the formulas on the previous page, determine the Fourier series for the function f. Begin by sketching f for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges

89. (1 pt.) To apply the formulas given on the previous page we choose ℓ = _____.___ A B C D E
A) $\ell = 1$ B) $\ell = 2$ C) $\ell = 3$ D) $\ell = 4$ E) $\ell = -1$ AB) $\ell = -2$ AC) $\ell = -3$
AD) $\ell = -4$ AE) None of the above

Next write down the formulas for a Fourier series and its coefficients using this value of ℓ. After computing the a_n's and the b_n's, note what they are for n odd and n even. Then answer the questions below and on the next two pages.

90. (3 pts.) We have a_0 = __________. _____ A B C D E A) 0 B) 1 C) 2 D) 3
E) 4 AB) −1 AC) −2 AD) −3 AE) −4 BC) None of the above

a_0 =

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Let f(x) be as on the previous page. Continue the computation of the Fourier Series coefficients.

91. (3 pts.) For a_n with n odd (n = 1, 3, 5, ...) so that for k = 0,1,2,3,.. we have

a_{2k+1} =_________________________. ______ A B C D E A) 0 B) 1/(2k+1)
C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π]
AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1)
CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π]
ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] BCD)None of the above

a_n =

92. (3 pts.) For a_n with n even (n= 2, 4, 6, ...) so that for k = 1,2,3,... we have

a_{2k} =_________________________. _____ A B C D E A) 0 B) 1/(2k) C) 1/k
D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ)
BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ)
ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) BCD)None of the above

93. (3 pts.) For b_n with n odd (n = 1, 3, 5, ...) so that for k = 0,1,2,3,... we have

b_{2k+1} =_________________________. ____ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π]
AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1)
CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π]
ABE) −8/[(2k+1)π] BCD)None of the above

b_n =

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ______________________(_______________) ID No. _______________
Last Name, First Name MI, What you wish to be called

Let f(x) be as on the page before the previous page. Continue the computation of the Fourier Series coefficients.

94. (3 pts.)For b_n with n even (n = 2, 4, 6, ...) so that for k = 1,2,3,...

we have b_{2k} =____________________. _____ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) BCD)None of the above

95. (3 pts.) Thus the Fourier series for f(x) may be written as

f(x) = ____.______ A B C D E

A) $1 + \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ B) $1 + \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

C) $1 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ D) $1 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \cos\frac{(2k+1)\pi}{2} x)$

E) $1 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ AB) $1 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

AC) $2 + \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ AD) $2 + \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

AE) $2 + \sum_{k=1}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ BC) $2 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

BD) $2 + \sum_{k=1}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ BE) $2 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

CD) $1 - \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ CE) $1 - \sum_{k=0}^{\infty} \frac{1}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

DE) $1 - \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ ABC) $1 - \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

ABD) $1 - \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ ABE) $1 - \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

ACD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Consider the Partial Differential Equation (PDE): $t\,u_{tt} - x\,u_{xx} = 0$

96. (4 pts.) Using the method of separation of variables with separation constant λ, one of the following sets of two Ordinary Differential Equations (ODE's) can be obtained from this PDE. Recall that the process does not yield a unique set of ODE's. Following the advice given in class as to the choice of separation constant (attendance is mandatory) we may obtain the set of

ODE's ____. ____ A B C D E

A) $x X'' + \lambda X = 0,\quad t T'' - \lambda T = 0$
B) $X'' + \lambda x X = 0,\quad T'' - \lambda t T = 0$
C) $X'' + \lambda x X = 0,\quad t T'' - \lambda T = 0$
D) $\lambda X'' + x X = 0,\quad \lambda T'' - t T = 0$
E) $x X'' + \lambda X = 0,\quad t T'' + \lambda T = 0$
AB) $X'' + \lambda x X = 0,\quad T'' + \lambda t T = 0$
AC) $X'' - \lambda x X = 0,\quad t T'' - \lambda T = 0$
AD) $\lambda X'' + x X = 0,\quad \lambda T'' + t T = 0$
AE) $X'' + \lambda X = 0,\quad t T'' - \lambda T = 0$
BC) $X'' + \lambda X = 0,\quad T'' - \lambda t T = 0$
BD) $X'' + \lambda x X = 0,\quad T'' - \lambda T = 0$
BE) $\lambda X'' + x X = 0,\quad \lambda T'' - T = 0$
CD) Separation of variables does not work on this PDE.
CE) Separation of variables works on this PDE, but none of the above is correct.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ____________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Recall the definition of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ given on page 22. Now let $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions, $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ be the functions in $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to $[0,\ell]$, and $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ be the subspace of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ for which the Fourier (sine) series is finite. Recall that $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ is a Hamel basis of $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ and a Schauder basis of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$. To formulate the heat conduction in a rod problem as a linear mapping problem we let D = $(-\ell,\ell)\times(0,\infty)$, $\bar{D} = [-\ell,\ell]\times[0,\infty)$, $\mathcal{A}_{HC}(\bar{D},\mathbf{R}) = \{u(x,t)\in\mathcal{F}(\bar{D},\mathbf{R})$: $u\in \mathcal{A}(D,\mathbf{R})\cap C(\bar{D},\mathbf{R})\}$ and $\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}) = \{u(x,t)\in\mathcal{A}_{HC}(\bar{D},\mathbf{R}))$: $u(0,t) = 0$ and $u(\ell,t) = 0$ for $t > 0\}$. Now let $L_B:\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}) \to \mathcal{A}(D,\mathbf{R})$ be defined by $L_B[u] = u_t - \alpha^2 u_{xx}$, Thus we incorporate the boundary conditions into the domain of the operator. Now let N_{L_B} be the null space of L_B and $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2,\ell)$ be the problem defined by

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
BC	$u(0,t) = 0$, $u(\ell,t) = 0$,	$t > 0$

so that the solution set of $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}),L_B[u] = 0; \alpha^2,\ell)$) is the null space of L_B. Now let $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ be the problem defined by

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
BC	$u(0,t) = 0$, $u(\ell,t) = 0$,	$t > 0$
IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

and $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L$: $u(x,0)\in PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)\}$ and $\mathcal{A}_{HC,0,o,fs}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L$: $u(x,0)\in PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)\}$. We claim that if $u_0(x)\in PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$, then the solution of $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ is in $\mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R})$. Hence $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell) \subseteq \mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R}) \subseteq N_L \subseteq \mathcal{A}_{HC,0}(\bar{D},\mathbf{R}) \subseteq \mathcal{A}_{HC}(\bar{D},\mathbf{R})$. Recall that $B_{AH,0,o,fs(\bar{D},\mathbf{R};\alpha^2,\ell)} = \{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ is a Hamel basis for $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell)$ and a Schauder basis for $\mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R})$.

97. (1pt.) The set (which may be thought of as a subset of $L^2([0,\ell],\mathbf{R})$) that we (attendance is mandatory) considered to be the state space for $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2,\ell)$

is____._____A B C D E

A) D B) $\bar{D}$ C) $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ D) $\mathcal{A}_{HC,0}(\bar{D},\mathbf{R})$ E) $\mathcal{A}(D,\mathbf{R})$ AB) $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell)$ AC) N_L
AC) $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ AD) $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ AE) $\mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R})$ BC) None of the above

98. (1pt.) A Schauder basis for the state space may be taken to

be____._____A B C D E A) $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ B) $\mathcal{A}_{HC,0}(\bar{D},\mathbf{R})$ C) $\mathcal{A}(D,\mathbf{R})$ D) $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell)$ E) N_L AB) $\mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R})$ AC) $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$
AD) $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ AE) $B_{AH,0,o,fs(\bar{D},\mathbf{R};\alpha^2,\ell)} = \{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ BC) $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$
BD) $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)} = \{1/2\}\cup\{\cos(\frac{n\pi}{\ell}) : n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}) : n\in\mathbf{N}\}$ BE) None of the above

Total points this page = 2. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Let L_B, N_{LB}, $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$, all of the function spaces, and all of the basis sets be as on the previous page.

99. (1pt.) A Schauder basis for N_{L_B}, the null space of L_B, may be taken to

be____.______A B C D E

A) $\mathcal{A}_{HC}(\bar{D},\mathbf{R})$ B) $\mathcal{A}_{HC,0}(\bar{D},\mathbf{R})$ C) $\mathcal{A}(D,\mathbf{R})$ D) $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell)$ E) N_L AB) $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$

AC) $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ AD) $\mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R})$ AE) $B_{AH,0,o,fs(\bar{D},\mathbf{R};\alpha^2,\ell)} = \{e^{-\left(\frac{\alpha n\pi}{\ell}\right)^2 t}\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$

BC) $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ BD) $B_{PC^1_{fs}(\mathbf{R},\mathbf{R},\ell)} = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$

BE) None of the above

100. (2 pts.) The "general" or formal solution of $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B(x,t) = 0; \alpha^2,\ell)$ is given

by u(x,t) =____. _____ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\sin(\frac{\ell}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t}\sin(\frac{n\pi}{\ell}x)$ D) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\sin(n\pi\ell x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t}\cos(\frac{n\pi}{\ell}x)$ AD) $\sum_{n=1}^{\infty} c_n \cos(\frac{n\pi}{\ell}x)$ AE) None of the above.

PRINT NAME ________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let L_B, N_{LB}, $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0; \alpha^2, \ell)$, all of the function spaces, and all of the basis sets be as on the page before the previous page.

101. (2 pts.) The "general" or formal solution of PDE $u_t = u_{xx}$ $0 < x < 2$, $t > 0$

BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

(we denote this problem by $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}),L_B[u] = 0; 1, 2)$)

is given by u(x,t) =____. ______ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{2}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$ D) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(2n\pi x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\cos(\frac{n\pi}{2}x)$ AD) $\sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2}x) + b_n\cos(\frac{n\pi}{2}x)$ AE) None of the above.

102. (4 pts.) The solution of

BVP for a PDE
PDE $u_t = u_{xx}$ $0 < x < 2$, $t > 0$
BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$
IC $u(x,0) = 6\sin(6\pi x)$ $0 < x < 2$

(we denote this problem by $Prob_D(\mathcal{A}_{HC,0}(\bar{D},\mathbf{R}),L_B[u] = 0, 6\sin(6\pi x); 1, 2)$)

is given by u(x,t) =____. _____A B C D E

A) $\sum_{n=1}^{\infty} 6e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ B) $6e^{-\frac{\pi^2}{4}t}\sin(\pi x)$ C) $6e^{-\frac{9\pi^2}{4}t}\sin(\frac{3\pi}{2}x)$ D) $6e^{-36\pi^2 t}\sin(6\pi x)$

E) $\sum_{n=1}^{\infty} 6e^{-36\pi^2 t}\sin(6\pi x)$ AB) $6e^{-12\pi^2 t}\sin(6\pi x)$ AC) $6e^{-12\pi^2 t}\sin(3\pi x)$ AD) $6e^{-144\pi^2 t}\sin(6\pi x)$

AE) None of the above.

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Let $\text{Prob}_D(A_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$ be the problem defined by

PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell,\ t > 0$

BC $u(0,t) = 0,\quad u(\ell,t) = 0,\quad t > 0$

IC $u(x,0) = u_0(x)$ $0 < x < \ell$

103. (2 pts.) The formula for the solution of $\text{Prob}_D(A_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = u_0(x) ; \alpha^2, \ell)$

is given by u(x,t) =____. _____ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{\ell}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2\pi^2\ell^2 t}\sin(\frac{n\pi}{\ell}x)$

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(n\pi\ell x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2\pi^2}{\ell^2}t}\cos(\frac{n\pi}{\ell}x)$ AD) $\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{\ell}x)$ AE) None of the above

104. (2 pts.) where the formula for c_n is c_n =___________________________. ____ A B C D E

A) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{\ell}{n\pi}x\right)dx$ B) $\frac{1}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ C) $\frac{2}{\ell}\int_{-\ell}^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ D) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ E) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\sin(n\pi\ell x)dx$ AB) $\int_0^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ AC) $\frac{1}{\ell}\int_{-\ell}^{\ell} u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ AD) $\frac{2}{\ell}\int_0^{\ell} u_0(x)\cos\left(\frac{n\pi}{\ell}x\right)dx$ AE) None of the above.

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Let $\text{Prob}_D(\mathcal{A}_{HC,0}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 2\ ; 1, 2)$ be the problem defined by

PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
IC	$u(x,0) = 2$	$0 < x < 2$

105. (2 pts.) The formula for the solution of $\text{Prob}_D(\mathcal{A}_{HC,0}(\bar{D}, \mathbf{R}), L_B[u] = 0, u(x,0) = 2\ ; 1, 2)$ is given by $u(x,t) =$____. ___ A B C D E

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{2}{n\pi}x)$ B) $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ C) $\sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t} \sin(\frac{n\pi}{2}x)$

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ E) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(2n\pi x)$ AB) $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \cos(\frac{n\pi}{2}x)$ AD) $\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{2}x)$ AE) None of the above.

106. (2 pts.) where the formula for c_n is $c_n =$____________________________. _____ A B C D E

A) $2\int_0^2 \sin\left(\frac{2}{n\pi}x\right)dx$ B) $\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ C) $2\int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$ D) $2\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

E) $2\int_0^2 \sin(2n\pi x)dx$ AB) $4\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AC) $\int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AD) $c_n = 2\int_0^2 \cos\left(\frac{n\pi}{2}x\right)dx$

AE) None of the above.

PRINT NAME ________________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let $Prob_D(A_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = 2 ; 1, 2)$ be as on the previous page.

107. (2pts.) Computing c_n using the formula on the previous page, for n odd (n = 1, 3, 5, ...) we have for k = 0,1,2,3,... that c_{2k+1} =_______________. ____ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] BCD)None of the above

c_n =

108. (2 pts.)For c_n with n even (n = 2, 4, 6, ...) we have for k = 1,2,3,...that

c_{2k} =____________________. ____ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) BCD)None of the above

109. (2 pts.) Hence the solution of $Prob_D(A_{HC,0}(\bar{D},\mathbf{R}), L_B[u] = 0, u(x,0) = 2 ; 1, 2)$ may be written

as u(x,t) = ____. ______ A B C D E

A) $\sum_{k=0}^{N}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ B) $\sum_{k=0}^{N}\frac{8}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k+1)\pi}{2}x)$ D) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

E) $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k+1)\pi}{2}x)$ AB) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{4}x)$

AC) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k+1)\pi}{4}x)$ AD) $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

AE) $\sum_{n=1}^{\infty}\frac{8}{(2k)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k)\pi}{2}x)$ BC) $\sum_{k=1}^{\infty}\frac{4}{k\pi}e^{-\frac{k^2\pi^2}{4}t}\sin(\frac{k\pi}{2}x)$ BD) $\sum_{k=1}^{\infty}\frac{2}{k\pi}e^{-\frac{k^2\pi^2}{2}t}\sin(\frac{k\pi}{2}x)$ BE) None of the above.

PRINT NAME ____________________________(____________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

108. (3 pts.) If both ends of a rod are insulated (recall that we assume that the lateral sides are also insulated so that the temperature does not vary over a cross section), then a good mathematical model of the physical heat conduction problem is given by:

BVP for a PDE	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
	BC	$u_x(0,t) = 0$, $u_x(\ell,t) = 0$,	$t > 0$
	IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

where $u_0(x)$ is the initial temperature distribution in the rod.

```
           ////////////////////////////////////////////
       -→ O____________________________________O ←-
     /    ////////////////////////////////////////////     \
 insulated                                          insulated
           *-------------------------------------*
         x = 0                                  x = ℓ
```

Applying the separation of variables process to the PDE results in the two ODE's:

1. $X'' + \lambda X = 0$
2. $T' + \alpha^2 \lambda T = 0$

where λ is the separation constant. The spacial eigenvalue problem that results from

applying the BC given above is ______________________________________ ._____ A B C D E

A) $X'' + \lambda X = 0$, $X(0) = 0$, $X(\ell) = 0$

B) $X'' + \lambda X = 0$, $X'(0) = 0$, $X(\ell) = 0$

C.) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(\ell) = 0$

D) $X'' + \lambda X = 0$, $X(0) = 1$, $X(\ell) = 0$

E) $X'' + \lambda X = 0$, $X'(0) = 0$, $X'(\ell) = 0$

AB) $X'' + \lambda X = 0$, $X'(0) = 1$, $X'\ell) = 0$

AC) $X'' + \lambda X = 0$, $X(0) = 1$, $X(\ell) = 1$

AD) $X'' + \lambda X = 0$, $X'(0) = 1$, $X(\ell) = 0$

AE) $X'' + \lambda X = 0$, $X(0) = 0$, $X'(\ell) = 0$

BC) None of the above.

PRINT NAME ___________________________(__________________) SS No. ____________________
Last Name, First Name MI, What you wish to be called

TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	$s > \|a\|$
cosh (at)	$\frac{s}{s^2 - a^2}$	$s > \|a\|$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
u(t)	$\frac{1}{s}$	$s > 0$
u(t - c)	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	F(s - c)	
f(ct) c > 0	$\frac{1}{c} F(\frac{s}{c})$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	

PRINT NAME ______________________(_________________) ID No. __________________
Last Name, First Name MI, What you wish to be called

PARTIAL TABLE OF ANTIDERIVATIVES

1. $\int x[\sin(ax)]dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + c$

2. $\int x[\cos(ax)]dx = \frac{1}{a^2}\cos(ax) - \frac{x}{a}\sin(ax) + c$

3. $\int x^2[\sin(ax)]dx = \frac{2x}{a^2}\sin(ax) - \frac{a^2x^2 - 2}{a^3}\cos(ax) + c$

4. $\int x^2[\cos(ax)]dx = \frac{2x}{a^2}\cos(ax) - \frac{a^2x^2 - 2}{a^3}\sin(ax) + c$

5. $\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

6. $\int \cos^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

7. $\int [\sin(ax)][\cos(ax)]dx = \frac{1}{2a}\sin^2(ax) + c$

8. $\int [\sin(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

9. $\int [\cos(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

10. $\int [\sin(ax)][\cos(bx)]dx = -\frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$