MATH 261 | MATH 261: Elementary Differential Equations | MATH 261
FALL 2005 | FINAL EXAM | FALL 2005
FINAL EXAM | EXAMINATION COVER PAGE | Professor Moseley

PRINT NAME____(______________________)
Last Name, First Name MI (What you wish to be called)

ID # ___________________________________ EXAM DATE Tuesday, May 3, 2004, 8:00 am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

ST1	75	

ST2	

_______________________________ _______________
SIGNATURE DATE

INSTRUCTIONS: 1.Besides this cover page, there are 33 pages of questions and problems on this exam. Page 34 contains Laplace transforms you need not memorize. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2.Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. 3.Pages 1-33 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit may be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. GOOD LUCK!!

Scores

page	points	score
1	25	
2	6	
3	7	
4	8	
5	5	
6	9	
7	---	
8	1 5	

Scores

page	points	score
9	---	
10	8	
11	9	
12	9	
13	12	
14	6	
15	5	
16	3	

Scores

page	points	score
17	---	
18	13	
19	10	
20	3	
21	---	
22	5	
23	15	
24	5	

Scores

page	points	score
25	---	
26	6	
27	5	
28	---	
29	6	
30	6	
31	5	
32	4	
33	---	
34		
35		
36		
37		
38		
39		
ST4	32	
ST1	75	
ST2	52	
ST3	51	
ST4	32	

Tot	200	

PRINT NAME ___________________________(______________) SS No. ___________________
Last Name, First Name MI, What you wish to be called

(25 pts.) Classify the following first order ordinary differential equations. The classification relates to the method of solution. Recall from class (attendance is mandatory) that the possible methods are:

A) First order linear (y as a function of x).- Integrating factor = $\mu = \exp(\int p(x)\,dx)$
B) First order linear (x as a function of y).- Integrating factor = $\mu = \exp(\int p(y)\,dy)$
C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).
E) Bernoulli, but not linear (y as a function of x). Use the substitution $v = y^{1-n}$.
AB) Bernoulli, but not linear (x as a function of y). Use the substitution $v = x^{1-n}$.
AC) Homogeneous, but not separable. Use the substitution $v = y/x$ or $v = x/y$.
AD) None of the above techniques works.

Also recall the following: a. In this context, exact means exact as given (in either of the forms discussed in class). b. Bernoulli is not a correct method of solution if the original equation is linear. c. Homogeneous (use the substitution v=y/x) is not a correct method of solution if it converts a separable equation into another separable equation. **Circle one** and only one answer. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is **WRONG**. **DO NOT SOLVE**.

1. $(y^2 + xy)dx + x\,dy = 0$ A B C D E AB AC AD

2. $(e^x + 2xy + x)dx + (x^2 + 2y)dy = 0$ A B C D E AB AC AD

3. $(y^2 + x^2)dx + x^2\,dy = 0$ A B C D E AB AC AD

4. $xye^{x+y}\,dx + dy = 0$ A B C D E AB AC AD

5. $(xy + \cos(x))dx + (1 + x^2)\,dy = 0$ A B C D E AB AC AD

PRINT NAME ______________________(______________) ID No. __________________
Last Name, First Name MI What you wish to be called

6. (1 pts.) To solve the first order linear Ordinary Differential Equation (ODE) $\mathbf{y' = 2y + x^2}$**,** we first put it in the standard form (for solving first order linear ODE's) (Circle the correct (first order linear) standard form for this ODE's. Be careful. No part credit for this problem. Hence if you miss this part, it may cause you to miss all parts):

A. $y' = 2y + x^2$ (It's already in the appropriate form for solving a first order linear ODE)

B. $y' + 2y = x^2$, C. $y' + 2y + x^2$, D. $y' - 2y = x^2$

E. $y' - 2y - x^2 = 0$ AB. None of the above

7. (2 pts.) To solve the first order linear Ordinary Differential Equation (ODE) above, we must find an integrating factor. An integrating factor μ for this linear ODE is (Circle the correct integrating factor. Be careful. No part credit for this problem):

A. $\mu = 2x$, B. $\mu = x^2$, C. $\mu = e^{2x}$, D. $\mu = e^{-2x}$ E. $\mu = e^{-x^2}$,

AB. $\mu = e^{x^2}$, AC. $\mu = e^{x}$, AD. $\mu = e^{-x}$, AE. None of the above

8. (3 pts.) In solving the linear Ordinary Differential Equation (ODE) above which of the following steps occurs. (Circle the step that is correct. Be careful. No part credit for this problem):

A. $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$, B. $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$, C. $\frac{d(ye^{2x})}{dx} = x^2e^{2x}$

D. $\frac{d(ye^{2x})}{dx} = 2xe^{2x}$, E. $\frac{d(ye^{-2x})}{dx} = x^2e^{-2x}$, AB. $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$,

AC. $\frac{d(ye^{2x})}{dx} = x^3$, AD. None of the above steps ever appears in any solution of this problem.

PRINT NAME ______________________(_________________) ID No. _______________
Last Name, First Name MI What you wish to be called

To solve the first order linear ODE, we isolate the unknown function on the left side of the equation. Recall that an ODE is really a “vector” equation with the infinite number of unknown variables being the values of the function for each value of the independent variable in the function’s domain. The isolation of the dependent variable (or function) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^x)}{dx} = xe^x$.

9. (2 pts.) Circle the theorem from calculus that allows you to integrate the **Left Hand Side** of this equation: A. Intermediate Value Theorem,   B. Mean Value Theorem
C. Rolle's Theorem,   D. Chain Rule,   E. Fundamental Theorem of Calculus,
AB. Product Rule,   AC. Integration by Parts.   AD. Partial Fractions, None off the above.

10. (5 pts.) Now complete the solution process to obtain y as a function of x. However, be sure that you have given all solutions; that is, that you have given parametrically the entire collection of solutions to this problem. Circle the correct solution or family of solutions below.

A. $y = x + 1 + c\,e^x$,   B. $y = -x + 1 + c\,e^x$,   C. $y = x - 1 + c\,e^x$,   D. $y = x + 1 + c\,e^{-x}$,

E. $y = -x + 1 + c\,e^{-x}$,   AB. $y = x + 2 + c\,e^{-x}$,   AC. $y = x - 1 + c\,e^{-x}$,   AD. $y = x + 1 + e^x +$ c,

AE. $y = -x + 1 + e^x + c$,   BC. $y = x - 1 + e^x + c$,   BD. $y = x + 1 + e^{-x} + c$,

BE. $y = -x + 1 + e^{-x} + c$,   CD. $y = x - 1 + e^{-x} + c$,   CE. $y = x + 1 - e^{-x} + c$,

DE. None of the above families of solutions is correct..

PRINT NAME ________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

You are to solve $\underset{2x2}{A}\ \underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ where $A = \begin{bmatrix} 1 & i \\ i & -1 \end{bmatrix}$, $\vec{b} = \begin{bmatrix} 1 \\ i \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$. Be sure you write your answer according to the directions given in class for these kinds of problems (attendance is mandatory).

11. (4 pts.) If $\left[A \middle| \vec{b} \right]$ is reduced to $\left[U \middle| \vec{c} \right]$ using Gauss elimination, then $\left[U \middle| \vec{c} \right]$ =______.

A. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & i \end{array}\right]$, B. $\left[\begin{array}{cc|c} 1 & i & 0 \\ 0 & 0 & 0 \end{array}\right]$, C. $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$, D. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 1 \end{array}\right]$, E. $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$,

AB. None of the above

12. (4 pts.) The solution of $\underset{2x2}{A}\ \underset{2x1}{\vec{x}} = \underset{2x1}{\vec{b}}$ is ________.

A. No Solution, B. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, C. $\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix}$, D. $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix}$, ED. $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$,

AB $\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$, AC. $\vec{x} = \begin{bmatrix} -i \\ 1 \end{bmatrix} + y\begin{bmatrix} 1 \\ 0 \end{bmatrix}$, AD. $\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$,

AE. None of the above.

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

(5 pts.) True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a vector space V to a vector space W. Now consider

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

Under these hypotheses, determine which of the following is true and which is false.
If true, circle True. If false, circle False.

True or False 13. If $\vec{b} = \vec{0}$, then (*) always has at least one solution.

True or False 14. The vector equation (*) may have exactly two solutions.

True or False 15. If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ and $\vec{b} \neq \vec{0}$, then the general solution of (*) is given by $\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where c_1, c_2, ..., c_n are arbitrary constants.

True or False 16. Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

True or False 17. If the null space of T is $N(T) = \{\vec{0}\}$ and $\vec{b}$ is in the range space of T, then (*) has a unique solution.

PRINT NAME ________________________(_____________) ID No. _________________
Last Name, First Name MI, What you wish to be called

(9 pts.) The **dimension of the null space** of the **linear operator** L[y] = $y'' - y'$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the **homogeneous equation** L[y] = 0 of the form $y = e^{rx}$ leads to the two **linearly independent solutions** $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that

$y_h = c_1 + c_2\ e^{x}$ is the **general solution** of $y'' - y' = 0$.

Use the method discussed in class to determine the **proper (most efficient) form** of the judicious guess for a **particular solution** y_p of the following ode's. Circle the correct (most efficient) form of the judicious guess for a particular solution y_p of the following ode's

18. $y'' - y' = 2xe^{-x}$ A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE

19. $y'' - y' = 3 \sin x$ A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE

20. $y'' - y' = -4e^{x}$ A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE

Possible Answers:

A. Ae^{x} B. Axe^{x} C Ax^2e^{x} D. $Axe^{x} + Be^{x}$ E. $Ax^2e^{x} + Bxe^{x}$

AB. Ae^{-x} AC. Axe^{-x} AD. Ax^2e^{-x} AE. $Axe^{-x} + Be^{-x}$ BC. $Ax^2e^{-x} + Bxe^{-x}$

BD. A sin x BE. cos x CD. A x sin x CE. A x cos x ABC. A sin x + B cos x

ABD. A x sin x + B x cos x ABE. None of the above

PRINT NAME _________________________(______________) SS No. ___________________
Last Name, First Name MI, What you wish to be called

You are to solve y" + y = tan(x) I = (0, π/2) (i.e. 0 < x < π/2). Let L[y] = y" + y.

21. (2 pts.) The general solution of L[y] = 0 is ___. A. c_1 cos(x) + c_2 sin(x), B. c_1 cos(2x) + c_2 sin(2x), C. $c_1 e^x + c_2 e^{-x}$, D. c_1 x + c_2, E. r = ±i, AB. r = ±1, AC r = ±2i, AD. None of the above.

22. (3 pts.) To find a particular solution y_p to L[y] = tan(x) using the technique of variation of parameters, we let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. Substituting into the ODE and making the appropriate assumption we obtain:

A. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$,
B. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$,
C. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x)$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$,
D $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sin(x)$,
E. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \cos(x)$,
AB. None of the above.

23. (3 pts.) Solving we obtain: A. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = \sin(x)$,
B. $u'_1(x) = \sin(x)$, $u'_2(x) = -\sin^2(x)/\cos(x)$, C. $u'_1(x) = 1$, $u'_2(x) = \sin(x)$,
D. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = 1$, E. $u'_1(x) = 0$, $u'_2(x) = \sin(x)$,
AB. None of the above.

24. (3 pts.) Hence we may choose : A. $u_1(x) = -\ln(\tan(x) + \sec(x))$, $u_2(x) = -\cos(x)$,
B. $u'_1(x) = -\cos(x)$, $u'_2(x) = -\ln(\tan(x) + \sec(x))$, C. $u_1(x) = x$, $u_2(x) = -\cos(x)$,
D. $u'_1(x) = -\ln(\tan(x) + \sec(x))$, $u'_2(x) = x$, E. $u_1(x) = 1$, $u'_2(x) = -\cos(x)$,
AB. None of the above.

25. (2 pts.) Hence a particular solution to L[y] = tan(x) is: A. $y_p(x) = -\ln(\tan(x) + \sec(x))$,
B. −[cos(x)] ln(tan(x) +sec(x)), C. −[sin(x)]ln(tan(x) +sec(x)),
D. −[tan(x)] ln(tan(x) +sec(x)), E. sin(x) cos(x), AB. 2 sin(x) cos(x),
AC. −[sin(x) cos(x)] ln(tan(x) +sec(x)),

26. (2 pts.) Hence the general solution of L[y] = tan(x) is:
A. −ln(tan(x) +sec(x)) + c_1 cos(x) + c_2 sin(x),
B. −[cos(x)] ln(tan(x) +sec(x)) + c_1 cos(x) + c_2 sin(x),
C. −[sin(x)]ln(tan(x) +sec(x))+c_1 cos(x) + c_2 sin(x),
D. sin(x) cos(x) + $c_1 e^x + c_2 e^{-x}$, E. −ln(tan(x) +sec(x)) + $c_1 e^x + c_2 e^{-x}$
AB. −[cos(x)] ln(tan(x) +sec(x)) + $c_1 e^x + c_2 e^{-x}$, AC. None of the above.

PRINT NAME __________________________(________________) IDo. ___________________
Last Name, First Name MI, What you wish to be called

You solved y" + y = tan(x) I = (−π/2 , π/2) (i.e −π/2 < x < π/2) on the previous page. Now let L: $\mathcal{A}((0,\pi/2),\mathbf{R})\to\mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by L[y] = y" + y and answer the following questions.

21. (2 pts.) The general solution of L[y] = 0 is ___.
A. $c_1 \cos(2x) + c_2 \sin(2x)$, B. $c_1 \cos(x) + c_2 \sin(x)$, C. $c_1 e^x + c_2 e^{-x}$, D. $c_1 x + c_2$,
E. $r = \pm i$, AB. $r = \pm 1$, AC. $r = \pm 2i$, AD. None of the above.

22. (3 pts.) To find a particular solution y_p to L[y] = tan(x) using the technique of variation of parameters, you let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. Substituting into the ODE and making the appropriate assumption you obtained:
A. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$,
B. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$,
C. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x)$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$,
D $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sin(x)$,
E. $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \cos(x)$,
AB. None of the above.

23. (3 pts.) Solving we obtain: A. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = \sin(x)$,
B. $u'_1(x) = \sin(x)$, $u'_2(x) = -\sin^2(x)/\cos(x)$, C. $u'_1(x) = 1$, $u'_2(x) = \sin(x)$,
D. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = 1$, E. $u'_1(x) = 0$, $u'_2(x) = \sin(x)$,
AB. $u'_1(x) = \sin(x)$, $u'_2(x) = 1$, AC. None of the above.

24. (3 pts.) Hence we may choose : A. $u_1(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, $u_2(x) = -\cos(x)$,
B. $u_1(x) = -\cos(x)$, $u_2(x) = -\ln(\tan(x) + \sec(x))$, C. $u_1(x) = x$, $u_2(x) = -\cos(x)$,
D. $u_1(x) = -\ln(\tan(x) + \sec(x))$, $u_2(x) = x$, E. $u_1(x) = 1$, $u_2(x) = -\cos(x)$,
AB. $u_1(x) = -\cos(x)$, $u_2(x) = x$, AC. None of the above.

25. (2 pts.) Hence a particular solution to L[y] = tan(x) is: A. $y_p(x) = -\ln(\tan(x) + \sec(x))$,
B. $-[\cos(x)]\ln(\tan(x) + \sec(x))$, C. $-[\sin(x)]\ln(\tan(x) + \sec(x))$,
D. $-[\tan(x)]\ln(\tan(x) + \sec(x))$,
E. $\sin(x)\cos(x)$, AB. $2\sin(x)\cos(x)$, AC. $-[\sin(x)\cos(x)]\ln(\tan(x) + \sec(x))$,
AD. None of the above.

26. (2 pts.) Hence the general solution of L[y] = tan(x) may be written as:
A. $-\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$,
B. $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$,
C. $-[\sin(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$, D. $\sin(x)\cos(x) + c_1 e^x + c_2 e^{-x}$,
E. $-\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$ AB. $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$,

AC. None of the above.

PRINT NAME __________________________(_______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Solve $y^{IV} - 4y''' + 4y'' = 0$ on this page and answer the questions on the next page. Be careful as once you make a mistake, the rest is wrong.

PRINT NAME _________________________(______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

You solved $y^{IV} - 4y''' + 4y'' = 0$ on the previous page. Now let $L: \mathcal{A}(\mathbf{R},\mathbf{R}) \to \mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$ and answer the following questions by circling the correct answers.

27. (1 pt). The order of the ODE is:
A. 1 B. 2 C. 3 D. 4 E. 5 AB. 6 AC. 7 AD) None of the above.

28. (1 pt). The dimension of the null space for the linear operator L is:
A. 1 B. 2 C. 3 D. 4 E. 5 AB. 6 AC. 7 AD) None of the above.

29. (1 pts). The auxiliary equation for the ODE is: A. $r^2 - 4r + 4 = 0$, B. $r^4 - 4r^2 + 4 = 0$,
C. $r^4 - 4r^3 + 4r^2 = 0$, D. $r^6 + 4r^3 + 4r^2 = 0$, E. $r^6 - 4r^3 + 4r^2 = 0$, AB. None of the above.

30. (2 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation are: A. r = 0, 2,
B. r = 0, 0, 2, 2, C. r = 2, 2, D. r = 0,4, E. r = 0, 2,4, AB. r = 0, 0, −2, −2
AC. None of the above.

31. (2 pts). A basis for the null space of the linear operator L is :
A. $\{1, x, e^{-2x}, xe^{-2x}\}$, B. $\{1, x, e^{2x}, xe^{2x}\}$, C. $\{1, x, x^2, e^{2x}\}$, D. $\{1, e^{2x}\}$,
E. $\{1, x, x^2, x^3\}$, AB. $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$, AC. $\{1, x, x^2, e^{-2x}\}$, AD. $\{1, e^{-2x}\}$,
AE. None of the above.

32. (1 pt). The general solution of $y^{IV} - 4y''' + 4y'' = 0$ is: A. $y(x) = c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$,
B. $y(x) = c_1 + c_2 x + c_3 e^{2x} + c_4 xe^{2x}$, C. $y)x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x}$, D. $y(x) = c_1 + c_2 e^{2x}$
E. $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$, AB. $y(x) = c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$
AC. $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD. $y(x) = c_1 + c_2 e^{-2x}$. AE. None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME ________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

(9 pts.) Let $L: A(\mathbf{R},\mathbf{R}) \to A(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y''' + y'$. The dimension of the null space of the linear operator L is 3. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the three linearly independent solutions $y_1 = 1$ and $y_2 = \cos(x)$ and $y_3 = \sin(x)$ so that a basis of the null space of L is $\{1, \cos(x), \sin(x)\}$. Hence we can deduce from the linear theory that

$$y_c = c_1 + c_2 \cos(x) + c_3 \sin(x) \quad \text{is the general solution of} \quad y''' + y' = 0 \text{ on } \mathbf{R}.$$

Use the method discussed in class (attendance is mandatory) to determine the **proper (most efficient) form** of the judicious guess for a particular solution y_p of the following ODE's. Circle the answer that corresponds to your **final guess**.

33. $y''' + y' = \sin(x)$ A B C D E AB AC AD AE BC BD BE CD CE DE

ABC ABD ABE BCD BCE BDE. CDE. ABCD

34. $y''' + y' = 4x^2$ A B C D E AB AC AD AE BC BD BE CD CE DE

ABC ABD ABE BCD BCE BDE. CDE. ABCD

35. $y''' + y' = -4xe^{-x}$ A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE BCD BCE BDE. CDE. ABCD

Possible Answers

A. A   B. Ax   C. Ax^2 D. Ax + B   E. $Ax^2 + Bx$ AB. $Ax^2 + Bx + C$
AC. Ae^x AD. Axe^x AE. Ax^2e^x BC. $Axe^x + Be^x$ BD. $Ax^2e^x + Bxe^x$
BE. Ae^{-x} CD. Axe^{-x} CE. Ax^2e^{-x} DE. $Axe^{-x} + Be^{-x}$ ABC. $Ax^2e^{-x} + Bxe^{-x}$
ABD. A sin x   ABE. cos x   BCD A x sin x   BCE A x cos x   BDE. A sin x + B cos x
CDE. A x sin x + B x cos x   ABCD None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

(9 pts.) Compute the Laplace transform of the following functions. Then circle the letter corresponding to the Laplace transform of the function.

36. f(t) = 2 + 3t   A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE.

37. $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE.

38 f(t) = 2 sin(2t) + 3 cos(3t)   A B C D E AB AC AD AE BC BD BE CD CE DE ABC ABD ABE.

Possible Answers

A. $\frac{2}{s}+\frac{3}{s^2}$,   B. $\frac{2}{s^2}+\frac{3}{s^3}$,   C. $\frac{2}{s^2}+\frac{3/2}{s^3}$,   D. $\frac{2}{s^2}+\frac{1}{s^3}$,   E. $\frac{1}{s}+\frac{1}{s^2}$,   AB. $2+\frac{3}{s}$

AC. $\frac{2}{s-2}+\frac{3}{s+3}$, AD. $\frac{2}{s+2}+\frac{3}{s-3}$, AE. $\frac{2}{s-3}+\frac{3}{s+2}$, BC. $\frac{2}{(s-3)^2}+\frac{3}{(s+3)^3}$,

BD. $\frac{2}{s-4}+\frac{4}{s+2}$, BE. $\frac{2}{s^2+1}+\frac{3s}{s^2+4}$, CD. $\frac{2s}{s^2+4}+\frac{3}{s+9}$, CE. $\frac{2}{s^2-1}+\frac{3}{s^2-4}$,

DE. $\frac{2}{s^2+4}+\frac{3}{s^2+9}$, ABC. $\frac{2}{s^2+4}+\frac{2s}{s^2+9}$, ABD. $\frac{2s}{s^2+4}+\frac{3}{s^2+9}$ ABE. None of the above.

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME __________________________(______________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

(12 pts.) Compute the inverse Laplace transform of the following functions:

39. F(s) $= \frac{2}{s} + \frac{3}{s+2}$ A B C D E AB AC AD AE BC BD BE CD CE DE

40. F(s) $= \frac{2s+4}{s^2+9}$ A B C D E AB AC AD AE BC BD BE CD CE DE

41. F(s) $= \frac{2s+3}{s^2-2s+2}$ A B C D E AB AC AD AE BC BD BE CD CE DE

Possible Answers

A. $2 + 2e^{-2t}$, B. $3 + 2e^{-2t}$, C. $2 + 2e^{-3t}$, D. $2 + 3e^{-2t}$, E. $2 + e^{-2t}$,
AB. 2 cos 3t + (4/3) sin 3t, AC. 2 cos 3t + 4 sin 3t, AD. 2 cos 2t + (4/3) sin 2t,
AE. 3 cos 3t + (4/3) sin 3t, BC. 2 cos 3t + 3 sin 3t, BD. $2e^t \cos 3t + 5e^t \sin 3t$,
BE. $2e^t \cos t + 5e^t \sin t$, CD. $2e^t \cos 3t + 2e^t \sin 3t$, CE. $5e^t \cos 3t + 5e^t \sin 3t$,
DE. None of the above

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Consider the matrix $A = \begin{bmatrix} i & 2 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

42. (1 pt.) The degree of the polynomial where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is _____. A. 1, B. 2, C. 3, D. 4, E. 5, AB. 6, AC.7, AD. None of the above.

43. (2 pt.) The polynomial $p(\lambda)$ where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is _____.. A. $p(\lambda) = (\lambda-1)(\lambda-i)$, B. $p(\lambda) = (\lambda-1)(\lambda+i)$, C. $p(\lambda) = (\lambda+1)(\lambda-i)$, D. $p(\lambda) = (\lambda+1)(\lambda+i)$, E. $p(\lambda) = (\lambda-2)(\lambda-i)$, AB. $p(\lambda) = (\lambda-2)(\lambda+I)$, AC. $p(\lambda) = (\lambda+2)(\lambda-i)$, AD. $p(\lambda) = (\lambda-2)(\lambda+i)$, AE. $p(\lambda) = (\lambda+2)(\lambda+2i)$, BC. $p(\lambda) = (\lambda+2)(\lambda+2i)$, BD. $p(\lambda) = (\lambda+2)(\lambda-2i)$, BE. $p(\lambda) = (\lambda-2)(\lambda+2i)$, CD. None of the above.

44. (1 pt.) Counting repeated roots, the matrix A given above has how many eigenvalues?
A. 0, B 1, C. 2, D. 3, E 4, AB. 5, AC. 6, AD. 7, AE. 8.

45. (2 pts.) The eigenvalues of A may be chosen as ______. Circle the correct answer.
A. $\lambda_1 = 2$, $\lambda_2 = i$, B. $\lambda_1 = 1$, $\lambda_2 = -i$, C. $\lambda_1 = -1$, $\lambda_2 = i$, D. $\lambda_1 = -1$, $\lambda_2 = -i$, E. $\lambda_1 = 2$, $\lambda_2 = i$, AB. $\lambda_1 = 2$, $\lambda_2 = -i$, AC. $\lambda_1 = -2$, $\lambda_2 = i$, AD. $\lambda_1 = -2$, $\lambda_2 = -i$, AE. $\lambda_1 = -2$, $\lambda_2 = i$, BC. $\lambda_1 = -2$, $\lambda_2 = -2i$, BD. $\lambda_1 = -2$, $\lambda_2 = 2i$, BE. $\lambda_1 = 2$, $\lambda_2 = -2i$,
CD. None of the above

PRINT NAME ________________________(____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

$\lambda = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$

46. (5 pts.) Using the conventions discussed in class (attendance is mandatory), find a basis B for the eigenspace associated with this eigenvalue. Circle the correct answer.

A. $B = \{[1,1]^T, [4,4]^T\}$, B. $B = \{[1,1]^T\}$, C. $B = \{[1,2]^T\}$, D. $B = \{[1,2]^T, [4,8]^T\}$
E. $B = \{[2,1]^T\}$ AB. $B = \{[1,3]^T\}$ AC. $B = \{[1,4]^T\}$, AD. $B = \{[4,1]^T\}$
AE. $B = \{[3,1]^T\}$, BC. $B = \{[1,-1]^T, [4,4]^T\}$, BD. $B = \{[1,-1]^T\}$, BE. $B = \{[1,-2]^T\}$, CD. $B = \{[1,-2]^T, [4,8]^T\}$, CE. $B = \{[2,1]^T\}$ DE. $B = \{[1,3]^T\}$ ABC. $B = \{[1,-4]^T\}$, ABD. $B = \{[4,-1]^T\}$ ABE. $B = \{[3,-1]^T\}$, BC. None of the above.

PRINT NAME __________________________(______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

47. (3 pts.) Consider the scalar equation $u'' + 4u' - 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du'/dt$). Convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y; you may think of x as the position and y as the velocity of a point particle). Now write this system of two scalar equations in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. Be sure to give A explicitly. Choose the correct answer from the following:

A. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, B. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, C. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

D. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, E. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AB. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

AC. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AD. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AE. None of the above.

PRINT NAME ___________________________(_______________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Solve $\vec{x}' = A\vec{x}$ where $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$ and $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ on this page. Provide appropriate steps and a table as explained in class (attendance is mandatory). Remember, once you make a mistake, the rest is wrong. Then answer the questions on the next page.

PRINT NAME ________________________(____________) ID No. _________________

Last Name, First Name MI, What you wish to be called

Let $A = \begin{bmatrix} 3 & -2 \\ 2 & -2 \end{bmatrix}$ and $L: \mathcal{A}(\mathbf{R},\mathbf{R}^2) \to \mathcal{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}] = \vec{x}' - A\vec{x}$. Let r_1 be an eigenvalue of A with associated eigenvector $\vec{\xi}_1$. Similarly, let r_2 be a second eigenvalue with associated eigenvector $\vec{\xi}_2$.

48. (3 pts.) We may choose r_1 and r_2 as follows: A. $r_1 = 1$, $r_2 = 2$, B. $r_1 = 1$, $r_2 = -2$, C. $r_1 = -1$, $r_2 = 2$, D. $r_1 = -1$, $r_2 = -2$, E. $r_1 = 1$, $r_2 = -1$, AB. $r_1 = 1$, $r_2 = 1$, AC. $r_1 = -1$, $r_2 = -1$, AD. $r_1 = 1$, $r_2 = 3$, AE. $r_1 = 1$, $r_2 = -3$, BC. $r_1 = -1$, $r_2 = 3$, BD. None of the above are possible.

49. (5 pts.) Following the conventions discussed in class (attendance is mandatory) the eigenvectors associated with the eigenvalues selected above should be chosen as:

A. $\vec{\xi}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, B. $\vec{\xi}_1 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, C. $\vec{\xi}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, D. $\vec{\xi}_1 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, E. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, AB. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, AC. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, AD. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, AE. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, BC. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$, BD. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$, BE. $\vec{\xi}_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$, $\vec{\xi}_2 = \begin{bmatrix} 2 \\ -1 \end{bmatrix}$ CD. None of the above are the eigenvectors for the eigenvalues chosen in question 48 above. (This includes the case where BD was chosen in question 48.)

50. (3 pts.) A basis for the null space of the linear operator L is: A. , $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}, \right\}$,

B. $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}, \right\}$, C. $B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}, \right\}$, D. $B = \left\{ \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{2t}, \right\}$,

E. $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}, \right\}$, AB. $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t}, \right\}$, AC. $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-2t}, \right\}$, AD. $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t},\right\}$, AE. $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t},\right\}$, BC $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t},\right\}$

BD. $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t},\right\}$, CD. None of the above

51. (2 pts.) The general solution of $\vec{x}' = A\vec{x}$ is: A. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$,

B. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\-2\end{bmatrix}e^{2t}$, C. $\vec{x}(t)=c_1\begin{bmatrix}2\\-1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$, D. $\vec{x}(t)=c_1\begin{bmatrix}2\\-1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\-2\end{bmatrix}e^{2t}$

E. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, AB. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, AC. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$,

AD. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, AE. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, BC. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$,

BD. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, BE. $\vec{x}(t)=c_1\begin{bmatrix}2\\1\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$, CD. None of the above.

Total points this page = 13. TOTAL POINTS EARNED THIS PAGE ____________

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

(5 pts.) True or False. Odd, even, and periodic functions. Let f and g be **real valued functions of a real variable**; that is, f:**R** → **R** and g:**R** → **R**. Circle True if the statement is true. Circle False if the statement is false.

True False 52. The function f is even if $f(-x) = -f(x)\ \forall\ x \in \mathbf{R}$.

True False 53 The function f is odd if $f(-x) = f(x)\ \forall\ x \in \mathbf{R}$.

True False 54. If f and g are both odd functions, then the product of f and g is an odd function.

True False 55. The function f is periodic of period T if $f(x + T) = f(x)\ \forall\ x \in \mathbf{R}$.

True False 56. If f and g are even functions, then we know that $\int_{-\ell}^{\ell} f(x)g(x)dx = 0$.

(5 pts.) Circle Odd if the function is odd. Circle Even if the function is even. Circle Neither if the function is neither odd nor even.

57. $f(x) = -x$ A. Odd B. Even C. Neither

58. f(x) = -3 A. Odd B. Even C. Neither

59. f(x) = sin(x) A. Odd B. Even C. Neither

60. f(x) = *-x* A. Odd B. Even C. Neither

61. f(x) = $-e^x$ A. Odd B. Even C. Neither

PRINT NAME ________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Let $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ = {f∈F(**R**,**R**): f is periodic of period 2ℓ, f and f' are piecewise continuous on [)ℓ,ℓ], and f(x) =(f(x+) + f(x))/2 at points of discontinuity}, $PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)$ be the functions in $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to $[-\ell,\ell]$ and $PC^1_{ffs}([-\ell,\ell],\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)$ for which the Fourier series is finite. Recall that $B_{PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)} = \{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$ is a Hamel basis of $PC^1_{ffs}([-\ell,\ell],\mathbf{R};\ell)$ and a Schauder basis of $PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)$.

62. (1 pt.) A Hamel basis for $PC^1_{ffs}([-2,2],\mathbf{R};2)$ is A. $\{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$,

B. $\{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$, C. $\{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$

D. $\{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$, E. $\{1/2\}\cup\{\cos(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}\cup\{\sin(\frac{k\pi}{\ell}) : k\in\mathbf{N}\}$

AB. None of te above.

63. (2 pts.) Determine the Fourier series for the function f(x) which has period 4 and is defined on
the interval [-2,2) by f(x) = 3 + 2 cos(2πx) + 2 sin(3π x).

A. $f(x) = 3 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$, B. $f(x) = 1 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$,

C. f(x) = $3 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$, D. f(x) = $1 + \sum_{n=1}^{\infty} \frac{1}{k\pi} \sin(k\pi x)$,

E. f(x) = $1 + \sum_{n=1}^{\infty} \frac{1}{k\pi} \cos(k\pi x)$, AB. f(x) = $2 + \sum_{n=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$,

AC. f(x) = $1 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2} x)$, AD. f(x) = $2 + \sum_{n=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2} x)$,

AE. 3 + 3 cos(2πx) + 2 sin(3π x), BC. 3 + 2 cos(2πx) + 3 sin(3π x),

BD. 3 + 2 cos(πx) + 2 sin(3π x). BE. 3 + 2 cos(2πx) + 2 sin(π x),

CD. 2 + 2 cos(2πx) + 2 sin(3π x), CE. 2 + 2 cos(πx) + 2 sin(3π x).

DE. 3 + 2 cos(2πx) + 2 sin(3π x), ABC. None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

Determine the Fourier series for the function f(x) which has period 4 and is defined on the interval [-2,2) by

$$f(x) = \begin{cases} 0 & -2 \le x \le 0 \\ 2 & 0 < x < 2 \end{cases}$$

Sketch f for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges. Next write down the general formulas for a Fourier series and its coefficients that you memorized.

PRINT NAME _________________________(______________) ID No. __________________

Last Name, First Name MI, What you wish to be called

64. (2 pts.) Suppose that a function f is periodic with period 2 ℓ. Then the formula for the general Fourier series for f **given in our text** is

A. $f(x) = a_0 + \sum_{n=1}^{N} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$,

B. $f(x) = a_0 + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$,

C. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$,

D. $f(x) = \frac{a_0}{2} + \sum_{n=0}^{N} a_n \cos(n\pi x) + b_n \sin(n\pi x)$

E. $f(x) = \frac{a_0}{2} + \sum_{n=0}^{\infty} a_n \cos(\frac{n\pi}{\ell}x) + b_n \sin(\frac{n\pi}{\ell}x)$,

AB. $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos(n\pi x) + b_n \cos(n\pi x)$ AC. None of the above.

65. (2pts.) where

A. $a_n = \frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ..., $b_n = \frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

B. $a_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$,n = 0, 1, 2, ... $b_n = \frac{2}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

C. $a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$, n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

D. $a_n = \frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$,n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ n = 1, 2, ...

E. $a_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(n\pi x)dx$, n = 0, 1, 2, ... $b_n = \frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(n\pi x)dx$ n = 1, 2, ...

AB $a_n = \frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\cos(x)dx$,n = 0, 1, 2, ... $b_n = \frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\sin(x)dx$ n = 1, 2, ...

AC. None of the above.

66. (1 pt.) On the previous page you determined the Fourier series for the function f(x) which has period 4 and is defined on the interval [-2,2) by $f(x) = \begin{cases} 0 & -2 \le x \le 0 \\ 2 & 0 < x < 2 \end{cases}$. To apply the above formulas, we see that we should choose: A. $\ell = 1$, B. $\ell = 2$, C. $\ell = 3$, D. $\ell = 4$, E. $\ell = -1$, AB. $\ell = -2$, AC. $\ell = -3$, AD. $\ell = -4$, AE. None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

On the page before the previous page, you determined the Fourier series (i.e., the Fourier series coefficients) as given in your text for the function f(x) which has period 4 and is defined on the interval [-2,2) by $f(x) = \begin{cases} 0 & -2 \le x \le 0 \\ 2 & 0 < x < 2 \end{cases}$.

67.(3 pts.) a_0 = _____. A. 0, B. 1, C. 2, D. 1/2, E. 2/3, AB. −1, AC. −2, AD. −1/2 AE. −2/3 BC. None of the above.

68. (5 pts.) a_n =_____. A. 0, B.1/n, C. 2/n, D. a_{2k}=1/(2k), a_{2k+1}=1/(2k+1)
E. a_{2k}=π/(2k), a_{2k+1}=π/(2k+1), AB. a_{2k}=1/(2kπ), a_{2k+1}=1/[(2k+1)π],
AC. a_{2k}=2/(2kπ), a_{2k+1}=2/[(2k+1)π], AD. a_{2k}=1/(2k), a_{2k+1}= 0
AE. a_{2k}=π/(2k), a_{2k+1}= 0, BC. a_{2k}=1/(2kπ), a_{2k+1}= 0, BD. a_{2k}=2/(2kπ), a_{2k+1}= 0,
BE. a_{2k}= 0, a_{2k+1}=1/(2k+1), CD. a_{2k}= 0, a_{2k+1}=π/(2k+1),
CE. a_{2k}=0, a_{2k+1}=1/[(2k+1)π], DE. a_{2k}= 0, a_{2k+1}=2/[(2k+1)π], ABC. None of the above.

69. (5 pts.) b_n =_____. A. 0, B.1/n, C. 2/n, D. b_{2k}=1/(2k), b_{2k+1}=1/(2k+1)

E. $b_{2k}=\pi/(2k)$, $b_{2k+1}=\pi/(2k+1)$, AB. $b_{2k}=1/(2k\pi)$, $b_{2k+1}=1/[(2k+1)\pi]$,
AC. $b_{2k}=2/(2k\pi)$, $a_{2k+1}=2/[(2k+1)\pi]$, AD. $b_{2k}=1/(2k)$, $a_{2k+1}= 0$
AE. $b_{2k}=\pi/(2k)$, $b_{2k+1}= 0$, BC. $b_{2k}=1/(2k\pi)$, $a_{2k+1}= 0$, BD. $b_{2k}=2/(2k\pi)$, $b_{2k+1}= 0$,
BE. $b_{2k}= 0$, $b_{2k+1}=1/(2k+1)$, CD. $b_{2k}= 0$, $b_{2k+1}=\pi/(2k+1)$, CE. $b_{2k}=0$, $b_{2k+1}=1/[(2k+1)\pi]$,
DE. $b_{2k}= 0$, $b_{2k+1}=2/[(2k+1)\pi]$, ABC. None of the above.

70. (2 pts.) The Fourier series may be written as: A. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$,

B. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$, C. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

D. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$, E. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AB. $f(x) = 2 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$, AC. $f(x) = 1 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AD. $f(x) = 2 + \sum_{n=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$, AE. $f(x) = 2 + \sum_{n=1}^{\infty}\frac{2}{(2k)\pi}\cos(\frac{(2k)\pi}{2}x)$

BC. $f(x) = 2 + \sum_{n=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$, BD. $f(x) = 1 + \sum_{n=1}^{\infty}\frac{4}{(2k)\pi}\cos(\frac{(2k)\pi}{2}x)$

BE. None of the above.

Total points this page = 15. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

71. (5 pts.) Using the **method of separation of variables** with separation constant λ, one of the following sets of <u>two</u> **Ordinary Differential Equations** (ODE's) can be obtained from the following **Partial Differential Equation** (PDE):

PDE $t\,u_{xx} + x\,u_{tt} = 0$

Circle a correct set of ODE's from the following answers. Recall that the process does not yield a unique set of ODE's. Follow the advice given in class as to the choice of separation constant (attendance is mandatory).

A. $x X'' + \lambda X = 0$, $t T'' - \lambda T = 0$, B. $X'' + \lambda x X = 0$, $T'' - \lambda t T = 0$,
C. $X'' + \lambda x X = 0$, $t T'' - \lambda T = 0$, D. $\lambda X'' + x X = 0$, $\lambda T'' - t T = 0$,
E. $x X'' + \lambda X = 0$, $t T'' + \lambda T = 0$, AB. $X'' + \lambda x X = 0$, $T'' + \lambda t T = 0$,
AC. $X'' - \lambda x X = 0$, $t T'' - \lambda T = 0$, AD. $\lambda X'' + x X = 0$, $\lambda T'' + t T = 0$,
AE. Separation of variables does not work on this PDE. BC. None of the above

PRINT NAME __________________________(______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

Recall the definitions of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$, $PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)$, $PC^1_{ffs}([-\ell,\ell],\mathbf{R};\ell)$, and $B_{PC^1_{fs}([-\ell,\ell],\mathbf{R};\ell)}$ given on page 20. Now let $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ be the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions, $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ be the functions in $PC^1_{fs,o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to $[0,\ell]$, and $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ be the subspace of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$ for which the Fourier series is finite. Recall that $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is a Hamel basis of $PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)$ and a Schauder basis of $PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)$.

To formulate the heat conduction in a rod problem as a linear mapping problem we let D $= (-\ell,\ell)\times(0,\infty)$, $\bar{D} = [-\ell,\ell]\times[0,\infty)$, $\mathcal{A}_{HC}(\bar{D},\mathbf{R};\ell) = \{u(x,t)\in\mathcal{F}(\bar{D},\mathbf{R})\colon u\in\mathcal{A}(D,\mathbf{R})\cap C(\bar{D},\mathbf{R})\}$ and $\mathcal{A}_{HC,0}(\bar{D},\mathbf{R};\ell) = \{u(x,t)\in\mathcal{A}_{HC}(\bar{D},\mathbf{R}))\colon u(0,t) = 0 \text{ and } u(\ell,t) = 0 \text{ for } t > 0\}$. Now let $L\colon \mathcal{A}_{HC,0}(\bar{D},\mathbf{R}) \to \mathcal{A}(D,\mathbf{R})$ be defined by $L[u] = u_t - \alpha^2 u_{xx}$. Thus we incorporate the boundary conditions into the domain of the operator. Now let N_L be the null space of L and $\mathcal{A}_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L\colon u(x,0)\in PC^1_{ffs,o}([0,\ell],\mathbf{R};\ell)\}$ and $\mathcal{A}_{HC,0,0,fs}(\bar{D},\mathbf{R};\alpha^2,\ell) = \{u(x,t)\in N_L\colon u(x,0)\in PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)\}$. Hence we see that $\mathcal{A}_{HC,0,0,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell) \subseteq \mathcal{A}_{HC,o,fs}(\bar{D},\mathbf{R}) \subseteq N_L \subseteq$

$A_{HC,0}(\bar{D},\mathbf{R}) \subseteq A_{HC}(\bar{D},\mathbf{R};\ell)$. Now recall that $B_{PC^1_{fs,o}([0,\ell],\mathbf{R};\ell)} = \{e^{-\left(\frac{\alpha k\pi}{\ell}\right)^2} \sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is a Hamel basis for $A_{HC,0,o,ffs}(\bar{D},\mathbf{R};\alpha^2,\ell)$ and a Schauder basis for $A_{HC,o,fs}(\bar{D},\mathbf{R})$.

You are to solve: the following problem on this sheet and then answer the questions on the next two sheets.

	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 6 \sin(6\pi x)$	$0 < x < 2$

.

PRINT NAME ___________________________(_______________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

72. (3 pts.) The "general" or formal solution of

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell, \quad t > 0$
BC	$u(0,t) = 0, \quad u(\ell,t) = 0,$	$t > 0$

is given by _____.

A. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{\ell}{n\pi} x)$, B. $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

C. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell} x)$, D. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

E. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(n\pi\ell x)$, AB. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

AC. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \cos(\frac{n\pi}{\ell} x)$, AD. $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{\ell} x) + b_n \cos(\frac{n\pi}{\ell} x)$

AE. None of the above.

73. (3 pts.) The "general" or formal solution of PDE $u_t = u_{xx}$ $0 < x < 2$, $t > 0$

BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

is given by _____.

A. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{2}{n\pi}x)$, B. $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

C. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t} \sin(\frac{n\pi}{2}x)$, D. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

E. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(2n\pi x)$, AB. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

AC. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \cos(\frac{n\pi}{2}x)$, AD. $u(x,t) = \sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2}x) + b_n \cos(\frac{n\pi}{2}x)$

AE. None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME _________________________(______________) ID No. __________________

Last Name, First Name MI, What you wish to be called

74. (5 pts.) The solution of

	PDE	$u_t = u_{xx}$	$0 < x < 2$, $t > 0$
BVP for a PDE	BC	$u(0,t) = 0$, $u(2,t) = 0$,	$t > 0$
	IC	$u(x,0) = 6 \sin(6\pi x)$	$0 < x < 2$

is______.

A. $\sum_{n=1}^{\infty} 6 e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ B. $6 e^{-\frac{\pi^2}{4}t} \sin(\pi x)$

C. $6 e^{-\frac{9\pi^2}{4}t} \sin(\frac{3\pi}{2}x)$ D. $6 e^{-36\pi^2 t} \sin(6\pi x)$

E. $\sum_{n=1}^{\infty} 6 e^{-36\pi^2 t} \sin(6\pi x)$ AB. $6 e^{-12\pi^2 t} \sin(6\pi x)$

AC. $6\,e^{-12\pi^2 t}\sin(3\pi x)$ AD. $6\,e^{-144\pi^2 t}\sin(6\pi x)$

AE. None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______

MATH 261 FINAL EXAM Fall 2005 Professor Moseley Page 28

PRINT NAME __________________________(_______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

On this page you are to solve: the following problem:

	PDE	$u_t = u_{xx}$	$0 < x < 2,\quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0,\quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 2$	$0 < x < 2$

Begin by writing down the general solution for the PDE and the BC's. Then answer the questions on the next three pages.

PRINT NAME ___________________________(_______________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

75. (3 pts.) The solution of PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$
BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$
IC $u(x,0) = u_0(x)$
is given by _____.

A. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{\ell}{n\pi} x)$, B. $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

C. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell} x)$, D. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

E. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(n\pi\ell x)$, AB. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$

AC. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \cos(\frac{n\pi}{\ell} x)$, AD. $u(x,t) = \sum_{n=1}^{\infty} a_n \cos(\frac{n\pi}{\ell} x) + b_n \sin(\frac{n\pi}{\ell} x)$

AE. None of the above.

76. (3 pts.) where

A. $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{\ell}{n\pi}x\right)dx$ B. $c_n = \frac{1}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

C. $c_n = \frac{2}{\ell}\int_{-\ell}^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$, D. $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

E. $c_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin(n\pi\ell x)dx$, AB. $c_n = \int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

AC. $c_n = \frac{1}{\ell}\int_{-\ell}^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$, AD. $a_n = \frac{2}{\ell}\int_0^\ell u_0(x)\cos\left(\frac{n\pi}{\ell}x\right)dx$ and $b_n = \frac{2}{\ell}\int_0^\ell u_0(x)\sin\left(\frac{n\pi}{\ell}x\right)dx$

AE. None of the above.

PRINT NAME ________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

77. (3 pts.) The solution of

PDE	$u_t = u_{xx}$	$0 < x < \ell$, $t > 0$
BC	$u(0,t) = 0$,	$u(\ell,t) = 0$, $t > 0$
IC	$u(x,0) = 2$	

is given by _____.

A. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{2}{n\pi}x)$, B. $u(x,t) = \sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

C. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t}\sin(\frac{n\pi}{2}x)$, D. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

E. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\sin(2n\pi x)$, AB. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t}\sin(\frac{n\pi}{2}x)$

AC. $u(x,t) = \sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t}\cos(\frac{n\pi}{2}x)$, AD. $u(x,t) = \sum_{n=1}^{\infty} a_n\sin(\frac{n\pi}{2}x) + b_n\cos(\frac{n\pi}{2}x)$

AE. None of the above.

78. (3 pts.) where

A. $c_n = 2\int_0^2 \sin\left(\frac{2}{n\pi}x\right)dx$ B. $c_n = \int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

C. $c_n = 2\int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$, D. $c_n = 2\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

E. $c_n = 2\int_0^2 \sin(2n\pi x)dx$, AB. $c_n = 4\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

AC. $c_n = \int_{-2}^2 \sin\left(\frac{n\pi}{2}x\right)dx$, AD. $a_n = 2\int_0^2 \cos\left(\frac{n\pi}{2}x\right)dx$ and $b_n = 2\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$

AE. None of the above.

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

79. (5 pts.) The solution of

	PDE	$u_t = u_{xx}$	$0 < x < 2, \quad t > 0$
BVP for a PDE	BC	$u(0,t) = 0, \quad u(2,t) = 0,$	$t > 0$
	IC	$u(x,0) = 2$	$0 < x < 2$

is______.

A. $u(x,t) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$,

B. $u(x,t) = \sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$,

C. $u(x,t) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{2}x)$,

D. $u(x,t) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$,

E. $u(x,t) = \sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{2}x)$,

AB. $u(x,t) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{4}x)$,

AC. $u(x,t) = \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k+1)\pi}{4}x)$,

AD. $u(x,t) = \sum_{k=0}^{\infty} \frac{8}{(2k+1)\pi} e^{-\frac{(2k+1)^2\pi^2}{4}t} \sin(\frac{(2k+1)\pi}{2}x)$,

AE. $u(x,t) = \sum_{n=1}^{\infty} \frac{8}{(2k)\pi} e^{\frac{(2k+1)^2\pi^2}{4}t} \cos(\frac{(2k)\pi}{2}x)$,

BC. $u(x,t) = \sum_{k=1}^{\infty} \frac{4}{k\pi} e^{-\frac{k^2\pi^2}{4}t} \sin(\frac{k\pi}{2}x)$,

BD. $u(x,t) = \sum_{k=1}^{\infty} \frac{2}{k\pi} e^{-\frac{k^2\pi^2}{2}t} \sin(\frac{k\pi}{2}x)$,

BE. None of the above.

PRINT NAME _________________________(______________) SS No. __________________
Last Name, First Name MI, What you wish to be called

80. (4 pts.) If both ends of a rod are insulated (recall that we assume that the lateral sides are also insulated so that the temperature does not vary over a cross section), then a good mathematical model of the physical heat conduction problem is given by:

	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell$, $t > 0$
BVP for a PDE	BC	$u_x(0,t) = 0$, $u_x(\ell,t) = 0$,	$t > 0$
	IC	$u(x,0) = f(x)$	$0 < x < \ell$

where f(x) is the initial temperature distribution in the rod.

```
          ////////////////////////////////////////////
     -→ O__________________________O ←-
   /      ////////////////////////////////////////////      \
insulated                                              insulated
          *-----------------------------------*
```

x = 0 x = ℓ

Applying the separation of variables process to the PDE results in the two ODE's:

1. $X'' + \sigma X = 0$
2. $T' + \alpha^2 \sigma T = 0$

where σ is the separation constant. Determine the **spacial eigenvalue problem** that results from applying the BC given above. Circle the correct answer.

A. $X'' + \sigma X = 0$, $X(0) = 0,\ X(\ell) = 0$

B. $X'' + \sigma X = 0$, $X'(0) = 0,\ X(\ell) = 0$

C. $X'' + \sigma X = 0$, $X(0) = 0,\ X'(\ell) = 0$

D. $X'' + \sigma X = 0$, $X(0) = 1,\ X(\ell) = 0$

E. $X'' + \sigma X = 0$, $X'(0) = 0,\ X'(\ell) = 0$

AB. $X'' + \sigma X = 0$, $X'(0) = 1,\ X'\ell) = 0$

AC. $X'' + \sigma X = 0$, $X(0) = 1,\ X(\ell) = 1$

AD. $X'' + \sigma X = 0$, $X'(0) = 1,\ X(\ell) = 0$

AE. $X'' + \sigma X = 0$, $X(0) = 0,\ X'(\ell) = 0$

BC. None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(______________) SS No. ___________________
Last Name, First Name MI, What you wish to be called

TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	s > *a *
cosh (at)	$\frac{s}{s^2 - a^2}$	s > *a *
$e^{at}\sin(bt)$	$\frac{b}{\quad}$	$s > a$

	$(s - a)^2 + b^2$	
$e^{at}\cos(bt)$	$\dfrac{s - a}{(s - a)^2 + b^2}$	s > a
$t^n e^{at}$ n = positive integer	$\dfrac{n!}{(s - a)^{n+1}}$	s > a
u(t)	$\dfrac{1}{s}$	s > 0
u(t - c)	$\dfrac{e^{-cs}}{s}$	s > 0
$e^{ct}f(t)$	F(s - c)	
f(ct) c > 0	$\dfrac{1}{c} F\left(\dfrac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	