MATH 261
FALL 2006
FINAL EXAM

MATH 261: Elementary Differential Equations
FINAL EXAM
EXAMINATION COVER PAGE

MATH 261
FALL 2006
Professor Moseley

PRINT NAME____(________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Wednesday, December 13, 2006, 8 am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

ST1	75	

ST2

_________________________________ _______________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 32 pages of questions and problems on this exam. Page 33 contains Laplace transforms you need not memorize. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! See me if you need scratch paper Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on the front of all sheets. Pages 1-32 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Every thought you have should be expressed in your best mathematics on this paper. Partial credit may be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

Scores

page	points	score
1	25	
2	6	
3	7	
4	8	
5	5	
6	9	
7	---	
8	15	

Scores

page	points	score
9	---	
10	8	
11	9	
12	9	
13	12	
14	6	
15	5	
16	3	

Scores

page	points	score
17	13	
18	11	
19	3	
20	4	
21	---	
22	10	
23	9	
24	5	

Scores

page	points	score
25	6	
26	---	
27	5	
28	6	
29	---	
30	6	
31	5	
32	4	
33	---	
ST4	32	
ST1	75	
ST2	52	
ST3	55	
ST4	32	

Tot.	214	

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Last Name, First Name MI, What you wish to be called

For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. You are to classify the first order ordinary differential equations given below. The classification relates to the method of solution. Recall from class (attendance is mandatory) the possible methods listed below. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is wrong. DO NOT SOLVE. Also recall the following: a. In this context, exact means exact as given (in either of the forms discussed in class). b. Bernoulli is not a correct method of solution if the original equation is linear.

c. Homogeneous (use the substitution v=y/x) is not a correct method of solution if it converts a separable equation into another separable equation.

1. (5 pts.) $xye^{x+y}\,dx + dy = 0$ _____ A B C D E

2.(5 pts.) $(y^2 + xy)dx + x\,dy = 0$ _____ A B C D E

3. (5 pts.) $(e^x + 2xy + x)dx + (x^2 + 2y)dy = 0$ _____ A B C D E

4.(5 pts.) $(xy + \cos(x))dx + (1 + x^2)\,dy = 0$ _____ A B C D E

5. (5 pts.) $(y^2 + x^2)dx + x^2\,dy = 0$ _____ A B C D E

Possible answers this page.

A) First order linear (y as a function of x). B) First order linear (x as a function of y).

C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).

E) Bernoulli, but not linear (y as a function of x).
AB) Bernoulli, but not linear (x as a function of y)
AC) Homogeneous, but not separable.   AD) None of the above techniques works.

Total points this page = 25. TOTAL POINTS EARNED THIS PAGE _______

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Write each answer in the blank provided and then circle the letter or letters that corresponds to your answer from the answers listed. Then circle this letter or letters. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss all parts. Consider the Ordinary Differential Equation (ODE) $y' = 2y + x^2$ (*)

6. (1 pts.) The correct standard form for solving the first order linear ODE (*)

is ____. _____ A B C D E

A) $y' = 2y + x^2$ (It's already in the appropriate form for solving a first order linear ODE)
B) $y' + 2y = x^2$ C) $y' + 2y + x^2$ D). $y' - 2y = x^2$
E) $y' - 2y - x^2 = 0$ AB) None of the above

7. (2 pts.) An integrating factor μ for (*) is _________________________. ______ A B C D E

A) $\mu = 2x$ B) $\mu = x^2$ C) $\mu = e^{2x}$ D) $\mu = e^{-2x}$ E) $\mu = e^{-x^2}$
AB) $\mu = e^{x^2}$ AC) $\mu = e^x$ AD) $\mu = e^{-x}$ AE) None of the above

8. (3 pts.) In solving (*) which of the following step occurs:

____. ______ A B C D E

A) $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$ B) $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$ C) $\frac{d(ye^{2x})}{dx} = x^2e^{2x}$
D) $\frac{d(ye^{2x})}{dx} = 2xe^{2x}$ E) $\frac{d(ye^{-2x})}{dx} = x^2e^{-2x}$ AB) $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$
AC) $\frac{d(ye^{2x})}{dx} = x^3$ AD) None of the above steps ever appears in any solution of this problem.

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Then circle this letter or letters. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss all parts. To solve a first order linear ODE, we isolate the unknown function on the left side of the equation. Recall that an ODE is really a “vector” equation with the infinite number of unknown variables being the values of the function for each value of the independent variable in the function’s domain. The isolation of the dependent variable (or function) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^{x})}{dx} = xe^{x}$ (*).

9. (2 pts.) The theorem from calculus that allows you to integrate the **Left Hand Side** of

this equation is ____. ____ A B C D E

A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem D) Chain Rule E) Fundamental Theorem of Calculus AB) Product Rule
AC) Integration by Parts AD) Partial Fractions AE) None off the above

10. (5 pts.) The solution or family of solutions to the ODE (*)

is ____. _______ A B C D E

A) $y = x + 1 + c\,e^{x}$ B) $y = -x + 1 + c\,e^{x}$ C) $y = x - 1 + c\,e^{x}$ D) $y = x + 1 + c\,e^{-x}$
E) $y = -x + 1 + c\,e^{-x}$ AB) $y = x + 2 + c\,e^{-x}$ AC) $y = x - 1 + c\,e^{-x}$ AD) $y = x + 1 + e^{x} + c$
AE) $y = -x + 1 + e^{x} + c$ BC) $y = x - 1 + e^{x} + c$ BD) $y = x + 1 + e^{-x} + c$
BE) $y = -x + 1 + e^{-x} + c$ CD) $y = x - 1 + e^{-x} + c$ CE) $y = x + 1 - e^{-x} + c$
DE) None of the above solutions or families of solutions is correct.

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Then circle this letter or letters. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss both parts. You are to solve $\underset{2x2}{A}\ \underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ where $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$. Be sure you write your answer according to the directions given in class for these kinds of problems (attendance is mandatory).

11. (4 pts.) If $\left[A\middle|\vec{b}\right]$ is reduced to $\left[U\middle|\vec{c}\right]$ using Gauss elimination, then

$\left[U\middle|\vec{c}\right]$ = ________________________________. ______ A B C D E A) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & i \end{array}\right]$

B) $\left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$ C) $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ D) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$ E) $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ AB). None of the above

12. (4 pts.) The solution of $\underset{2x2}{A}\ \underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ is ____________________________. ____A B C D E

A) No Solution B) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ C) $\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ D) $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$ E) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

AB) $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ AC) $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ AD) $\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$

AE) None of the above

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True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a vector space V to a vector space W. Now consider

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

13. (1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) always has at least one solution.

14. (1 pt.) A)True or B)False The vector equation (*) may have exactly two solutions.

15.(1 pt.) A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} \neq \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where $c_1, c_2, \ldots, c_n$ are arbitrary constants.

16. (1 pt.) A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

17. (1 pt.) A)True or B)False If the null space of T is $N(T) = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has a unique solution.

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The dimension of the null space of the linear operator $L[y] = y'' - y'$ that maps $A(\mathbf{R},\mathbf{R})$ to $A(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that

$y_c = c_1 + c_2 e^{x}$ is the general solution of $y'' - y' = 0$.

Use the method discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

18. (3 pts.) $y'' - y' = 2xe^{-x}$ First guess: y_p = ________________________
Second guess (if needed): y_p = ___________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . _______ A B C D E

19.(3 pts.) $y'' - y' = 3 \sin x$ First guess: y_p = ________________________
Second guess (if needed): y_p = ___________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . ______ A B C D E

20. (3 pts.) $y'' - y' = -4e^{x}$ First guess: y_p = ________________________
Second guess (if needed): y_p = ___________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . _____ A B C D E

Possible Answers:
A) Ae^{x} B) Axe^{x} C) Ax^2e^{x} D) $Axe^{x} + Be^{x}$ E) $Ax^2e^{x} + Bxe^{x}$ AB) Ae^{-x} AC) Axe^{-x}
AD) Ax^2e^{-x} AE) $Axe^{-x} + Be^{-x}$ BC) $Ax^2e^{-x} + Bxe^{-x}$ BD) $A \sin x$ BE) $\cos x$
CD) $A x \sin x$ CE) $A x \cos x$ ABC) $A \sin x + B \cos x$ ABD) $A x \sin x + B x \cos x$
ABCDE) None of the above

PRINT NAME __________________________(_________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

You are to solve y'' + y = tan(x) I = (−π/2, π/2) (i.e. −π/2 < x < π/2) on this page. To obtain a particular solution let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. Remember, once you make a mistake, the rest is wrong. This page will not be graded. After working this problem, answer the questions on the next page. They will be graded. On the next page, write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss all parts.

PRINT NAME _________________________(_________________) IDo. __________________

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You solved y'' + y = tan(x) (*) I = (−π/2 , π/2) (i.e −π/2 < x < π/2) on the previous page.

Now let L: $\mathcal{A}((0,\pi/2),\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by L[y] = y'' + y and answer the following questions.

21. (2 pts.) The general solution of L[y] = 0 is __________________________. ____ A B C D E

A) $c_1 \cos(2x) + c_2 \sin(2x)$ B) $c_1 \cos(x) + c_2 \sin(x)$ C) $c_1 e^x + c_2 e^{-x}$ D) $c_1 x + c_2$

E) r = ±i AB) r = ±1 AC). r = ±2i AD) None of the above.

22. (3 pts.) To find a particular solution y_p to (*)you let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$.
Substituting into the ODE and making the appropriate assumption you obtained the two equations:

____ . ____A B C D E

A) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

B) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$

C) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x)$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$

D) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sin(x)$

E) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \cos(x)$

AB) None of the above.

23. (3 pts.) Solving these two equations we obtain:

____. _____ A B C D E

A) $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = \sin(x)$ B) $u'_1(x) = \sin(x)$, $u'_2(x) = -\sin^2(x)/\cos(x)$

C) $u'_1(x) = 1$, $u'_2(x) = \sin(x)$ D) $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = 1$

E) $u'_1(x) = 0$, $u'_2(x) = \sin(x)$ AB) $u'_1(x) = \sin(x)$, $u'_2(x) = 1$ AC) None of the above.

24. (3 pts.) Hence we may choose ____________________________________. ____A B C D E A) $u_1(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, $u_2(x) = -\cos(x)$

B) $u_1(x) = -\cos(x)$, $u_2(x) = -\ln(\tan(x) + \sec(x))$ C) $u_1(x) = x$, $u_2(x) = -\cos(x)$

D) $u_1(x) = -\ln(\tan(x) + \sec(x))$, $u_2(x) = x$ E) $u_1(x) = 1$, $u_2(x) = -\cos(x)$

AB) $u_1(x) = -\cos(x)$, $u_2(x) = x$ AC) None of the above

25. (2 pts.) Hence $y_p(x)$ = ____.___ ___A B C D E

A) $-\ln(\tan(x) + \sec(x))$ B) $-[\cos(x)]\ln(\tan(x) + \sec(x))$, C. $-[\sin(x)]\ln(\tan(x) + \sec(x))$

D) $-[\tan(x)]\ln(\tan(x) + \sec(x))$ E) $\sin(x)\cos(x)$ AB) $2\sin(x)\cos(x)$

AC) $-[\sin(x)\cos(x)]\ln(\tan(x) + \sec(x))$ AD) None of the above.

26. (2 pts.) Hence the general solution of (*) is__________________________. _____A B C D E

A) $-\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$ B) $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$,

C) $-[\sin(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$ D) $\sin(x)\cos(x) + c_1 e^x + c_2 e^{-x}$

E) $-\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$ AB) $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$

AC) None of the above.

Total points this page = 15. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Solve $y^{IV} - 4y''' + 4y'' = 0$ on this page and answer the questions on the next page. Be careful as once you make a mistake, the rest is wrong.. On the next page, write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss all parts.

PRINT NAME ___________________________(__________________) ID No. ___________________

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer. Be careful. No part credit for this problem. Hence if you miss one part, it may cause you to miss all parts. You solved $y^{IV} - 4y''' + 4y'' = 0$ (*) on the previous page. Now let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$ and answer the following questions by circling the correct answers.

27. (1 pt). The order of the ODE (*) is __________________. ____A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

28. (1 pt). The dimension of the null space of L is __________________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

29. (1 pts). The auxiliary equation for the ODE (*) is ______________________ . ___ A B C D E
A) $r^2 - 4r + 4 = 0$ B) $r^4 - 4r^2 + 4 = 0$ C) $r^4 - 4r^3 + 4r^2 = 0$ D) $r^6 + 4r^3 + 4r^2 = 0$
E) $r^6 - 4r^3 + 4r^2 = 0$ AB) None of the above.

30. (2 pts). Listing repeated roots, the roots of the auxiliary equation are _____________________.
A) r = 0, 2 B) r = 0, 0, 2, 2 C) r = 2, 2 D) r = 0,4 E) r = 0, 2,4
AB) r = 0, 0, −2, −2 AC) None of the above.

31. (2 pts). A basis for the null space of L is ________________________________. ___ A B C D E
A) $\{1, x, e^{-2x}, xe^{-2x}\}$ B) $\{1, x, e^{2x}, xe^{2x}\}$ C) $\{1, x, x^2, e^{2x}\}$ D) $\{1, e^{2x}\}$
E) $\{1, x, x^2, x^3\}$ AB) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ AC). $\{1, x, x^2, e^{-2x}\}$ AD) $\{1, e^{-2x}\}$
AE) None of the above

32. (1 pt). The general solution of (*) is __________________________________ . ___ A B C D E
A) $y(x) = c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$ B) $y(x) = c_1 + c_2 x + c_3 e^{2x} + c_4 xe^{2x}$
C) $y)x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x}$ D) $y(x) = c_1 + c_2 e^{2x}$
E) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$ AB) $y(x) = c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$
AC) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $y(x) = c_1 + c_2 e^{-2x}$ AE) None of the above

Points this page = 8. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME __________________________(__________________) ID No. ___________________
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Let $L:\mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y''' + y'$. The dimension of the null space of the linear operator L is 3. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the three linearly independent solutions $y_1 = 1$ and $y_2 = \cos(x)$ and $y_3 = \sin(x)$ so that a basis of the null space of L is $\{1, \cos(x), \sin(x)\}$. Hence we can deduce from the linear theory that

$y_c = c_1 + c_2 \cos(x) + c_3 \sin(x)$ is the general solution of $y''' + y' = 0$ on **R**.

Use the method discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

33. (3 pts.) $y''' + y' = \sin(x)$ First guess: y_p = __________________________
Second guess (if needed): y_p = _____________________________
Third guess (if needed): y_p = ______________________________

Final guess ____ . _____ A B C D E

34. (3 pts.) $y''' + y' = 4x^2$ First guess: y_p = __________________________
Second guess (if needed): y_p = _____________________________
Third guess (if needed): y_p = ______________________________

Final guess ____ . _____ A B C D E

35. (3 pts.) $y''' + y' = -4xe^{-x}$ First guess: y_p = __________________________
Second guess (if needed): y_p = _____________________________
Third guess (if needed): y_p = ______________________________

Final guess ____ . ______ A B C D E

Possible Answers
A) A B) Ax C) Ax^2 D) $Ax + B$ E) $Ax^2 + Bx$ AB) $Ax^2 + Bx + C$ AC) Ae^x
AD) Axe^x AE) Ax^2e^x BC) $Axe^x + Be^x$ BD) $Ax^2e^x + Bxe^x$ BE) Ae^{-x} CD) Axe^{-x}
CE) Ax^2e^{-x} DE) $Axe^{-x} + Be^{-x}$ ABC) $Ax^2e^{-x} + Bxe^{-x}$ ABD) $A \sin x$ ABE) $\cos x$
BCD) $A x \sin x$ BCE) $A x \cos x$ BDE) $A \sin x + B \cos x$ CDE) $A x \sin x + B x \cos x$ ABCD) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(__________________) ID No. ____________________
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Compute the Laplace transform of the following functions. Then circle the letter corresponding to the Laplace transform of the function.

36. (3 pts.) $f(t) = 2 + 3t$ $\mathcal{L}\{f\}$ = ____ A B C D E .

37. (3 pts.) $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ $\mathcal{L}\{f\}$ = ___________________________________ A B C D E.

38 (3 pts.) $f(t) = 2\sin(2t) + 3\cos(3t)$ $\mathcal{L}\{f\}$ = ___________________________ A B C D E

Possible Answers

A) $\frac{2}{s}+\frac{3}{s^2}$ B) $\frac{2}{s^2}+\frac{3}{s^3}$ C) $\frac{2}{s^2}+\frac{3/2}{s^3}$ D) $\frac{2}{s^2}+\frac{1}{s^3}$ E) $\frac{1}{s}+\frac{1}{s^2}$ AB) $2+\frac{3}{s}$

AC) $\frac{2}{s-2}+\frac{3}{s+3}$ AD) $\frac{2}{s+2}+\frac{3}{s-3}$ AE) $\frac{2}{s-3}+\frac{3}{s+2}$ BC) $\frac{2}{(s-3)^2}+\frac{3}{(s+3)^3}$

BD) $\frac{2}{s-4}+\frac{4}{s+2}$ BE) $\frac{2}{s^2+1}+\frac{3s}{s^2+4}$ CD) $\frac{2s}{s^2+4}+\frac{3}{s+9}$ CE) $\frac{2}{s^2-1}+\frac{3}{s^2-4}$

DE) $\frac{2}{s^2+4}+\frac{3}{s^2+9}$ ABC). $\frac{2}{s^2+4}+\frac{2s}{s^2+9}$ ABD) $\frac{2s}{s^2+4}+\frac{3}{s^2+9}$ ABE). None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ___________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed below.

Compute the inverse Laplace transform of the following functions:

39. (4 pts.) $F(s) = \frac{2}{s} + \frac{3}{s+2}$. $\mathcal{L}^{-1}\{F\}$ = _______________________________ . ____ A B C D E

40. (4 pts.) $F(s) = \frac{2s+4}{s^2+9}$. $\mathcal{L}^{-1}\{F\}$ = __________________________________. ____A B C D E

41. (4 pts.) $F(s) = \frac{2s+3}{s^2-2s+2}$. $\mathcal{L}^{-1}\{F\}$ = ________________________________ . ____A B C D E

Possible Answers
A) $2 + 2e^{-2t}$ B) $3 + 2e^{-2t}$ C) $2 + 2e^{-3t}$ D) $2 + 3e^{-2t}$ E) $2 + e^{-2t}$ AB) 2 cos 3t + (4/3) sin 3t
AC) 2 cos 3t + 4 sin 3t AD) 2 cos 2t + (4/3) sin 2t AE) 3 cos 3t + (4/3) sin 3t
BC) 2 cos 3t + 3 sin 3t BD) $2e^t \cos 3t + 5e^t \sin 3t$ BE) $2e^t \cos t + 5e^t \sin t$
CD) $2e^t \cos 3t + 2e^t \sin 3t$ CE) $5e^t \cos 3t + 5e^t \sin 3t$ DE. None of the above
Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ________________________(________________) ID No. _________________
Last Name, First Name MI, What you wish to be called

Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

Consider the matrix $A = \begin{bmatrix} i & 2 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

42. (1 pt.) The degree of the polynomial where the solution of $p(\lambda) = 0$ yields the eigenvalues of

A is __________. ___ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC)7 AD) None of the above

43. (2 pt.) The polynomial $p(\lambda)$ where the solution of $p(\lambda) = 0$ yields the eigenvalues of A may be

written as $p(\lambda)$ = ____. ____ A B C D E

A) $p(\lambda) = (\lambda-1)(\lambda - I)$ B) $p(\lambda) = (\lambda-1)(\lambda + i)$ C) $p(\lambda) = (\lambda+1)(\lambda - i)$
D) $p(\lambda) = (\lambda+1)(\lambda + I)$ E) $p(\lambda) = (\lambda-2)(\lambda - I)$ AB) $p(\lambda) = (\lambda-2)(\lambda + I)$
AC) $p(\lambda) = (\lambda+2)(\lambda - i)$ AD) $p(\lambda) = (\lambda-2)(\lambda + I)$ AE) $p(\lambda) = (\lambda+2)(\lambda + 2i)$
BC) $p(\lambda) = (\lambda+2)(\lambda + 2i)$ BD) $p(\lambda) = (\lambda+2)(\lambda - 2i)$ BE) $p(\lambda) = (\lambda-2)(\lambda + 2i)$
CD) None of the above

44. (1 pt.) Counting repeated roots, the number of eigenvalues that the matrix A has

is ______________ ____ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) None of the above

45. (2 pts.) The eigenvalues of A may be chosen as ____________________. ____ A B C D E

A) $\lambda_1 = 2,\ \lambda_2 = i$ B) $\lambda_1 = 1,\ \lambda_2 = -i$ C) $\lambda_1 = -1,\ \lambda_2 = i$ D) $\lambda_1 = -1,\ \lambda_2 = -i$
E) $\lambda_1 = 2,\ \lambda_2 = i$ AB) $\lambda_1 = 2,\ \lambda_2 = -i$ AC) $\lambda_1 = -2,\ \lambda_2 = i$ AD) $\lambda_1 = -2,\ \lambda_2 = -i$
AE) $\lambda_1 = -2,\ \lambda_2 = i$ BC) $\lambda_1 = -2,\ \lambda_2 = -2i$ BD) $\lambda_1 = -2,\ \lambda_2 = 2i$ BE) $\lambda_1 = 2,\ \lambda_2 = -2i$
CD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

$\lambda = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$

46. (5 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with this eigenvalue is ______________________. ___A B C D E

A) $B = \{[1,1]^T, [4,4]^T\}$ B) $B = \{[1,1]^T\}$ C) $B = \{[1,2]^T\}$ D) $B = \{[1,2]^T, [4,8]^T\}$
E) $B = \{[2,1]^T\}$ AB) $B = \{[1,3]^T\}$ AC) $B = \{[1,4]^T\}$ AD) $B = \{[4,1]^T\}$
AE) $B = \{[3,1]^T\}$ BC) $B = \{[1,-1]^T, [4,4]^T\}$ BD) $B = \{[1,-1]^T\}$ BE) $B = \{[1,-2]^T\}$
CD) $B = \{[1,-2]^T, [4,8]^T\}$ CE) $B = \{[2,1]^T\}$ DE) $B = \{[1,3]^T\}$ ABC) $B = \{[1,-4]^T\}$ ABD) $B = \{[4,-1]^T\}$ ABE) $B = \{[3,-1]^T\}$ BC) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

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Write your answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

Consider the scalar equation $u'' + 4u' - 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du'/dt$). As was done in class (attendance is mandatory) convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y:you may think of x as the position and y as the velocity of a point particle).

47. (3 pts.) This system of two scalar equations can be written in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix as ____. ____ A B C D E:

A) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ B) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ C) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

D). $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ E) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AB). $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

AC) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AD) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AE) None of the above

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

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Write your answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

Let the 2x2 matrix have the eigen value table

TABLE	
Eigenvalues	Eigenvectors
$r_1 = -1$	$\vec{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
$r_2 = 2$	$\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Let $L: \mathcal{A}(\mathbf{R},\mathbf{R}^2) \to \mathcal{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}] = \vec{x}' - A\vec{x}$.

48. (1 pt). The dimension of the null space of L is _________________. _____ A B C D E

A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

49. (3 pts.) A basis for the null space of L is____________________________ . ____ A B C D E

A) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} \right\}$ B) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t} \right\}$ C) $B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} \right\}$ D) $B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t} \right\}$

E) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$ AB) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t} \right\}$ AC) $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t} \right\}$

AD). $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$ AE) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ BC) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$

BD) $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ CD) None of the above

50. (2 pts.) The general solution of $\vec{x}' = A\vec{x}$ is ________________________________. ___ A B C D E

A) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ B) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$ C) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

D) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}$ E) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ AB) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$

AC) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ AD) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ AE) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$

BC) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ BD) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ BE) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ CD)None of the above

PRINT NAME ________________________(________________) ID No. _________________
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True or False. Let f and g be real valued functions of a real variable; that is, f:**R** → **R** and g:**R** → **R**. Circle True if the statement is true. Circle False if the statement is false.

51. (1 pt.) A)True B)False The function f is even if $f(-x) = -f(x)\ \forall\ x \in \mathbf{R}$.

52.(1 pt.) A)True B)False The function f is odd if $f(-x) = f(x)\ \forall\ x \in \mathbf{R}$.

53. (1 pt.) A)True B)False If f and g are both odd functions, then the product of f and g is an odd function.

54. (1 pt.) A)True B)False The function f is periodic of period T if $f(x + T) = f(x)\ \forall\ x \in \mathbf{R}$.

55.(1 pt.) A)True B)False If f and g are even functions, then we know that $\int_{-\ell}^{\ell} f(x)g(x)dx = 0$.

Circle Odd if the function is odd, but not even. Circle Even if the function is even, but not odd. Circle Both if the function is both Odd and Even. Circle Neither if the function is neither odd nor even.

56. (1 pt.) $f(x) = -x$ A) Odd B) Even C) Both D)Neither

57. (1 pt.) $f(x) = -3$ A) Odd B) Even C) Both D) Neither

58.(1 pt.) $f(x) = \sin(x)$ A) Odd B) Even C) Both D) Neither

59. (1 pt.) f(x) = *-x* A) Odd B) Even C) Both D). Neither

60. (1 pt.) $f(x) = -e^x$ A) Odd B). Even C) Both D) Neither

61. (1 pt.) $f(x) = 0$ A) Odd B). Even C) Both D) Neither

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

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Write your answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

Let $PC^1_{ls}(\mathbf{R},\mathbf{R};\ell)$ = {f∈F(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [ ) ℓ,ℓ], and f(x) = (f(x+) + f(x))/2 at points of discontinuity} and $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{ls}(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that $B = \{1/2\} \cup \{\cos(\frac{k\pi}{\ell}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is an orthogonal Hamel basis of

.

62. (1 pt.) An orthogonal Hamel basis for

is ____. ___A B C D E Hint: What is ℓ?

A) $\{\cos(\frac{k\pi}{2}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{2}) : k \in \mathbf{N}\}$ B) $\{1/2\} \cup \{\cos(\frac{k\pi}{2}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{2}) : k \in \mathbf{N}\}$

C) $\{1/2\} \cup \{\cos(\frac{k\pi}{4}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{4}) : k \in \mathbf{N}\}$ D) $\{1/2\} \cup \{\cos(\frac{k\pi}{3}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{3}) : k \in \mathbf{N}\}$

E) $\{(1/2)x\} \cup \{\cos(\frac{k\pi}{2}) : k \in \mathbf{N}\} \cup \{\sin(\frac{k\pi}{2}) : k \in \mathbf{N}\}$ AB) None of the above.

63. (2 pts.) The Fourier series for the function f(x) which has period 4 and is defined on the interval

[-2,2) by f(x) = 3 + 2 cos(2πx) + 2 sin(3π x) is__________________________. ___A B C D E

A) $f(x) = 3 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ B) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

C) $f(x) = 3 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$ D) $f(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{k\pi} \sin(k\pi x)$

E) $f(x) = 1 + \sum_{n=1}^{\infty} \frac{1}{k\pi} \cos(k\pi x)$ AB) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

AC) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$ AD) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ AE) 3 + 3 cos(2πx) + 2 sin(3π x) BC) 3 + 2 cos(2πx) + 3 sin(3π x)

BD) 3 + 2 cos(πx) + 2 sin(3π x) BE) 3 + 2 cos(2πx) + 2 sin(π x)

CD) 2 + 2 cos(2πx) + 2 sin(3π x) CE) 2 + 2 cos(πx) + 2 sin(3π x)

DE) 3 + 2 cos(2πx) + 2 sin(3π x) ABC) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

Let f;**R**→**R** be in $PC_s^1(\mathbf{R},\mathbf{R};\ell)$ = {f∈F(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [)ℓ,ℓ], and f(x) =(f(x+) + f(x))/2 at points of discontinuity} so that its Fourrier series exists.

64. (2 pts.) The formula for the general Fourier series for f ∈ $PC_s^1(\mathbf{R},\mathbf{R};\ell)$ given in our text

is ____ . ______ A B C D E

A) $f(x) = a_0 + \sum_{n=1}^{N} a_n\cos(\frac{n\pi}{\ell}x) + b_n\sin(\frac{n\pi}{\ell}x)$ B) $f(x) = a_0 + \sum_{n=0}^{\infty} a_n\cos(\frac{n\pi}{\ell}x) + b_n\sin(\frac{n\pi}{\ell}x)$

C) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(\frac{n\pi}{\ell}x) + b_n\sin(\frac{n\pi}{\ell}x)$ D) $f(x) = \frac{a_0}{2} + \sum_{n=0}^{N} a_n\cos(n\pi x) + b_n\sin(n\pi x)$

E) $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n\cos(\frac{n\pi}{\ell}x) + b_n\sin(\frac{n\pi}{\ell}x)$ AB) $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n\cos(n\pi x) + b_n\cos(n\pi x)$ AC) None of the above

65. (2pts.) where ____. ____ A B C D E

A) $a_n = \frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ..., $b_n = \frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

B) $a_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ... $b_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

C.) $a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

D). $a_n = \frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

E). $a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(n\pi x)dx$, n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(n\pi x)dx$ n = 1, 2, ...

AB) $a_n = \frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\cos(x)dx$, n = 0, 1, 2, ... $b_n = \frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\sin(x)dx$ n = 1, 2, ...

AC). None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ______

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Using the formula from the previous page, determine the Fourier series for the function f(x) which has period 4 and is defined on the interval [-2,2) by

Sketch f for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges. Next write down the general formulas for a Fourier series and its coefficients that you memorized. After computing the a_n's and the b_n's, note what they are for n odd and n even. Then answer the questions on the next two pages.

PRINT NAME __________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Write your answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer. On the previous page, you determined the Fourier series (including the Fourier series coefficients) for the function f(x) which

has period 4 and is defined on the interval [-2,2) by .

66. (1 pt.) To apply the formulas given on page 20 you chose ℓ = ____________.______ A B C D E
A) $\ell = 1$ B) $\ell = 2$ C) $\ell = 3$ D) $\ell = 4$ E) $\ell = -1$ AB) $\ell = -2$ AC) $\ell = -3$
AD). $\ell = -4$ AE) None of the above

67.(3 pts.) You then computed a_0 = __________. ______ A B C D E A) 0 B) 1 C) 2 D) 1/2
E) 2/3 AB) −1 AC) −2 AD) −1/2 AE) −2/3 BC) None of the above

68. (3 pts.)You then computed the odd a_n coefficients with n = 1, 3, 5, ... so that for k = 0,1,2,3,..

we have a_{2k+1} =____. _______ A B C D E
A) $a_{2k+1} = 0$ B)$a_{2k+1} = 1/(2k+1)$ C) $a_{2k+1} = 2/(2k+1)$ D) $a_{2k+1}=1/(2k+1)$
E) $a_{2k+1}=1/[(2k+1)\pi$ AB) $a_{2k+1}=1/[(2k+1)\pi]$ AC) $a_{2k+1}=2/[(2k+1)\pi]$
AD) $a_{2k+1}=\pi/(2k+1)]$ AE) $a_{2k+1}= [(2k+1)\pi]/2$ BC) $a_{2k+1}= [(2k+1)\pi]/3$
BD) $a_{2k+1}= [(2k+1)\pi]/4$ BE) $a_{2k+1}= [(2k+1)\pi$ CD) $a_{2k+1}=\pi/(2k+1)$
CE) $a_{2k+1}=1/(2k+1)$ DE) $a_{2k+1}=2/[(2k+1)\pi]$ ABC) None of the above

69. (3 pts.)You then computed the even a_n coefficients with n = 2, 4, 6, ... so that for k = 1,2,3,...

we have a_{2k} =____. ____ A B C D E A) $a_{2k}=0$
B) $a_{2k}=1/(2k)$ C) $a_{2k}= 2/(2k)$ D) $a_{2k}=1/(2k)$
E) $a_{2k}=\pi/(2k)$ AB) $a_{2k}=1/(2k\pi)$ AC) $a_{2k}=2/(2k\pi)$ AD) $a_{2k}=1/(2k)$
AE) $a_{2k}=\pi/(2k)$ BC) $a_{2k}=1/(2k\pi)$ BD) $a_{2k}=2/(2k\pi)$ BE) $a_{2k}=1/(2k+1)$
CD) $a_{2k}=\pi/(2k+1)$ CE) $a_{2k}=1/[(2k+1)\pi]$ DE) $a_{2k}=2/[(2k+1)\pi]$ ABC) None of the above

Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ________

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer. On the page before the previous page, you determined the Fourier series for the function f(x) which has period 4 and is

defined on the interval [-2,2) by .

70. (3 pts.)You computed the odd b_n coefficients with n = 1, 3, 5, ... so that for k = 0,1,2,3,...

we have b_{2k+1} =____. ____ A B C D E

A) $b_{2k+1} = 0$ B) $b_{2k+1} = 1/(2k+1)$ C) $b_{2k+1} = 2/(2k+1)$ D) $b_{2k+1}=1/(2k+1)$
E) $b_{2k+1}=1/[(2k+1)\pi$ AB) $b_{2k+1}=1/[(2k+1)\pi]$ AC) $b_{2k+1}=2/[(2k+1)\pi]$
AD) $b_{2k+1}=\pi/(2k+1)]$ AE) $b_{2k+1}= [(2k+1)\pi]/2$ BC) $b_{2k+1}= [(2k+1)\pi]/3$
BD) $b_{2k+1}= [(2k+1)\pi]/4$ BE) $b_{2k+1}= [(2k+1)\pi$ CD) $b_{2k+1}=\pi/(2k+1)$
CE) $b_{2k+1}=1/(2k+1)$ DE) $b_{2k+1}=2/[(2k+1)\pi]$ ABC) None of the above

71. (3 pts.)You then computed the even b_n coefficients with n = 2, 4, 6, ... so that for k = 1,2,3,...

we have b_{2k} =____. ____ A B C D E A) $b_{2k}=0$

B) $b_{2k}=1/(2k)$ C) $b_{2k}= 2/(2k)$ D) $b_{2k}=1/(2k)$
E) $b_{2k}=\pi/(2k)$ AB) $b_{2k}=1/(2k\pi)$ AC) $b_{2k}=2/(2k\pi)$ AD) $b_{2k}=1/(2k)$
AE) $b_{2k}=\pi/(2k)$ BC) $b_{2k}=1/(2k\pi)$ BD) $b_{2k}=2/(2k\pi)$ BE) $b_{2k}=1/(2k+1)$
CD) $b_{2k}=\pi/(2k+1)$ CE) $b_{2k}=1/[(2k+1)\pi]$ DE).$b_{2k}=2/[(2k+1)\pi]$ ABC) None of the above

72. (3 pts.) Thus the Fourier series may be written

as ____._____ A B C D E

A) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$ B) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

C) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$ D) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

E) $f(x) = 1 + \sum_{n=0}^{\infty} \frac{1}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$ AB) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

AC) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$ AD) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$

AE) $f(x) = 2 + \sum_{n=1}^{\infty} \frac{2}{(2k)\pi} \cos(\frac{(2k)\pi}{2} x)$ BC) $f(x) = 2 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$

BD) $f(x) = 1 + \sum_{n=1}^{\infty} \frac{4}{(2k)\pi} \cos(\frac{(2k)\pi}{2} x)$ BE) None of the above

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On thepage before the previous page, you determined the Fourier series (i.e., the Fourier series coefficients) as given in your text for the function f(x) which has period 4 and is defined on the interval [-2,2) by $f(x)=\begin{cases}0 & -2\le x\le 0\\ 2 & 0<x<2\end{cases}$.

70. (3 pts.)You computed the odd b_n coefficients with $n = 1, 3, 5, \ldots$ as

b_{2k+1} =______________________________________ with k = 0,1,2,3,...

A) $b_{2k+1} = 0$ B) $b_{2k+1} = 1/(2k+1)$ C) $b_{2k+1} = 2/(2k+1)$ D) $b_{2k+1}=1/(2k+1)$
E) $b_{2k+1}=1/[(2k+1)\pi$ AB).$b_{2k+1}=1/[(2k+1)\pi]$ AC) $b_{2k+1}=2/[(2k+1)\pi]$
AD) $b_{2k+1}=\pi/(2k+1)]$ AE) $b_{2k+1}= [(2k+1)\pi]/2$ BC) $b_{2k+1}= [(2k+1)\pi]/3$
BD) $b_{2k+1}= [(2k+1)\pi]/4$ BE) $b_{2k+1}= [(2k+1)\pi$ CD) $b_{2k+1}=\pi/(2k+1)$
CE) $b_{2k+1}=1/(2k+1)$ DE). $b_{2k+1}=2/[(2k+1)\pi]$ ABC) None of the above.

71. (3 pts.)You then computed the even b_n coefficients with $n = 2, 4, 6, \ldots$ as

b_{2k} =____ with k = 1,2,3,...

A) $b_{2k}=0$ B) $b_{2k}=1/(2k)$ C) $b_{2k}= 2/(2k)$ D) $b_{2k}=1/(2k)$
E) $b_{2k}=\pi/(2k)$ AB). $b_{2k}=1/(2k\pi)$ AC) $b_{2k}=2/(2k\pi)$ AD) $b_{2k}=1/(2k)$
AE) $b_{2k}=\pi/(2k)$ BC) $b_{2k}=1/(2k\pi)$ BD) $b_{2k}=2/(2k\pi)$ BE) $b_{2k}=1/(2k+1)$
CD) $b_{2k}=\pi/(2k+1)$ CE) $b_{2k}=1/[(2k+1)\pi]$ DE).$b_{2k}=2/[(2k+1)\pi]$
ABC) None of the above.

72. (3 pts.) Thus the Fourier series may be written as ____.

A) $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ B) $f(x) = 1 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

C) $f(x) = 1 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$ D) $f(x) = 1 +\sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

E) $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$ AB) $f(x) = 2 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

AC). $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$ AD) $f(x) = 2 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$

AE). $f(x) = 2 + \sum_{n=1}^{\infty}\frac{2}{(2k)\pi}\cos(\frac{(2k)\pi}{2}x)$ BC) $f(x) = 2 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

BD) $f(x) = 1 + \sum_{n=1}^{\infty}\frac{4}{(2k)\pi}\cos(\frac{(2k)\pi}{2}x)$ BE) None of the above.

Total points this page = 15. TOTAL POINTS EARNED THIS PAGE _______

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

74. (3 pts.) The "general" or formal solution of PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$

BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$

is given by ____. _____ A B C D E

A) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{\ell}{n\pi}x)$ B) $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$

C) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell}x)$ D) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$

E) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(n\pi\ell x)$ AB) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$

AC) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \cos(\frac{n\pi}{\ell}x)$ AD) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{\ell}x) + b_n \cos(\frac{n\pi}{\ell}x)$

AE) None of the above.

75. (3 pts.) The "general" or formal solution of PDE $u_t = u_{xx}$ $0 < x < 2$, $t > 0$

BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

is given by ____. ______ A B C D E

A) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4}t} \sin(\frac{2}{n\pi}x)$ B) $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{n^2 \pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

C) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-4n^2 \pi^2 t} \sin(\frac{n\pi}{2}x)$ D) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

E) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4}t} \sin(2n\pi x)$ AB) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{n^2 \pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

AC) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2 \pi^2}{4}t} \cos(\frac{n\pi}{2}x)$ AD) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2}x) + b_n \cos(\frac{n\pi}{2}x)$

AE) None of the above.

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Recall the definition of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ given on page 20. Now let $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of containing only odd functions, $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ be the functions in $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to $[0,\ell]$, and $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ be the subspace of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ for which the Fourier (sine) series is finite. Recall that $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is a Hamel basis of $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ and a Schauder basis of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$. To formulate the heat conduction in a rod problem as a linear mapping problem we let $D = (-\ell,\ell)\times(0,\infty)$, $\bar{D} = [-\ell,\ell]\times[0,\infty)$, $A_{HC}(\ ,\mathbf{R}) = \{u(x,t)\in F(\ ,\mathbf{R}): u\in A(D,\mathbf{R})\cap C(\ ,\mathbf{R})\}$ and $A_{HC,0}(\ ,\mathbf{R}) = \{u(x,t)\in A_{HC}(\ ,\mathbf{R})): u(0,t) = 0 \text{ and } u(\ell,t) = 0 \text{ for } t > 0\}$. Now let $L:A_{HC,0}(\bar{D},\mathbf{R}) \to A(D,\mathbf{R})$ be defined by $L[u] = u_t - \alpha^2 u_{xx}$. Thus we incorporate the boundary conditions into the domain of the operator. Now let N_L be the null space of L and $A_{HC,0,o,ffs}(\ ,\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L: u(x,0)\in PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)\}$ and $A_{HC,0,o,fs}(\ ,\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L: u(x,0)\in PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)\}$. Hence we see that $A_{HC,0,o,ffs}(\ ,\mathbf{R};\alpha^2,\ell) \subseteq A_{HC,o,fs}(\ ,\mathbf{R}) \subseteq N_L \subseteq A_{HC,0}(\ ,\mathbf{R}) \subseteq A_{HC}(\ ,\mathbf{R})$. Now recall that $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{e^{-\left(\frac{\alpha k\pi}{\ell}\right)^2} \sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is a Hamel basis for $A_{HC,0,o,ffs}(\ ,\mathbf{R};\alpha^2,\ell)$ and a Schauder basis for $A_{HC,o,fs}(\bar{D},\mathbf{R})$.

Solve the following problem on this sheet and then answer the questions on the next two sheets.

	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 6 \sin(6\pi x)$	$0 < x < 2$

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

76. (5 pts.) The solution of

BVP for a PDE	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 6\sin(6\pi x)$	$0 < x < 2$

is____. ______A B C D E

A) $\sum_{n=1}^{\infty} 6e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$ B) $6e^{-\frac{\pi^2}{4}t}\sin(\pi x)$

C) $6e^{-\frac{9\pi^2}{4}t}\sin(\frac{3\pi}{2}x)$ D) $6e^{-36\pi^2 t}\sin(6\pi x)$

E) $\sum_{n=1}^{\infty} 6e^{-36\pi^2 t}\sin(6\pi x)$ AB) $6e^{-12\pi^2 t}\sin(6\pi x)$

AC) $6e^{-12\pi^2 t}\sin(3\pi x)$ AD) $6e^{-144\pi^2 t}\sin(6\pi x)$

AE) None of the above.

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

77. (3 pts.) The solution of PDE $u_t = \alpha^2 u_{xx}$ $\quad 0 < x < \ell, \quad t > 0$

BC $u(0,t) = 0, \quad u(\ell,t) = 0, \quad t > 0$

IC $u(x,0) = u_0(x)$

is given by ____. _____ A B C D E

A) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{\ell}{n\pi} x)$ B) $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

C) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell} x)$ D) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

E) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(n\pi\ell x)$ AB) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

AC) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \cos(\frac{n\pi}{\ell} x)$ AD) $u(x,t) = \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{\ell} x) + b_n \sin(\frac{n\pi}{\ell} x)$

AE) None of the above

78. (3 pts.) where ____. ____ A B C D E

A) $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{\ell}{n\pi}x\right)dx$ B) $c_n = \frac{1}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ C) $c_n = \frac{2}{\ell}\int_{-\ell}^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

D) $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ E) $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(n\pi\ell x\right)dx$ AB) $c_n = \int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

AC) $c_n = \frac{1}{\ell}\int_{-\ell}^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$ AD) $a_n = \frac{2}{\ell}\int_0^\ell u_0(x)\cos\left(\frac{n\pi}{\ell}x\right)dx$ and $b_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

AE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

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On this page you are to solve: the following problem:

	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 2$	$0 < x < 2$

Begin by writing down the general solution for the PDE and the BC's. After you solve the problem, answer the questions on the next page.

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Last Name, First Name MI, What you wish to be called

Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

79. (3 pts.) The solution of

PDE	$u_t = u_{xx}$	$0 < x < \ell$,	$t > 0$
BC	$u(0,t) = 0,$	$u(\ell,t) = 0,$	$t > 0$
IC	$u(x,0) = 2$		

is given by ____. ___ A B C D E

A) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{2}{n\pi}x)$ B) $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

C) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$ D). $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

E) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(2n\pi x)$ AB) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

AC) $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\cos(\frac{n\pi}{2}x)$ AD) $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2}x) + b_n\cos(\frac{n\pi}{2}x)$

AE) None of the above.

80. (3 pts.) where ____. ____ A B C D E

A) $c_n = 2\int_0^2 \sin\left(\frac{2}{n\pi}x\right)dx$ B) $c_n = \int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ C) $c_n = 2\int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$ D) $c_n = 2\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

E) $c_n = 2\int_0^2 \sin(2n\pi x)dx$ AB) $c_n = 4\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AC) $c_n = \int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$

AD) $a_n = 2\int_0^2 \cos\left(\frac{n\pi}{2}x\right)dx$ and $b_n = 2\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ AE) None of the above.

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81. (5 pts.) The solution of

	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 2$	$0 < x < 2$

is u(x,t) = ____. ______ A B C D E

A) $\sum_{k=0}^{N} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$

B) $\sum_{k=0}^{N} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{2}x)$

D) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$

E) $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{2}x)$

AB) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{4}x)$

AC) $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{4}x)$

AD) $\sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$

AE) $\sum_{n=1}^{\infty} \frac{8}{(2k)\pi} e^{\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k)\pi}{2}x)$

BC) $\sum_{k=1}^{\infty} \frac{4}{k\pi} e^{-\frac{k^2\pi^2}{4}t} \sin(\frac{k\pi}{2}x)$

BD) $u(x,t) = \sum_{k=1}^{\infty} \frac{2}{k\pi} e^{-\frac{k^2\pi^2}{2}t} \sin(\frac{k\pi}{2}x)$

BE) None of the above.

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Write each answer in the blank provided and then circle the letter or letters that correspond to your answer from the answers listed. Also circle the correct answer.

82. (4 pts.) If both ends of a rod are insulated (recall that we assume that the lateral sides are also insulated so that the temperature does not vary over a cross section), then a good mathematical model of the physical heat conduction problem is given by:

	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell,\quad t > 0$
BVP for a PDE	BC	$u_x(0,t) = 0,\quad u_x(\ell,t) = 0,$	$t > 0$
	IC	$u(x,0) = f(x)$	$0 < x < \ell$

where f(x) is the initial temperature distribution in the rod.

```
          //////////////////////////////////////////////
      -→ O__________________________O ←-
    /        //////////////////////////////////////////////        \
insulated                                        insulated
         *-------------------------------------*
       x = 0                                  x = ℓ
```

Applying the separation of variables process to the PDE results in the two ODE's:

1. $X'' + \sigma X = 0$
2. $T' + \alpha^2 \sigma T = 0$

where σ is the separation constant. The spacial eigenvalue problem that results from

applying the BC given above is ________________________________ .____ A B C D E

A) $X'' + \sigma X = 0$, $X(0) = 0,\ X(\ell) = 0$

B) $X'' + \sigma X = 0$, $X'(0) = 0,\ X(\ell) = 0$

C.) $X'' + \sigma X = 0$, $X(0) = 0,\ X'(\ell) = 0$

D) $X'' + \sigma X = 0$, $X(0) = 1,\ X(\ell) = 0$

E) $X'' + \sigma X = 0$, $X'(0) = 0,\ X'(\ell) = 0$

AB) $X'' + \sigma X = 0$, $X'(0) = 1,\ X'\ell) = 0$

AC) $X'' + \sigma X = 0$, $X(0) = 1,\ X(\ell) = 1$

AD) $X'' + \sigma X = 0$, $X'(0) = 1,\ X(\ell) = 0$

AE) $X'' + \sigma X = 0$, $X(0) = 0,\ X'(\ell) = 0$

BC) None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME _________________________(_________________) SS No. __________________
Last Name, First Name MI, What you wish to be called

TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	$s > \lvert a \rvert$
cosh (at)	$\frac{s}{s^2 - a^2}$	$s > \lvert a \rvert$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
u(t)	$\frac{1}{s}$	$s > 0$
u(t - c)	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	$F(s - c)$	
$f(ct)$ $c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	