FALL2014 MATH 261: Elementary Differential Equations MATH 261
FINAL EXAM -A4 10:00 am FINAL EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(_________________________)
Last Name, First Name MI (What you wish to be called)

ID # ___________________________________ EXAM DATE Monday, December. 16,2014

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

_____________________________ ______________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 39 pages on this exam. Read through the entire exam. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! Ask for scratch paper if you need it. You may remove the staple. Print your name on all sheets. Pages 1-33 are Fill-in-the Blank/Multiple Choice or True/False. Expect no partial credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Your thoughts should be expressed in your best mathematics on this paper. Proofread as time allows. Pages 34 -39 contain information. GOOD LUCK!!

Scores

page	points	score
1	20	
2	7	
3	6	
4	8	
5	6	
6	5	
7	5	
8	9	
ST1	66	

Scores

page	points	score
9	3	
10	10	
11	7	
12	2	
13	12	
14	4	
15	9	
16	9	
ST2	56	

Scores

page	points	score
17	6	
18	5	
19	4	
20	5	
21	11	
22	6	
23	3	
24	4	
ST3	44	

Scores

page	points	score
25	12	
26	3	
27	4	
28	5	
29	6	
30	4	
31	4	
32	6	
33	3	
34	----	
35	----	
36	---	
37	---	
38	---	
ST4	47	
ST1	66	
ST2	56	
ST3	44	
ST4	47	
Tot.	213	

PRINT NAME _______________________________(________________) ID No. __________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. You are to classify the first order ordinary differential equations given below. The classification relates to the method of solution. Recall from class (attendance is mandatory) the possible methods listed below. Do not put more than one answer. If more than one method works, then any correct answer will receive full credit. Also remember that if I cannot read your answer, it is wrong. DO NOT SOLVE. Also recall the following:

a. In this context, exact means exact as given (in either of the forms discussed in class).

b. Bernoulli is not a correct method of solution if the original equation is linear.

c. Homogeneous (use the substitution $v = y/x$) is not a correct method of solution if it converts a separable equation into another separable equation.

1. (4 pts.) $(5e^x + 2xy + x)dx + (x^2 + 6y)dy = 0$ _______ A B C D E

2. (4 pts.) $(3xy + 5\cos(x))dx + 6x^2\, dy = 0$ ________ A B C D E

3. (4 pts.) $(3y^2 + x^2)dx + 5x^2\, dy = 0$ ________ A B C D E

4. (4 pts.) $3xye^{x+y}\, dx + 4x^2y^3 dy = 0$ _______ A B C D E

5.(4 pts.) $(y^2 + 3x^2y)\, dx + x\, dy = 0$ ________ A B C D E

Possible answers this page.

A) First order linear (y as a function of x). B) First order linear (x as a function of y).

C) Separable. D) Exact Equation (Must be exact in one of the two forms discussed in class).

E) Bernoulli, but not linear (y as a function of x).

AB) Bernoulli, but not linear (x as a function of y)

AC) Homogeneous, but not separable. ABCDE) None of the above

Possible points on page 1 is 20. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

Consider the first order linear ODE $y' = -y - 2x$ which we denote by (*). To solve (*), you may need to change it to a standard form.

1. (1 pts.) The correct standard form for (*) is ______________________________. ________ A B C D E
A) $y' + y = x$ B) $y' + y = -x$ C) $y' - y = x$ D) $y' - y = -x$ E) $y' + 2y = x$ AB) $y' + 2y = -x$
AC) $y' - 2y = x$ AD) $y' - 2y = -x$ AE) $y' + y = 2x$ BC) $y' + y = -2x$ BD) $y' - y = 2x$
BE) $y' - y = -2x$ CD) $y' + 2y = 2x$ CE) $y' + 2y = -2x$ DE) $y' - 2y = 2x$ ABC) $y' - 2y = -2x$
ABCDE) None of the above

2. (2 pts.) An integrating factor for (*) is μ=______________________________. _________ A B C D E
A) x B) $-x$ C) x^2 D) $-x^2$ E) $2x$ AB) $-2x$ AC) $2x^2$ AD) $-2x^2$ AE) e^x AD) e^{-x}
AE) e^{2x} BC) e^{-2x} BD) e^{x^2} E e^{-x^2} ABCDE) None of the above

3. (3 pts.) In solving (*) as we did in class (attendance is mandatory), the following step occurs:

____. _________ A B C D E

A) $\frac{d(ye^x)}{dx} = xe^x$ $\frac{d(ye^x)}{dx} = -xe^x$ $\frac{d(ye^x)}{dx} = 2xe^x$ $\frac{d(ye^x)}{dx} = -2xe^x$ $\frac{d(ye^{-x})}{dx} = xe^{-x}$ E) $\frac{d(ye^{-x})}{dx} = -xe^{-x}$ AB)
AD) $\frac{d(ye^{-x})}{dx} = 2xe^{-x}$ $\frac{d(ye^{-x})}{dx} = -2xe^{-x}$ $\frac{d(ye^{2x})}{dx} = xe^{2x}$ $\frac{d(ye^{2x})}{dx} = -xe^{2x}$ $\frac{d(ye^{2x})}{dx} = 2xe^{2x}$ BD)
CD) $\frac{d(ye^{2x})}{dx} = -2xe^{2x}$ $\frac{d(ye^{-2x})}{dx} = xe^{-2x}$) $\frac{d(ye^{-2x})}{dx} = -xe^{-2x}$ $\frac{d(ye^{-2x})}{dx} = 2xe^{-2x}$ $\frac{d(ye^{-2x})}{dx} = -2xe^{-2x}$ ABC)
ABCDE) None of the above

4. (1pt.) Let (**) be the initial value problem consisting of (*) and the initial condition $y(0) = 0$.

The number of solutions to (**) is ____________________. __________A B C D E A) 0 B) 1
C)2 D)3 E) 4 AB) 5 AC) Countable infinite number of solutions
AD) Uncountably infinite number of solutions ABCDE) None of the above

Possible points on page 2 is 7. TOTAL POINTS EARNED _________

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

An ODE may be considered to be a "vector" equation with the infinite number of unknowns in the vector being the values of the function for each value of the independent variable in the function's domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE, call it (*), of the standard form L[y] = g(x) where L is of the form L[y] = y′ + p(x)y, an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^{-x})}{dx} = -xe^{-x}$, call it (**). Recall that if a problem has an infinite number of solutions, the form of the solution is not unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

1. (2 pts.) The theorem from calculus that allows you to integrate the Left Hand Side of (**)

is ____. ________ A B C D E

A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem D) Chain Rule
E) Fundamental Theorem of Calculus AB) Product Rule
AC) Integration by Parts AD) Partial Fractions ABCDE) None of the above

2. (4 pts.) The solution (or family of solutions) to the ODE (*) may be written

as____. __________ A B C D E

A) $y = x + 1 + c\,e^{x}$ B) $y = -x + 1 + c\,e^{x}$ C) $y = x - 1 + c\,e^{x}$ D) $y = -x - 1 + c\,e^{x}$
E) $y = x + 1 + c\,e^{-x}$ AB) $y = -x + 1 + c\,e^{-x}$ AC) $y = x - 1 + c\,e^{-x}$ AD) $y = -x - 1 + ce^{-x}$
AE) $y = 2x + 2 + ce^{x}$ BC) $y = -2x + 2 + ce^{x}$ BD) $y = 2x - 2 + ce^{x}$ BE) $y = -2x - 2 + c\,e^{x}$
CD) $y = 2x + 2 + ce^{-x}$ CE) $y = -2x + 2 + ce^{-x}$ DE) $y = 2x - 2 + ce^{-x}$ ABC) $y = -2x - 2 + ce^{-x}$
ABCDE) None of the above

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Let L, p(x), g(x), (*), and (**) be as on the previous page.

3. (1 pts.) The set of solutions for the ODE (*) on the previous page may be written as

S = ____. ________ A B C D E

A) $\{y = x + 1 + c\,e^x : c \in \mathbf{R}\}$ B) $\{y = -x + 1 + c\,e^x : c \in \mathbf{R}\}$ C) $\{y = x - 1 + c\,e^x : c \in \mathbf{R}\}$
D) $\{y = -x - 1 + c\,e^x : c \in \mathbf{R}\}$ E) $\{y = x + 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AB) $\{y = -x + 1 + c\,e^{-x} : c \in \mathbf{R}\}$
AC) $\{y = x - 1 + c\,e^{-x} : c \in \mathbf{R}\}$ AD) $\{y = -x - 1 + ce^{-x} : c \in \mathbf{R}\}$ AE) $\{y = 2x + 2 + e^x + c : c \in \mathbf{R}\}$
BC) $\{y = -2x + 2 + ce^x : c \in \mathbf{R}\}$ BD) $\{y = 2x - 2 + ce^x : c \in \mathbf{R}\}$ BE) $\{y = -2x - 2 + ce^x : c \in \mathbf{R}\}$
CD) $\{y = 2x + 2 + ce^{-x} : c \in \mathbf{R}\}$ CE) $\{y = -2x + 2 + ce^{-x} : c \in \mathbf{R}\}$ DE) $\{y = 2x - 2 + ce^{-x} : c \in \mathbf{R}\}$
ABC) $\{y = -2x - 2 + ce^{-x} : c \in \mathbf{R}\}$ DE) None of the above

4. (1 pt.) A basis for the nullspace of L is B = ________________________. ________A B C D E
A) $\{1\}$ B) $\{x\}$ C) $\{1,x\}$ D) $\{e^x\}$ E) $\{e^{-x}\}$ AB) $\{e^x, e^{-x}\}$ ABCDE) None of the above

5. (1 pt.) The general solution of $L[y] = 0$ is $y_c(x)$ =______________________. ________A B C D E
A) c B) cx C) $c_1 + c_2x$ D) ce^x E) ce^{-x} AB) $c_1e^x + c_2e^{-x}$ ABCDE) None of the above

6. (1 pt.)) Using the linear theory, a particular solution of $L[y] = g(x)$ is given by

$y_p(x)$ =____. _________A B C D E
A) 1 B) x C) x + 1 D) x − 1 E) 1 − x AB) −x −1 AC) e^x AD) e^{-x}
ABCDE) None of the above

7. (1 pt.) The number of solutions to (*) is ______________________________. ______A B C D E
A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Countable infinite number of solutions
AD) Uncountably infinite number of solutions ABCDE) None of the above

8. (2 pts.) Let (***) be the initial value problem consisting of (*) and the initial condition $y(0) = 0$. The solution (or family of solutions) to (***) may be written

as y =____. _________ A B C D E
A) $y = x + 1 - e^x$ B) $y = -x + 1 - e^x$ C) $y = x - 1 + e^x$ D) $y = x + 1 - e^{-x}$
E) $y = -x + 1 - e^{-x}$ AB) $y = x + 2 - 2e^{-x}$ AC) $y = x - 1 + e^{-x}$ AD) $y = x + 1 + e^x - 2 + c$
AE) $y = x + 1 + e^x$ BC) $y = x - 1 + e^x$ BD) $y = x + 1 + e^{-x} - 2$ BE) $y = -x + 1 + e^{-x} - 2$
CD) $y = x - 1 + e^{-x}$ CE) $y = x + 1 - e^{-x}$ ABCDE)None of the above .

9. (1 pt.) The number of solutions to (***) is __________.______A B C D E
A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Countable infinite number of solutions
AD) Uncountably infinite number of solutions ABCDE) None of the above

Possible points on page 4 is 8. TOTAL POINTS EARNED _________

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let $A = \begin{bmatrix} 1 & -2i \\ -i & -2 \end{bmatrix}$ $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$, $\vec{b} = \begin{bmatrix} -2 \\ 2i \end{bmatrix}$, $T(\vec{x}) = A\vec{x}$. Also let so

Prob($\mathbf{C}^2, A\vec{x} = \vec{b}$); that is, solve the mapping $T(\vec{x}) = \vec{b}$ (i.e., solve t $A\vec{x} = \vec{b}$ equation

form of the answer may not be unique. To obtain the answer listed, follow the directions given in class (attendance is mandatory). Also, be careful. If you miss a question on this page, it may cause you to miss questions on the next page.

1. (3 pts.) If $\left[A \middle| \vec{b}\right]$ is red $\left[U \middle| \vec{c}\right]$ using Gauss elimination we obtain

$\left[U \middle| \vec{c}\right]$ =_________________________________. ________A B C D $\left[\begin{array}{cc|c} 1 & i & 1 \\ 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & i & -1 \\ 0 & 0 & 0 \end{array}\right]$ B)

C) $\left[\begin{array}{cc|c} 1 & -i & 1 \\ 0 & 0 & 1 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & -i & -1 \\ 0 & 0 & 0 \end{array}\right]$ $\left[\begin{array}{cc|c} 1 & -2i & -2 \\ 0 & 0 & 1 \end{array}\right]$ E $\left[\begin{array}{cc|c} 0 & 0 & 0 \\ 0 & 0 & 0 \end{array}\right]$ AB) ABCI

2. (3 pts.) The solution of $A\vec{x} = \vec{b}$ may be written as

$\vec{x} =$ _______________________________._______A B C D E A) No Solution $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ C)

D) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AB) $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix}$ $\begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ $\begin{bmatrix} -2 \\ 0 \end{bmatrix} + y\begin{bmatrix} -2i \\ 1 \end{bmatrix}$ AD)

BC) $\begin{bmatrix} -2 \\ 0 \end{bmatrix} + y\begin{bmatrix} -2i \\ 1 \end{bmatrix}$ ABCDE) None of the above .

PRINT NAME _____________________________(_________________) ID No. ___________________
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Let $\text{Prob}(\mathbf{C}^2; A\vec{x} = \vec{b})$, A, $\vec{b}$, $\vec{x}$, and T be as on the previous page.

3. (1 pt.) The set of solutions for Prob($\mathbf{C}^2$, $A\vec{x} = \vec{b}$) may be written as

S =____.__________A B C D E A) $\varnothing$ B) $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$

C) $\left\{\vec{x} = y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$ $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ E) $\left\{\vec{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

AC) $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ $\left\{\vec{x} = \begin{bmatrix} -1 \\ 0 \end{bmatrix} + y\begin{bmatrix} -i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

AE) $\left\{\vec{x} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + y\begin{bmatrix} 2i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$ $\left\{\vec{x} = \begin{bmatrix} -2 \\ 0 \end{bmatrix} + y\begin{bmatrix} -2i \\ 1 \end{bmatrix} \in \mathbf{C}^2 : y \in \mathbf{C}\right\}$

ABCDE) None of the above correctly describes the set of solutions for this problem.

4. (1 pt.) A basis for the null space of the operator T is B =_______________________. _________A B C D E

A) $\varnothing$ B) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} 0 \\ 1 \end{bmatrix}\right\}$) $\left\{\begin{bmatrix} i \\ 1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} i \\ -1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} -i \\ -1 \end{bmatrix}\right\}$ AB) $\left\{\begin{bmatrix} i \\ -1 \end{bmatrix}\right\}$ AC)

AE) $\left\{\begin{bmatrix} -2i \\ 1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -i \\ 1 \end{bmatrix}\right\}$ $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -2i \\ 1 \end{bmatrix}\right\}$ BD) $\left\{\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -i \\ 2 \end{bmatrix}\right\}$ BE)

ABCDE) None of the above correctly describes a basis for the null space for this problem

5. (1 pt.) The general solution of $A\vec{x} = \vec{0}$ may be written as $\vec{x}_c$ =_____________. __________A B C D E

A) No Solution B) $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -i \\ 1 \end{bmatrix}$ $y\begin{bmatrix} i \\ 1 \end{bmatrix}$ D) $y\begin{bmatrix} i \\ -1 \end{bmatrix}$ E) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ AB)

AC) $y\begin{bmatrix} -i \\ 1 \end{bmatrix}$ $y\begin{bmatrix} 2i \\ 1 \end{bmatrix}$ $y\begin{bmatrix} -2i \\ 1 \end{bmatrix}$ AE) ABCDE) None of the above

6. (1 pt.) Using the linear theory, a particular solution of $A\vec{x} = \vec{b}$ is given by

$\vec{x}_p$ =________________________. __________A B C D E $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$ B) $\begin{bmatrix} i \\ 1 \end{bmatrix}$ C) $\begin{bmatrix} i \\ -1 \end{bmatrix}$ D)

E) $\begin{bmatrix} 2 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -i \\ -1 \end{bmatrix}$ $\begin{bmatrix} 1 \\ i \end{bmatrix}$ AC) $\begin{bmatrix} 1 \\ -i \end{bmatrix}$ AD) $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$ $\begin{bmatrix} -1 \\ -i \end{bmatrix}$) $\begin{bmatrix} i \\ i \end{bmatrix}$ BC) $\begin{bmatrix} -2 \\ 0 \end{bmatrix}$ BD)

CD) $A\vec{x} = \vec{b}$ has solutions but none are listed $A\vec{x} = \vec{b}$ E) has no solutions ABCDE) None of the ab

7. (1 pt.) The number of solutions to Prob($\mathbf{C}^2$; $A\vec{x} = \vec{b}$)
is ____.________A B C D E A) 0 B) 1 C)2 D)3 E) 4
AB) 5 AC) Countable infinite number of solutions AD) Uncountably infinite number of solutions
ABCDE) None of the above

Possible points on page 6 is 5. TOTAL POINTS EARNED THIS PAGE _______

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True or false. Solution of Abstract Linear Equations (having either **R** or **C** as the field of scalars). Assume T: V → W is a linear operator from a (real or complex) vector space V to a (real or complex) vector space W. Now consider the mapping problem defined by the vector equation

$$T(\vec{x}) = \vec{b} \quad .$$

Under these hypotheses, determine which of the following is true and which is false. If true, circle True. If false, circle False.

1. (1 pt.) A)True or B)False If $\vec{b} = \vec{0}$, then (*) has an infinite number of solutions.

2. (1 pt.) A)True or B)False The vector equation (*) may have exactly five solutions.

3.(1 pt.) A)True or B)False If the null space of T has a basis $B = \{\vec{x}_1, \vec{x}_2, \ldots, \vec{x}_n\}$ $\vec{b}=\vec{0}$ and then the general solution of (*) is given by
$\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2 + \cdots + c_n\vec{x}_n$ where $c_1, c_2, \ldots, c_n$ are arbitrary

4. (1 pt.) A)True or B)False Either (*) has no solutions, exactly one solution, or an infinite number of solutions.

5. (1 pt.) A)True or B)False If the null space of T is $N(T) = \{\vec{0}\}$ $\vec{b}$ and is in the range space of T, then (*) has a unique solution.

Possible points on page 7 is 5. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet.

The dimension of the null space N_L of the linear operator $L[y] = y'' - y'$ that maps $\mathscr{A}(\mathbf{R},\mathbf{R})$ to $\mathscr{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that $B_{N_L} = \{1, e^{x}\}$ is a basis of N_L so that

$y_c = c_1 + c_2\, e^{x}$ is the general solution of $y'' - y' = 0$.

Use the method of undetermined coefficients as discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Begin with a first guess. If needed provide additional guesses. Place your final guess in the space provided. Then circle the letter or letters that correspond to your answer from the answers listed below.

1. (3 pts.) $y'' - y' = 2x$ First guess: y_p = ________________________________
Second guess (if needed): y_p = ____________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . __________ A B C D E

2. (3 pts.) $y'' - y' = 2 \sin x$ First guess: y_p = ____________________________
Second guess (if needed): y_p = ____________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . __________ A B C D E

3. (3 pts.) $y'' - y' = 4e^{-x}$ First guess: y_p = ____________________________
Second guess (if needed): y_p = ____________________________
Third guess (if needed): y_p = ____________________________

Final guess ____ . __________ A B C D E

Possible Answers this page.
A) Ae^x B) Axe^x C) Ax^2e^x D) $Axe^x + Be^x$ E) $Ax^2e^x + Bxe^x$ AB) Ae^{-x} AC) Axe^{-x}
AD) Ax^2e^{-x} AE) $Axe^{-x} + Be^{-x}$ BC) $Ax^2e^{-x} + Bxe^{-x}$ BD) $A \sin x$ BE) $A\cos x$
CD) $A\, x \sin x$ CE) $A\, x \cos x$ DE) $A \sin x + B \cos x$ ABC) $A\, x \sin x + B\, x \cos x$
ABD) A ABE) Ax ACD) $Ax + B$ ACE) $Ax^2 + Bx$ ADE) $Ax^2 + Bx + C$
BCD) None of the above

Possible points on page 8 is 9. TOTAL POINTS EARNED THIS PAGE _______

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Let Prob($\mathscr{A}$((−π/2,π/2),**R**), (*)) be the problem defined by the ODE

$$y'' + y = -\sec(x) \quad I = (-\pi/2, \pi/2) \qquad (*)$$

Let L: $\mathscr{A}$((−π/2,π/2),**R**)→$\mathscr{A}$((−π/2,π/2),**R**) be defined by L[y] = y" + y. The general solution to L[y] = 0 is $y_c = c_1 \cos(x) + c_2 \sin(x)$. To obtain a particular solution of L[y] = tan(x) we let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. You are to find y_p. Be careful!! Remember, once you make a mistake, the rest is wrong.

1. (3 pts.) Starting with $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ and making the appropriate assumption(s) you obtain the two equations which are:

____ . _______A B C D E

A) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$
B) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\tan(x)$
C) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
D) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\tan(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
E) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sec(x)$
AB) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0, \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = -\sec(x)$
AC) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \sec(x), \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
AD) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = -\sec(x) \quad -u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
ABCDE) None of the above.

Possible points on page 9 is 3. TOTAL POINTS EARNED THIS PAGE ______

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Let Prob($\mathscr{A}((-\pi/2,\pi/2),\mathbf{R})$, (*)), (*), L and $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$ be as defined on the previous page.
2. (2 pts.) Solving the set of equations for $u'_1(x)$ and $u'_2(x)$ on the previous page we obtain

$u'_1(x)$ =____. _________A B C D E

3. (2 pts.) And $u'_2(x)$ = ____. ________A B C D E

4. (2 pts.) Hence we may choose $u_1(x)$ =___________________________________. _________A B C D E

5. (2 pts.) And $u_2(x)$ =____. __________A B C D E
6 (2 pts.) Hence a particular solution to (*) is

$y_p(x)$ = ____. ________A B C D E

Possible answers this page.
A)0 B)1 C)−1 D) x E) −x AB) sin x AC) −sin x AD) cos x AE) −cos x BC) tan x BD) −tan x
BE)sin(x) cos(x) CD) −sin(x) cos(x) CE) $\sin^2(x)/\cos(x)$ DE) − $\sin^2(x)/\cos(x)$ ABC)ln(sin x)
ABD) −ln(sin x) ABE) ln(cos x) ACD) −ln(cos x) ACE) [sin(x)]ln(tan(x)+sec(x))
ADE)[cos(x)]ln(tan(x)+sec(x)) BCD) −[cos(x)]ln(tan(x)+sec(x)) BCE) ln(sin x) BDE)−ln(sin x)
CDE) ln(cos x) ABCD) −ln(cos x) ABCE) (cos(x)) [ln(sin(x))] + x sin(x)
ABDE) −(cos(x)) [ln(sin(x))] + x sin(x) ACDE) (cos(x)) [ln(cos(x))] + x sin(x)
BCDE) −(cos(x)) [ln(cos(x))] − x sin(x) ABCDE)None of the above.
Possible points on page 10 is 10. TOTAL POINTS EARNED THIS PAGE ______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. If you miss one part, it may cause you to miss other parts.

Consider $y^V - 4y''' = 0$ which we denote by (*) Also let $L: \mathscr{A}(\mathbf{R},\mathbf{R}) \to \mathscr{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^V - 4y'''$ and N_L be the null space of L.

1. (1 pt). The dimension of N_L is ____________________. _________ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) Countably infinite AE) Uncountably infinite ABCDE) None of the above.

2. (1 pts). The auxiliary equation for (*) is ____________________________________ . _______ A B C D E
A) $r^5 + 4r^3 = 0$ B) $r^5 - 4r^3 = 0$ C) $r^5 + 4r^4 + 4r^3 = 0$ D) $r^4 - 4r^3 + 4r^2 = 0$
E) $r^4 + 4r^3 = 0$ AB) $r^4 - 4r^3 = 0$ AC) $r^6 + 4r^3 + 4r^2 = 0$ ABCDE) None of the above.

3. (2 pts). Listing repeated roots, the roots of the auxiliary equation

are ________________________________. ________ A B C D E A) 0,0,0,2,2 B)0,0,0,−2,−2 C)0,0,0, 2,−2 D) 0,0,0, 2i,−2i E)0,0,0, 2,−2 AB) r =0,0,0,−2,−2i ABCDE) None of the above.

4. (1 pts). A basis for the null space of L is B = ________________________________. ________ A B C D E
A)$\{1,x,x^2,e^{2x},xe^{2x}\}$ B)$\{1,x,x^2,e^{-2x},xe^{-2x}\}$ C)$\{1,x,x^2,e^{2x},e^{-2x}\}$ D)$\{1,x,x^2,\sin 2x, \cos 2x\}$
E)$\{1,x,x^2e^{2x}, \sin 2x\}$ AB)$\{1,x, x^2, e^{-2x}, \sin 2x\}$ AC) $\{1, x, x^2, e^{-2x}\}$ AD)$\{1, x, x^2, x^3, x^4\}$
AE) $\{1,e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ ABCDE) None of the above

5. (2 pt). The general solution of (*) is y(x) = ____________________________________ . _______ A B C D E
A)$c_1 + c_2 x + c_3x^2 + c^4 e^{2x} + c_5 xe^{2x}$ B)$c_1 + c_2 x + c_3x^2 + c_4 e^{-2x} + c_5 xe^{-2x}$ C)$c_1 + c_2 x + c_3x^2 + c_4 e^{2x} + c_5 e^{-2x}$
D) $c_1 + c_2x + c_3x^2 + c_4 \sin 2x + c_5 \cos 2x$ E)$c_1+c_2x + c_3x^2 + c_4x^3 + c_5 e^{2x}$ AB)$c_1+c_2 x + c_3 x^2 + c_4 e^{2x} + c_5 e^{2x}$
AC) $c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $c_1+c_2 x + c_3 x^2 + c_4 x^3$ AE) $c_1e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$
ABCDE)None of the above

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Let (*) and L be as on the previous page.

6. (1pt.) The set of solutions for (*) may be written as

S = ____. _________A B C D E

A) $\{y(x) = c_1 + c_2 x + c_3x^2 + c_4 e^{2x} + c_5 e^{-2x} : c_1, c_2, c_3, c_4, c_5, \in \mathbf{R}\}$

B) $\{y(x) = c_1 + c_2 x + c_3x^2 + c_4 e^{-2x} + c_5 xe^{-2x}: c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$

C) $\{y(x) = c_1 + c_2 x + c_3x^2 + c_4 e^{2x} + c_5x e^{2x}: c_1, c_2, c_3, c_4, c_5 \in \mathbf{R}\}$

D) $\{y(x) = c_1 + c_2 x + + c_3x^2 + c_4 e^{-2x} + c_5x e^{-2x}: c_1, c_2, c_3, c_4 , c_5 \in \mathbf{R}\}$

E) $\{y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x} + c_5x e^{2x}: c_1, c_2, c_3, c_4 , c_5 \in \mathbf{R}\}$

AB) $\{y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x} + c_5 xe^{-2x}: c_1, c_2, c_3, c_4 , c_5 \in \mathbf{R}\}$

AC) $\{y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}: c_1, c_2, c_3, c_4 \in \mathbf{R}\}$

AD) $\{y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x} : c_1, c_2, c_3, c_4 \in \mathbf{R}\}$ ABCDE) None of the above

7. (1 pt.) The number of solutions to (*) is _____________________________________._______A B C D E

A) 0 B) 1 C)2 D)3 E) 4 AB) 5 AC) Countable infinite number of solutions
AD) Uncountably infinite number of solutions ABCDE) None of the above

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Answer questions using the instructions on the Exam Cover Sheet. Also circle your answers. Be careful!! Remember, once you make a mistake, the rest is wrong.

Consider the ODE $y''' - y'' = 6x + 4e^x$ which we will call (*). Now let $L[y] = y''' - y''$.

1. (3 pts.) The general solution of $y''' - y'' = 0$ is

$y_c(x) =$ ____. _______ A B C D E

A)$c_1+c_2x+c_3e^x$ B)$c_1+c_2x+c_3e^{-x}$ C)$c_1+c_2e^x+c_3e^{-x}$ D)$c_1e^x+c_2\sin(x)+c_3\cos(x)$
E)$c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$ ABCDE) None of the above

2. (4 pts.) A particular solution of $y''' + y'' = 6x$ is

$y_{p1}(x) =$ ____. _________ A B C D E

A)1+x B)2+x C)2+2x D)2+3x E)3+3x AB)3+4x AC)4+3x AD)4+4x AE)4+5x BC)5+4x
BD) −3−x BE)1+3x CD)2+3x CE)1+4x DE)3+x ABC)3+2x ABD)4+2x ABE)5+2x ACD)2+5x
ACE)6+x ADE)2+2x BCD)2+2x BCE)2+2x BDE)2+2x CDE)2+2x ABCD)2+2x ABDE)2+2x
ACDE)2+2x BCDE)2+2x ABCDE) None of the above

3. (4 pts.) A particular solution of $y''' + y'' = 4e^x$ is

$y_{p2}(x) =$ ____. _________ A B C D E

A)$2+ e^x$ B) e^x C) $2e^x$ D)$3e^x$ E)$4xe^x$ AB)$5e^x$ AC)$6e^x$ AD)e^x+xe^x AE)$2e^x+xe^x$ BC)$2e^x+2xe^x$ BD)$2e^x+ 3xe^x$
BE)$3e^x +3xe^x$ CD)$3+2x+ e^x$ CE)$2+2x+ e^x$ DE)$2+2x +e^x$ ABC)$2+2x+ e^x$ ABD)$2+2x e^x$ ABE)$2+2x+ e^x$
ACD)$2+2x+ e^x$ ACE)$2+2x+ e^x$ ADE)$2+2x +e^x$ ABCDE) None of the above

4. (1 pts.) A particular solution of (*) is

$y_p(x) =$ ____. __________ A B C D E

A)$1+x+ e^x$ B)$-1+x +2e^x$ C)$1+2x+ 3e^x$ D)$2+2x+ 3xe^x$ E)$3+x+4xe^x$ AB)$2+2x+3xe^x$ AC)$-1+2x+ 3xe^x$
AD)$-3-x+ 4xe^x$ AE)$2+2x + xe^x$ BC)$2+2x+ e^x$ BD)$2+2x+ e^x$ BE)$2+2x+ e^x$ CD)$2+2x+3 e^x$ CE)$2+2x + 4e^x$
DE)$2+x +3e^x$ ABC)$2+2x+ e^x$ ABD)$2+x e^x$ ABE)$2+2x+ e^x$ ACD)$2+2x+ e^x$ ACE)$2+2x+ 5e^x$
ADE)$2+2x + 6e^x$ BCD)$2+2x+ 7e^x$ BCE)$2+2x + 8e^x$ BDE)$2+2x+ 9e^x$ CDE)$2+3x +4e^x$
ABCD)$2+4x+9e^x$ ABDE)2+5x +ex ACDE)$2+6x+ e^x$ BCDE)$2+7x +e^x$ ABCDE) None of the above

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Let (*) and L be as on the previous page.

5. (2 pts.) The general solution of (*) is

y(x) = ____. ________ A B C D E

A) $c_1 + c_2x + c_3e^x + x + 1 + xe^x$ B) $c_1 + c_2x + c_3e^x + x + 1 + 2xe^x$ C) $c_1 + c_2x + c_3e^x + x + 2 + xe^x$
D) $c_1 + c_2x + c_3e^x + x + 2 + 2xe^x$ E) $c_1 + c_2x + c_3e^x - x - 3 + 4xe^x$ AB) $c_1 + c_2x + c_3e^x + 2x + 1 + 2xe^x$
AB) $c_1 + c_2x + c_3e^x + 2x + 2 + xe^{x\,x}$ AC) $c_1 + c_2x + c_3e^x + 2x + 2 + 2xe^x$ AD) $c_1 + c_2x + c_3e^x + 2x + 2 + 3xe^x$
AE) $c_1 + c_2x + c_3e^x + 2x + 3 + 2xe^x$ BC) $c_1 + c_2x + c_3e^x + 2x + 3 + xe^x$ BD) $c_1 + c_2x + c_3e^x + x + 3 + 4xe^x$
BE) $c_1 + c_2x + c_3e^x + x + 3 + 4xe^x$ CD) $c_1 + c_2x + c_3e^x + 2x + 3 + 4xe^x$ CE) $c_1 + c_2x + c_3e^x + 3x + 3 + 4xe^x$
DE) $-2xe^{-x} + 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ ABC) $-2xe^{-x} + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}$
ABD) $-2xe^{-x} - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ ABE) $-2xe^{-x} - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^{-x}$
ACD) $-2e^x - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x}$ ACE) $2e^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x$
ADE) $2e^x + c_1\sin(x) + c_2\cos(x) + c_3e^{-x}$ CDE) $2e^x + 2\sin(x) + 2\cos(x) + c_1e^x + c_2e^{-x} + c_3x$
ABCD) $2e^{-x} + 2\sin(x) + \cos(x) + c_1xe^x + c_2e^{-x} + c_3xe^{-x}$ ABCE) $2e^x + 2\sin(2x) + \cos(2x) + c_1e^x + c_2xe^x + c_3$
ABCDE) None of the above.

6. (1pt.) The set of solutions for (*) may be written as

S = ____. _________A B C D E

A) $\{c_1 + c_2x + c_3e^x + x + 1 + xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ B) $\{c_1 + c_2x + c_3e^x + x + 1 + 2xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
C) $\{c_1 + c_2x + c_3e^x + x + 2 + xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ D) $\{c_1 + c_2x + c_3e^x + x + 2 + 2xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
E) $\{c_1 + c_2x + c_3e^x - x + 3 + 4xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ AB) $\{c_1 + c_2x + c_3e^x + 2x + 1 + 2xe^x\}$
AB) $\{c_1 + c_2x + c_3e^x + 2x + 2 + xe^{x\,x} : c_1, c_2, c_3 \in \mathbf{R}\}$ AC) $\{c_1 + c_2x + c_3e^x + 2x + 2 + 2xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
AD) $\{c_1 + c_2x + c_3e^x + 2x + 2 + 3xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ AE) $\{c_1 + c_2x + c_3e^x + 2x + 3 + 2xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
BC) $\{c_1 + c_2x + c_3e^x + 2x + 3 + xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ BD) $c_1 + c_2x + c_3e^x + x + 3 + 4xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
BE) $\{c_1 + c_2x + c_3e^x + x + 3 + 4xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$ CD) $\{c_1 + c_2x + c_3e^x + 2x + 3 + 4xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
CE) $\{c_1 + c_2x + c_3e^x + 3x + 3 + 4xe^x : c_1, c_2, c_3 \in \mathbf{R}\}$
DE) $\{y(x) = 2xe^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
ABC) $\{y(x) = 2xe^x - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
ABD) $\{y(x) = 2xe^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
ABE) $\{y(x) = -2xe^x + 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
ACD) $\{y(x) = -2xe^x + 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
ADE) $\{y(x) = -2xe^x - 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
BCD) $\{y(x) = -2xe^x - 2\sin(x) - 2\cos(x) + c_1 + c_2x + c_3e^x : c_1, c_2, c_3 \in \mathbf{R}\}$
BCE) $\{y(x) = 2xe^{-x} + 2\sin(x) + 2\cos(x) + c_1 + c_2x + c_3e^{-x} : c_1, c_2, c_3 \in \mathbf{R}\}$ ABCDE) None of the above

7. (1 pt.) The number of solutions to (*) is ____._____A B C D E
A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) Countable infinite number of solutions
AD) Uncountably infinite number of solutions ABCDE) None of the above

Possible points on page 14 is 4. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet. .

Compute the Laplace transform of the following functions in $PC[0,\infty)\cap Exp \subseteq \mathbf{T}$.

1. (3 pts.) $f(t) = 2 - 3t^2 \Rightarrow \mathcal{L}(f) =$ ___________________________________. __________ A B C D E

2. (3 pts.) $f(t) = 2e^{2t} - 3e^{-3t} \Rightarrow \mathcal{L}(f) =$ ________________________________. ___________ A B C D E

3. (3 pts.) $f(t) = 2\sin(3t) - 3\cos(2t) \Rightarrow \mathcal{L}(f) =$ __________________________. __________ A B C D E

Possible answers this page

A) $\frac{2}{s}+\frac{3}{s^2}$ $\frac{2}{s}-\frac{3}{s^2}$ $-\frac{2}{s}+\frac{3}{s^2}$ C) $-\frac{2}{s}-\frac{3}{s^2}$ $\frac{2}{s}+\frac{6}{s^3}$ D) $\frac{2}{s}-\frac{6}{s^3}$ $-\frac{2}{s^2}+\frac{6}{s^3}$ $-\frac{2}{s^2}-\frac{6}{s^3}$ B) AC

AE) $\frac{2}{s+2}+\frac{3}{s+3}$ $\frac{2}{s+2}-\frac{3}{s+3}$ $-\frac{2}{s+2}+\frac{3}{s+3}$ BI $-\frac{2}{s+2}-\frac{3}{s+3}$ BE)

CD) $\frac{2}{s-2}+\frac{3}{s+3}$ $\frac{2}{s-2}-\frac{3}{s+3}$ $-\frac{2}{s-2}+\frac{3}{s+3}$ D $\frac{2}{s+2}-\frac{3}{s-3}$ ABC)

ABD) $\frac{2}{s^2+4}+\frac{3s}{s^2+9}$ $\frac{4}{s^2+4}-\frac{3s}{s^2+9}$ $\frac{6}{s^2+4}-\frac{3s}{s^2+9}$ $-\frac{4}{s^2+4}-\frac{3s}{s^2+9}$ ACE)

ADE) $\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ $\frac{4}{s^2-4}-\frac{3s}{s^2-9}$ $-\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ I $-\frac{4}{s^2-4}-\frac{3s}{s^2-9}$ CD

ABCD) $-\frac{2}{(s-2)^2}+\frac{3}{(s+3)^2}$ $\frac{2}{s^2+2}+\frac{3s}{s^2+3}$ \BCE) $\frac{2s}{s^2+2}+\frac{3}{s^2+3}$ ABDE)

ACDE) $\mathcal{L}\{f\}$ exists but none of the above is $\mathcal{L}\{f\}$ BCDE) $\mathcal{L}\{f\}$ does not exist.

ABCDE)None of the above.

Possible points on page 15 is 9. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions if they are in **F**:

1. (3 pts.) $F(s) = \frac{2}{s} \quad \frac{3}{s+2}$ $\Rightarrow$ $\mathcal{L}^{-1}\{F\}$ = ________________________________. _______ A B C D E

2. (3 pts.) $F(s) = \frac{2s+3}{s^2+9}$ $\Rightarrow$ $\mathcal{L}^{-1}\{F\}$ = ____________________________________. _________ A B C D E

3. (3 pts.) $F(s) = \frac{-2s+3}{s^2-2s+2}$ $\Rightarrow$ $\mathcal{L}^{-1}\{F\}$ = ________________________________. ________ A B C D

Possible answers this page.

A) $2 + 3e^{2t}$ B) $2 - 3e^{2t}$ C) $-2 + 3e^{2t}$ D) $-2 - 3e^{2t}$ E) $2 + 3e^{-2t}$ AB) $2 - 3e^{-2t}$ AC) $-2 + 3e^{-2t}$
AD) $-2 - 3e^{-2t}$ AE) $\cos 3t + \sin 3t$ BC) $2\cos 3t - \sin 3t$ BD) $2\cos 3t + \sin 3t$
BE) $2\cos 3t + \sin 3t$ CD) $2\cos t + (4/3)\sin 3t$ CE) $2\cos 3t - (4/3)\sin 3t$
DE) $-2\cos 3t + (4/3)\sin 3t$ ABC) $-2\cos 3t - (4/3)\sin 3t$ ABD) $2e^t \cos t + 5e^t \sin t$
ABE) $2e^t \cos t - e^t \sin t$ ACD) $-2e^t \cos t + e^t \sin t$ ACE) $-2e^t \cos t - e^t \sin t$
ADE) $2e^t \cos t + e^t \sin t$ BCD) $2e^t \cos t - e^t \sin t$ BCE) $-2e^t \cos t + 2e^t \sin t$
BDE) $-2e^t \cos t - 2e^t \sin t$ CDE) $\mathcal{L}^{-1}\{f\}$ exists but none of the above is $\mathcal{L}\{f\}$
ABCD) $\mathcal{L}^{-1}\{f\}$ does not exist. ABCDE) None of the above.

Possible points on page 16 is 9. TOTAL POINTS EARNED THIS PAGE ________

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Using the procedure illustrated in class (attendance is mandatory), find the eigenvalues of

$A = \begin{bmatrix} -i & 3 \\ 0 & -4 \end{bmatrix} \in \mathbf{C}^{2x2}$.

1. (2 pts.) We have that $p(\lambda) = \det(A-\lambda I)$ in factored form is

$p(\lambda) =$ ____. __________ A B C D E

A) $(i+\lambda)(1+\lambda)$ B) $(i+\lambda)(1-\lambda)$ C) $(i-\lambda)(1+\lambda)$ D) $(i-\lambda)(1-\lambda)$ E) $(i+\lambda)(2+\lambda)$ AB) $(i+\lambda)(2-\lambda)$ AC) $(i-\lambda)(2+\lambda)$ AD) $(i-\lambda)(2-\lambda)$ AE) $(2i+\lambda)(1+\lambda)$ BC) $(2i+\lambda)(1-\lambda)$ BD) $(2i-\lambda)(1+\lambda)$ BE) $(2i-\lambda)(1-\lambda)$ CD) $(2i+\lambda)(2+\lambda)$ CE) $(2i+\lambda)(2-\lambda)$ DE) $(-i-\lambda)(-4-\lambda)$ ABC) $(2i-\lambda)(2-\lambda)$ ABD) $(3i-\lambda)(2+\lambda)$ ABCDE) None of the above.

2. (1 pt.) The degree of $p(\lambda)$ is ______________. ________ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 ` AE) 8 ABCDE) None of the above

3. (1 pt.) Counting repeated roots, the number of eigenvalues of A

is _____________. ________ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 ABCDE) None of the above

4. (2 pts.) The eigenvalues of A can be written as ____________________.____________ A B C D E

A) $\lambda_1 = 1, \lambda_2 = i$ B) $\lambda_1 = 1, \lambda_2 = -i$ C) $\lambda_1 = -1, \lambda_2 = i$ D) $\lambda_1 = -1, \lambda_2 = -i$ E) $\lambda_1 = 2, \lambda_2 = i$ AB) $\lambda_1 = 2, \lambda_2 = -i$ AC) $\lambda_1 = -2, \lambda_2 = i$ AD) $\lambda_1 = -2, \lambda_2 = -i$ AE) $\lambda_1 = 1, \lambda_2 = 2i$ BC) $\lambda_1 = 1, \lambda_2 = -2i$ BD) $\lambda_1 = -1, \lambda_2 = 2i$ BE) $\lambda_1 = -1, \lambda_2 = -2i$ CD) $\lambda_1 = 2, \lambda_2 = 2i$ CE) $\lambda_1 = 2, \lambda_2 = -2i$ DE) $\lambda_1 = -2, \lambda_2 = 2i$ ABC) $\lambda_1 = -4, \lambda_2 = -i$ ABCDE) None of the above

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answers.

Note that $\lambda_1 = 3$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & 0 \\ 1 & 2 \end{bmatrix}$

1. (4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with λ_1 is B = ___________________________________. _______A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$
AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$
BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$
ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[0,1]^T\}$
ACD) $\lambda = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix ABCDE) None of the above

2. (1pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, the eigenspace associated with λ_1 is often one dimensional. Hence conventions for selecting eigenvector(s) associated with λ_1 have been developed (by engineers). We say that the eigenvector(s) associated with λ_1

is (are) ___ ____. ________A B C D E

A) $[1,1]^T, [4,4]^T$ B) $[1,1]^T$ C) $\{[1,2]^T\}$ D) $[1,2]^T, [4,8]^T$ E) $[2,1]^T$
AB) $[1,3]^T$ AC) $[1,4]^T$ AD) $[4,1]^T$ AE) $[3,1]^T$ BC) $[1,-1]^T, [4,4]^T$
BD) $[1,-1]^T$ BE) $[1,-2]^T$ CD) $[1,-2]^T, [4,8]^T$ CE) $[2,1]^T$ DE) $[1,3]^T$
ABC) $[1,-4]^T$ ABD) $[4,-1]^T$ ABE) $[0,1]^T$
ACD) $\lambda_1 = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda_1 = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix ABCDE) None of the above .

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer. Consider the scalar equation $u'' - 5u' + 3u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du'/dt$). As was done in class (attendance is mandatory) convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y). You may think of x as the position and y as the velocity of a point particle). This system of two scalar equations can be written in the vector form $\vec{x}' = A\vec{x}$ $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. You ar

$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$; that is you are to find a, b, c, and d.

1. (1 pt.) a = __________________. ___________ A B C D E

2. (1 pt.) b = __________________. ___________ A B C D E

3. (1 pt.) c = ___________________ . ______________ A B C D E

4. (1 pt.) d = ______________________. ___________ A B C D E

Possible answers this page.
A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) 9
BD) −1 BE) −2 CD) −3 CE) −4 DE) −5 ABC) −6 ABD) −7 ABE) −8 ACD) −9
ACE) None of the above
Possible points on page 19 is 4. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let the 2x2 matrix A have the eigenvalue table

TABLE	
Eigenvalues	Eigenvectors
$r_1 =$	$\vec{\xi}_1 = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$
$r_2 = -2$	$\vec{\xi}_2 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

Let L:$\mathscr{A}$(**R**,**R**^{2})→$\mathscr{A}$(**R**,**R**^{2}) be defined by $L[\vec{x}] = \vec{x}\,' - A\vec{x}$

and let the null space of L be N_L

1. (1 pt). The dimension of N_L is ___________________________________. _________ A B C D E
   A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 ABCDE) None of the above.

2. (2 pts.) A basis for the null space of L is B =_________________________ . ____________ A B C D E

A) $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{t}, \begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{-t}, \begin{bmatrix}1\\2\end{bmatrix}e^{2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{t}, \begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{-t}, \begin{bmatrix}1\\2\end{bmatrix}e^{-2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{t}, \begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ D)

AB) $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{t}, \begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ $\left\{\begin{bmatrix}3\\2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ AI $\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\-1\end{bmatrix}e^{2t}\right\}$ AE)

BC) $\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ $\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$, $\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}, \begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ BE) ABC

3. (2 pts.) The general solution of $\vec{x}\,' = A\vec{x}$ $\vec{x}(t) =$ _______________________. ________ A B C D E

A) $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{t} + c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}1\\2\end{bmatrix}e^{2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}1\\2\end{bmatrix}e^{-2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ D)

AB) $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ $c_1\begin{bmatrix}3\\2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ $c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}$ AE

BC) $c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ $c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$) $c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t} + c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ BE)

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True or False. Let f and g be real valued functions of a real variable; that is, f:**R** → **R** and g:**R** → **R**. Circle True if the statement is true. Circle False if the statement is false.

1. (1 pt.) A)True B)False The function f is even if $f(-x) = f(x)\ \forall x \in \mathbf{R}$.

2. (1 pt.) A)True B)False The function f is odd if $f(-x) = -f(x)\ \forall x \in \mathbf{R}$.

3. (1 pt.) A)True B)False If f and g are both odd functions, then the product of f and g is an an even function.

4. (1 pt.) A)True B)False The function f is periodic of period T if $f(x+T) = f(T)\ \forall x \in \mathbf{R}$.

5. (1 pt.) A)True B)False If f is an odd function, then we know that $\int_{-L}^{L} f(x)\,dx = 2\int_{0}^{L} f(x)\,dx$.

For each of the following questions write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters

Classify the following function with regard to whether they are odd or even.

6. (1 pt.) $f(x) = x$ is ____. __________ A B C D E

7. (1 pt.) $f(x) = 0$ is ____. __________ A B C D E

8.(1 pt.) $f(x) = \sin(x)$ is ____. __________ A B C D E

9. (1 pt.) $f(x) = -|x|$ is ____. __________ A B C D E

10. (1 pt.) $f(x) = 3e^{-x}$ is ____. __________A B C D E

11. (1 pt.) $f(x) = 4$ is ____. __________ A B C D E

Possible answers for questions 78-83.
A) odd, but not even B) even, but not odd C) both odd and even
D) neither odd nor even ABCDE) none of the above
Possible points on page 21 is 11. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let f;**R**→**R** be in $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ = {f∈$\mathscr{F}$(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and $f(x)=\dfrac{f(x+)+f(x-)}{2}$ at points of discontinuity} so that its Fourier series exists.

1. (2 pts.) The formula for the general Fourier series for $f\in PC^1_s(\mathbf{R},\mathbf{R};\ell)$ given in our text is

f(x) = ____ . ________ A B C D E

A) $a_0+\sum_{n=1}^{N} a_n\cos(\frac{n\pi}{\ell}x)+b_n\sin(\frac{n\pi}{\ell}x)$ $\frac{a_0}{2}+\sum_{n=1}^{\infty} a_n\cos(\frac{n\pi}{\ell}x)+b_n\sin(\frac{n\pi}{\ell}x)$

C) $a_0+\sum_{n=0}^{\infty} a_n\cos(\frac{n\pi}{\ell}x)+b_n\sin(\frac{n\pi}{\ell}x)$ $\frac{a_0}{2}$ $\sum_{n=0}^{N} a_n\cos(n\pi x)+b_n\sin(n\pi x))$ +

E) $\frac{a_0}{2}$ $\sum_{n=1}^{\infty} a_n\cos(n\pi x)+b_n\cos(n\pi x)$ $\frac{a_0}{2}$ $\sum_{n=0}^{\infty} a_n\cos(\frac{n\pi}{\ell}x)+b_n\sin(\frac{n\pi}{\ell}x)$ +

ABCDE) None of the above

2. (2pts.) where for n = 0, 1, 2, ... we have a_n = ________________________________. _______ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ $\frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(n\pi x)dx$ C)

D) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\cos(x)dx$ Ǝ) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\cos(\frac{n\pi}{\ell}x)dx$ AB)

ABCDE) None of the above.

3. (2pts.) and for n = 1, 2, ... we have b_n = ________________________________. _______ A B C D E

A) $\frac{2}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ $\frac{2}{\ell}\int_{0}^{\ell} f(x)\sin(n\pi x)dx$ C) b_n =

D) $\frac{1}{\ell}\int_{0}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ $\frac{\ell}{2}\int_{-\ell}^{\ell} f(x)\sin(x)dx$)) $\frac{1}{\ell}\int_{-\ell}^{\ell} f(x)\sin(\frac{n\pi}{\ell}x)dx$ AB)

ABCDE) None of the above.

Possible points on page 22 is 6. TOTAL POINTS EARNED THIS PAGE ______

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Recall from the previous page that $PC^1_{\text{s}}(\mathbf{R},\mathbf{R};\ell)$ = {f∈$\mathscr{F}$(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and f(x) = (f(x+) + f(x−)/2 at points of discontinuity}. Now let $PC^1_{\text{ffs}}(\mathbf{R},\mathbf{R};\ell)$ the subspace of $PC^1_{\text{s}}(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance mandatory) that $PC^1_{\text{s}}(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{\text{ffs}}(\mathbf{R},\mathbf{R};\ell)$ d are inner product spaces wi $\int_{-\ell}^{\ell} f(x)g(x)\,dx$ t (f,g) = that $B_{PC^1_s(\mathbf{R},\mathbf{R},\ell)} = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ $PC^1_{\text{s}}(\mathbf{R},\mathbf{R};\ell)$ ın orthogonal Schaud $B_{PC^1_s(\mathbf{R},\mathbf{R},\ell)}$ is an orthogonal Hamel ǀ $PC^1_{\text{ffs}}(\mathbf{R},\mathbf{R};\ell)$.

4. (1 pt.) Using the notation given above, an orthogonal Hamel basis for $PC^1_{\text{ffs}}(\mathbf{R},\mathbf{R};3)$

is ____. ________A B C D E Hint: What is ℓ?

A) $\{\cos(\frac{n\pi}{2}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{2}) : n \in \mathbf{N}\}$ $\{1/2\} \cup \{\cos(\frac{n\pi}{2}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{2}) : n \in \mathbf{N}\}$

C) $\{1/2\} \cup \{\cos(\frac{n\pi}{3}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{3}) : n \in \mathbf{N}\}$ $\{1/2\} \cup \{\cos(\frac{n\pi}{4}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{4}) : n \in \mathbf{N}\}$

E) $\{(1/2)x\} \cup \{\cos(\frac{n\pi}{5}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{5}) : n \in \mathbf{N}\}$ ABCDE) None of the above.

5. (2 pts.) The Fourier series for the function f(x)∈ $PC^1_{\text{ffs}}(\mathbf{R},\mathbf{R};\pi)$ which has period 2π and is defined on the interval [−π,π) by f(x) = 3 + 3 cos(2x) + 3 sin(2x) is

f(x) =____. ___A B C D E
Hint: Think Hamel basis.

A) $3 + \sum_{k=0}^{\infty} \frac{2}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \sin(\frac{(2k+1)\pi}{2}x)$

C) $3 + \sum_{k=0}^{\infty} \frac{4}{(2k+1)\pi} \cos(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=1}^{\infty} \frac{1}{k\pi} \sin(k\pi x)$ D) 2 + E) 2+

AB)2+ 2cos(x) + 3sin(x) AC)2 + 3cos(x) + 3sin(x) AD) 3+ 3cos(x)+ 3sin(x)
AE) 2+ 2cos(2πx) + 2sin(2π x) BC) 2 + 2cos(2πx) + 3sin(2πx) BD)2 + 3cos(2x) + 3sin(2x)
BE)3 + 3cos(2x) + 3sin(2x) ABCDE) None of the above.

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Let $PC_s^1(\mathbf{R},\mathbf{R};\ell)$ $B_{PC_s^1(\mathbf{R},\mathbf{R};\ell)}$ and be as on the previous p $PC_s^1(\mathbf{R},\mathbf{R};\ell)$:t f(x) ∈ be domain is **R** which has period 4 and is defined on the interval (−1,1) by $f(x) = \begin{cases} 2 & -1 < x < 0 \\ 4 & 0 < x < 1 \end{cases}$. Using t formulas on the previous page, determine the Fourier series for the function f. Begin by sketching f for several periods. As discussed in class, indicate on your sketch the function to which the Fourier series converges.

6. (1 pt.) To apply the formulas given on the previous page we choose ℓ = __________.___ A B C D E
Next write down the formulas for a Fourier series and its coefficients using this value of ℓ and compute them. After computing the a_n's and the b_n's, note what they are for n odd and n even. Then answer the question below and those on the next two pages.

7. (3 pts.) We have a_0 = ________________________________. _________ A B C D E
Possible answers this page
A) 0 B)1 C)2 D) 3 E) 4 AB)5 AC) 6 AD) 7 AE) 8 BC) −1 BD) −2
ABCDE) None of the above
Possible points on page 24 is 4. TOTAL POINTS EARNED THIS PAGE _______

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Let f(x) be as on the previous page. Continue the computation of the Fourier Series coefficients.

8. (3 pts.) For a_n with n odd (n = 1, 3, 5, ...) so that if we let n = 2k+1 for k = 0,1,2,3,.. we have

a_{2k+1} =________________________________. __________ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] BCD)None of the above

9. (3 pts.) For a_n with n even (n= 2, 4, 6, ...) so that if we let n = 2k for k = 1,2,3,... we have

a_{2k} =___________________________. ________ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −4/(kπ) ABCDE)None of the above

10. (3 pts.) For b_n with n odd (n = 1, 3, 5, ...) so that if we let n = 2k+1 for k = 0,1,2,3,... we have

b_{2k+1} =__________________________. _________ A B C D E A) 0 B) 1/(2k+1) C) 2/(2k+1) D) 3/(2k+1) E) 1/[(2k+1)π] AB) 2/[(2k+1)π] AC) 3/[(2k+1)π] AD) 4/[(2k+1)π] AE) 8/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −12/[(2k+1)π] ABCDE)None of the above

11. (3 pts.)For b_n with n even (n = 2, 4, 6, ...) so that if we let n = 2k for k = 1,2,3,...

we have b_{2k} =___________________________. _________ A B C D E A) 0 B) 1/(2k) C) 1/k D) 3/(2k) E) 1/(2kπ) AB) 1/(kπ) AC) 3/(2kπ) AD) 2/(kπ) AE) 4/(kπ) BC) −1/(2k) BD) −1/k BE) −3/(2k) CD) −2/k CE) −1/(2kπ) DE) −1/(kπ) ABC) −3/(2kπ) ABD) −2/(2kπ) ABE) −6/(kπ) BCD)None of the above

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Let f(x) be as on the page before the previous page. Continue the computation of the Fourier Series of f.

12. (3 pts.) The Fourier series for f(x) may be written as

f(x) = ____. _________ A B C D E

A) $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

E) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AC) $1+\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

AE) $1+\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos\frac{(2k+1)\pi}{2}x)$

BD) $1+\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

CD) $2+\sum_{k=0}^{\infty}\frac{6}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$ -

DE) $2+\sum_{k=1}^{\infty}\frac{4}{(2k+1)\pi}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$ -

ABD) $3+\sum_{k=1}^{\infty}\frac{4}{(2k+1)\pi}\sin((2k+1)\pi x)$ $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}\cos(\frac{(2k+1)\pi}{2}x)$

ABCDE) None of the above

Possible points on page 26 is 3. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer.

Consider the Partial Differential Equation (PDE): $u_{tt} = t\,u_{xx}$

1. (4 pts.) Using the method of separation of variables with separation constant λ (or −λ if appropriate), one of the following sets of two Ordinary Differential Equations (ODE's) can be obtained from this PDE. Recall that the process does not yield a unique set of ODE's. (Note that we are looking for product solutions in the null space of the linear operator $L[u] = x\,u_{tt} - t\,u_{xx}$). Following the advice given in class as to how to choose the separation constant (attendance is mandatory) we may obtain the set of

ODE's ____. ________ A B C D E

Possible answers this page

A) $X'' + \lambda X = 0,\quad T'' + \lambda T = 0$
B) $X'' + \lambda X = 0,\quad T'' - \lambda T = 0$
C) $X'' + \lambda X = 0,\quad T'' + \lambda t T = 0$
D) $X'' + \lambda x X = 0,\quad T'' + \lambda T = 0$
E) $X'' + \lambda xX = 0,\quad T'' + \lambda t T = 0$
AB) $X'' + \lambda t X = 0,\quad T'' + \lambda x T = 0$
AC) $x X'' + \lambda X = 0,\quad t T'' + \lambda T = 0$
AD) $t X'' + \lambda X = 0,\quad xT'' + \lambda x T = 0$
AE) $x X'' + \lambda X = 0,\quad T'' + \lambda T = 0$
BC) $X'' + \lambda X = 0,\quad t T'' + \lambda T = 0$
BD) $X'' + \lambda x X = 0,\quad T'' + \lambda T = 0$
BE) $X'' + \lambda X = 0,\quad T'' + \lambda t T = 0$
CD) Separation of variables does not work on this PDE.
CE) Separation of variables works on this PDE, but none of the above is correct.
ABCDE) None of the above

Possible points on page 27 is 4. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ______________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Read pages 36-39. Let all of the problems, function spaces, and basis sets be as on pags 36-39.

1. (1pt.) The set (which may be thought of as a subset of $L^2([0,\ell],\mathbf{R})$) that we (attendance is mandatory) consider to be the state space for heat conduction in a rod

is____.________A B C D E

2. (1pt.) A Schauder basis for the state space for heat conduction in a rod may be taken to

be____.__________A B C D E

3. (1pt.) A Schauder basis for $N_{L_{Bfs(\alpha^2,\ell)}}$, the null s$L_{Hsz(\ell,\alpha^2)}$,

is____.__________A B C D E

4. (2 pts.) The "general" or formal solution of $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$,$L_{Hsz(\ell,\alpha^2)}$ [u] = 0; ℓ, α²) which is jus of the functions in the null space of $L_{Hsz(\ell,\alpha^2)}$ (which we $N_{L_{Bfssz(\ell,\alpha^2)}}$) is given

by u(x,t) =____. ______________ A B C D E

.

Possible answers this page

A) $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{fss}(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{fss}([0,\ell],\mathbf{R})$ $PC^1_{ffss}([0,\ell],\mathbf{R})$ D)

AB) $B_{fs(\ell)}=\{1/2\}\cup\{\cos(\frac{n\pi}{\ell}):n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}):n\in\mathbf{N}\}$ $B_{fss(\ell)}=\{\sin(\frac{k\pi}{\ell}):k\in\mathbf{N}\}$ AC)

AE) $\bar{D}(\ell)$ = [0,ℓ]×[0,∞) BC) $\mathscr{A}(\bar{D}(\ell)$,R) B $\bar{D}$ $\mathscr{A}_{fssz}$($L_{Bfssz(\ell,\alpha^2)}$) $L_{Bfss(\ell,\alpha^2)}$ CD)

CE) $N_{L_{Bfs(\alpha^2,\ell)}}$ $N_{L_{Bffs(\alpha^2,\ell)}}$) $B_{N_{L_{Bfss(\ell,\alpha^2)}}}=\{e^{-(\alpha n\pi/\ell)^2 t}\sin(n\pi/\ell):n\in\mathbf{N}\}$ $\sum_{n=1}^{\infty}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\sin(\frac{\ell}{n\pi}x)$

ABE) $\sum_{n=1}^{N}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ $\sum_{n=1}^{\infty}c_n e^{-\alpha^2n^2\pi^2\ell^2 t}\sin(\frac{n\pi}{\ell}x)$ $\sum_{n=1}^{\infty}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$

ADE) $\sum_{n=1}^{\infty}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\sin(n\pi\ell x)$ $\sum_{n=1}^{\infty}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\sin(\frac{n\pi}{\ell}x)$ $\sum_{n=1}^{\infty}c_n e^{-\frac{\alpha^2n^2\pi^2}{\ell^2}t}\cos(\frac{n\pi}{\ell}x)$ B

BDE) $\sum_{n=1}^{\infty}c_n\cos(\frac{n\pi}{\ell}x)$ ABCDE) None of the above

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ______________________________(________________) ID No. __________________

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Reread pages 36-39. Try to understand the notation.

5. (2 pts.) The "general" or formal solution of PDE $u_t = 4\, u_{xx}$ $0 < x < 2$, $t > 0$

BC $u(0,t) = 0$, $u(2,t) = 0$, $t > 0$

(we denote this problem by $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\mathbf{D}(2)$ $,\mathbf{L}_{Bfssz(2,4)}$ $[u] = 0; 2, 4)$

is given by u(x,t) =____. _________ A B C D E

6. (4 pts.) The solution of

	PDE	$u_t = 4\, u_{xx}$	$0 < x < 2$, $t > 0$
BVP for a PDE	BC	$u(0,t) = 0$, $u(2,t) = 0$,	$t > 0$
	IC	$u(x,0) = 6 \sin(6\pi x)$	$0 < x < 2$

(we denote this problem by $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\mathbf{D}(2)$ $,\mathbf{L}_{Bfssz(2,4)}$ $[u] = 0, u_0(x) = 6 \sin(6\pi); 2, 4)$

is given by u(x,t) =____. _________A B C D E

Posible answers this page

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{2}{n\pi}x)$ $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} c_n e^{-n^2\pi^2 t} \sin(\frac{n\pi}{2}x)$ C)

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(2n\pi x)$ $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ AB)

AC) $\sum_{n=1}^{\infty} c_n e^{-n^2\pi^2 t} \sin(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} a_n \sin(\frac{n\pi}{2}x) + b_n \cos(\frac{n\pi}{2}x)$

AE) $\sum_{n=1}^{\infty} 6 e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ BC) $6\, e^{-\frac{\pi^2}{4}t} \sin(\pi x)$ $6 e^{-9\pi^2 t} \sin(6\pi x)$ BE) $6\, e^{-36\pi^2 t} \sin(6\pi x)$

CD) $\sum_{n=1}^{\infty} 6\, e^{-36\pi^2 t} \sin(6\pi x)$ $6\, e^{-12\pi^2 t} \sin(6\pi x)$ E) $6\, e^{-12\pi^2 t} \sin(3\pi x)$ $6\, e^{-576\pi^2 t} \sin(6\pi x)$

ABCDE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME _______________________________(_________________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer. Reread pages 36-39.

Recall that $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$ $,\mathbf{IL}_{Hzz(\ell,\alpha^2)}$ $[u] = 0$ $u(x,0) = u_0(x)$; $\ell, \alpha^2)$ is the problem defined by

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell,\ \ t > 0$
BC	$u(0,t) = 0, \quad u(\ell,t) = 0,$	$t > 0$
IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

1\. (2 pts.) Recall that the formula for the solution of

$\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$ $,\mathbf{IL}_{Hzz(\ell,\alpha^2)}$ $[u] = 0$ $u(x,0) = u_0(x)$; $\ell, \alpha^2)$ is given by

u(x,t) =____. ________ A B C D E

2\. (2 pts.) where the formula for c_n is c_n =__________________________________. _______ A B C D E

Posible answerrs this page.

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{\ell}{n\pi} x)$ $\qquad \sum_{n=1}^{N} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$ $\qquad \sum_{n=1}^{\infty} c_n e^{-\alpha^2 n^2 \pi^2 \ell^2 t} \sin(\frac{n\pi}{\ell} x)$ C)

D) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$ $\qquad \sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(n\pi\ell x)$ $\qquad \sum_{n=1}^{\infty} c_n e^{\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \sin(\frac{n\pi}{\ell} x)$ AB)

AC) $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2} t} \cos(\frac{n\pi}{\ell} x)$ $\qquad \sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{\ell} x)$ AD) $\frac{2}{\ell}\int_0^{\ell} u_0(x) \sin\left(\frac{\ell}{n\pi} x\right) dx$ $\qquad \frac{1}{\ell}\int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$

BD) $\frac{2}{\ell}\int_{-\ell}^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$ $\qquad \frac{2}{\ell}\int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$ $\qquad \frac{2}{\ell}\int_0^{\ell} u_0(x) \sin(n\pi\ell x) dx$ $\qquad \int_0^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$

DE) $\frac{1}{\ell}\int_{-\ell}^{\ell} u_0(x) \sin\left(\frac{n\pi}{\ell} x\right) dx$ $\qquad \frac{2}{\ell}\int_0^{\ell} u_0(x) \cos\left(\frac{n\pi}{\ell} x\right) dx$ ABCDE) None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ____________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Recall that $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(2)$ $L_{Bfssz(2,1)}$ $[u] = 0$, $u(x,0) = 4$;2, 1) is the problem defined by

PDE	$u_t = u_{xx}$		$0 < x < 2$, $t > 0$
BC	$u(0,t) = 0$,	$u(2,t) = 0$,	$t > 0$
IC	$u(x,0) = 4$		$0 < x < 2$

3. (2 pts.) The formula for the solution of $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(2)$ $L_{Bfssz(2,1)}$ $[u] = 0$, $u(x,0) = 4$;2, 1) is given

by $u(x,t)$ =____. _______ A B C D E

4. (2 pts.) where the formula for c_n is c_n =___________________ ___________________. ________ A B C D E

Posible answers this page.

A) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{2}{n\pi}x)$ $\sum_{n=1}^{N} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} c_n e^{-4n^2\pi^2 t} \sin(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$

E) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \sin(2n\pi x)$ $\sum_{n=1}^{\infty} c_n e^{\frac{n^2\pi^2}{4}t} \sin(\frac{n\pi}{2}x)$3) $\sum_{n=1}^{\infty} c_n e^{-\frac{n^2\pi^2}{4}t} \cos(\frac{n\pi}{2}x)$ $\sum_{n=1}^{\infty} c_n \sin(\frac{n\pi}{2}x)$

AE) $2\int_0^2 \sin\left(\frac{2}{n\pi}x\right)dx$ $\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$) $2\int_{-2}^{2} \sin\left(\frac{n\pi}{2}x\right)dx$ BI $\frac{5}{2}\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ $3\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$)

CE) $4\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ $5\int_0^2 \sin\left(\frac{n\pi}{2}x\right)dx$ $2\int_0^2 \cos\left(\frac{n\pi}{2}x\right)dx$ ABC) ABCDE) None of t

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ______________________________(________________) ID No. __________________

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Reread pages 36-39. Let $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\mathrm{D}(2)$ $\mathbf{L}_{Bfssz(2,1)}$ [u] = 0, u(x,0) = 4 ;2, 1) be as on the previous pag

5. (2pts.) Computing c_n using the formula on the previous page, for n odd (n = 1, 3, 5, ...) so that if n = 2k+1

where k = 0,1,2,3,... then c_{2k+1} =______________________. _______ A B C D E A) 0 B) 1/[(2k+1)π] C) 2/[(2k+1)π] D) 3/[(2k+1)π] E) 4/[(2k+1)π] AB) 12/[(2k+1)π] AC) 16/[(2k+1)π] AD) 32/[(2k+1)π] AE) 64/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

6. (2 pts.)For c_n with n even (n = 2, 4, 6, ...) So that if n =2k where k = 1,2,3,...then

c_{2k} =_______________________. ________ A B C D E A) 0 B) 1/[(2k+1)π] C) 2/[(2k+1)π] D) 3/[(2k+1)π] E) 4/[(2k+1)π] AB) 12/[(2k+1)π] AC) 16/[(2k+1)π] AD) 32/[(2k+1)π] AE) 64/[(2k+1)π] BC) −1/(2k+1) BD) −2/(2k+1) BE) −3/(2k+1) CD) −4/(2k+1) CE) −1/[(2k+1)π] DE) −2/[(2k+1)π] ABC) −3/[(2k+1)π] ABD) −4/[(2k+1)π] ABE) −8/[(2k+1)π] ABCDE)None of the above

7. (2 pts.) Hence the solution of $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\mathrm{D}(2)$ $\mathbf{L}_{Bfssz(2,1)}$ [u] = 0, u(x,0) = 4 ;2, 1) may be written

as u(x,t) = ____. __**E**____ A B C D **E**

A) $\sum_{k=0}^{\infty}\frac{1}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{2}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$

C) $\sum_{k=0}^{\infty}\frac{4}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$)

E) $\sum_{k=0}^{\infty}\frac{16}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{2}x)$ $\sum_{k=0}^{\infty}\frac{32}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{4}x)$

AC) $\sum_{k=0}^{\infty}\frac{64}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\sin(\frac{(2k+1)\pi}{4}x)$ $\sum_{k=0}^{\infty}\frac{8}{(2k+1)\pi}e^{-\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k+1)\pi}{2}x)$

AE) $\sum_{n=1}^{\infty}\frac{8}{(2k)\pi}e^{\frac{(2k+1)^2\pi^2}{4}t}\cos(\frac{(2k)\pi}{2}x)$ $\sum_{k=1}^{\infty}\frac{4}{k\pi}e^{-\frac{k^2\pi^2}{4}t}\sin(\frac{k\pi}{2}x)$ I $\sum_{k=1}^{\infty}\frac{2}{k\pi}e^{-\frac{k^2\pi^2}{2}t}\sin(\frac{k\pi}{2}x)$

ABCDE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ___________________________(________________) ID No. __________________

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer

1. (3 pts.) Suspose both ends of a rod are held at different temperatures. (Recall that we assume that the lateral sides are insulated so that the temperature does not vary over a cross section), A good mathematical model of this physical heat conduction problem is given by:

	PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell,\quad t > 0$
BVP for a PDE	BC	$u(0,t) = T_1,\quad u(\ell,t) = T_2,$	$t > 0$
	IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

where $u_0(x)$ is the initial temperature distribution in the rod.

```
          ////////////////////////////////////////////
      -→ O___________________________O ←-
    /     ////////////////////////////////////////////     \
u = T1                                            u = T2
          |-------------------------------------|
        x = 0                                  x = ℓ
```

The general solution of the homogenous problem associated with this nonhomogeneous problem is $\sum_{n=1}^{\infty} c_n e^{-\frac{\alpha^2 n^2 \pi^2}{\ell^2}t} \sin(\frac{n\pi}{\ell}x)$. To obtain a particular solution of the nonhomogeneous problem we steady state solution which we compute to be

$u_p(x,t) = u_{ss}(x) =$ ____ .__________ A B C D E

Possible answers this page.

A) $T_1 + \frac{(T_1 - T_2)x}{\ell}$ $\quad T_2 + \frac{(T_2 - T_1)x}{\ell}$ B) $\quad T_1 + \frac{(T_2 - T_1)\ell}{x}$ $\quad T_1 + \frac{(T_2 - T_1)x}{\ell}$ D)

E) $T_1 + \frac{T_2 x}{\ell}$ AB) $T_1 + \frac{(T_2 - T_1)4}{\ell}$ $\quad T_1 + \frac{(T_2 - T_1)x}{4}$ AD) $\ell T_1 + \frac{(T_2 - T_1)x}{\ell}$

AE) $T_1 + \frac{(T_2 - T_1)}{\ell}$ ABCDE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) SS No. ___________________
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TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	$s > \lvert a \rvert$
cosh (at)	$\frac{s}{s^2 - a^2}$	$s > \lvert a \rvert$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
u(t)	$\frac{1}{s}$	$s > 0$
u(t - c)	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	$F(s - c)$	
$f(ct)$ $c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	

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PARTIAL TABLE OF ANTIDERIVATIVES

1. $\int x[\sin(ax)]dx = \frac{1}{a^2}\sin(ax) - \frac{x}{a}\cos(ax) + c$

2. $\int x[\cos(ax)]dx = \frac{1}{a^2}\cos(ax) - \frac{x}{a}\sin(ax) + c$

3. $\int x^2[\sin(ax)]dx = \frac{2x}{a^2}\sin(ax) - \frac{a^2x^2 - 2}{a^3}\cos(ax) + c$

4. $\int x^2[\cos(ax)]dx = \frac{2x}{a^2}\cos(ax) - \frac{a^2x^2 - 2}{a^3}\sin(ax) + c$

5. $\int \sin^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

6. $\int \cos^2(ax)dx = \frac{x}{2} - \frac{1}{4a}\sin(2ax) + c$

7. $\int [\sin(ax)][\cos(ax)]dx = \frac{1}{2a}\sin^2(ax) + c$

8. $\int [\sin(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} - \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

9. $\int [\cos(ax)][\cos(bx)]dx = \frac{\sin[(a-b)x]}{2(a-b)} + \frac{\sin[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

10. $\int [\sin(ax)][\cos(bx)]dx = -\frac{\cos[(a-b)x]}{2(a-b)} - \frac{\cos[(a+b)x]}{2(a+b)} + c \quad a^2 \neq b^2$

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SETS AND SPACES FOR FOURIER SERIES

1. $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ = {f∈$\mathscr{F}$(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and f(x) =(f(x+) + f(x−)/2 at points of discontinuity}. This is a space where the Fourier Series converges. At the points of discontinuity, the function is defined to be half way between the one-sided limits.
2. $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ is the s $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite. Recall from class discussions (attendance is mandatory) that $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ d are inner product spaces with inner product (f,g) = $\int_{-\ell}^{\ell} f(x)g(x)\,dx$. However, they are not Hilbert spaces. Why?
3. $B_{fs(\ell)} = \{1/2\} \cup \{\cos(\frac{n\pi}{\ell}) : n \in \mathbf{N}\} \cup \{\sin(\frac{n\pi}{\ell}) : n \in \mathbf{N}\}$ $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ ın orthogonal Schauder b and an orthogonal Hamel basis of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$.
4. $PC^1_{so}(\mathbf{R},\mathbf{R};\ell)$ is the sı $PC^1_s(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions. Hence a Fourier Serie $PC^1_{so}(\mathbf{R},\mathbf{R};\ell)$ contains only sine terms and is hence called a Fourier Sine Series.
5. $PC^1_{so}([0,\ell],\mathbf{R})$ is the set of ı $PC^1_{so}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to [0,ℓ]. This space is also called the space of Fourier Sine Series. Since it's domain is only [0,ℓ], it can be used as the state space for the heat conduction problem. Note that the dimension of the state space appears to be uncountably infinite as there are an uncountablly infinite number of temperatures on the interval [0,ℓ]. However, all of these temperatures can be expressed as a Fourier Sine Series. Hence the state space is actually only countably infinite.
6. $PC^1_{fso}([0,\ell],\mathbf{R};\ell)$ is th $PC^1_{so}([0,\ell],\mathbf{R};\ell)$ for which the Fourier sine series is finite.
7. $B_{fso(\ell)} = \{\sin(\frac{k\pi}{\ell}) : k \in \mathbf{N}\}$ is an orthogoı $PC^1_{so}([0,\ell],\mathbf{R};\ell)$ basis for and an orth Hamel basis of $PC^1_{fso}([0,\ell],\mathbf{R};\ell)$.

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SETS AND SPACES FOR THE HEAT CONDUCTION PROBLEM

8. Recall that we have established $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ as a function space where we can calculate Fourier Series ar let $PC^1_{ffs}(\mathbf{R},\mathbf{R};\ell)$ be the $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$` for which the Fourier $PC^1_{fss}(\mathbf{R},\mathbf{R};\ell)$ nite. Also, let the subspace of $PC^1_{fs}(\mathbf{R},\mathbf{R};\ell)$ containing only od $PC^1_{fss}([0,\ell],\mathbf{R})$ $PC^1_{fss}(\mathbf{R},\mathbf{R};\ell)$ functions in their domains restricted to $[0,\ell]$, and $PC^1_{ffss}([0,\ell],\mathbf{R})$ be the $PC^1_{fss}([0,\ell],\mathbf{R})$ for which the F Series is finite. Clearly $B_{fss(\ell)} = \{\sin(n\pi/\ell): n \in \mathbf{N}\}$ $PC^1_{ffss}([0,\ell],\mathbf{R})$ nel basis of $PC^1_{fss}([0,\ell],\mathbf{R})$.

We will see that we can view $PC^1_{fss}([0,\ell],\mathbf{R})$ as the state space for the problem of heat conduction in a rod. Recall that the heat equation is the pde, $u_t = \alpha^2 u_{xx}$. To formulate the heat conduction in a rod problem as a **linear mapping problem** we first let $D(\ell) = (0,\ell)\times(0,\infty)$ and $\bar{D}(\ell)$ $= [0,\ell]\times[0,\infty)$. We see that $D(\ell)$ is the open set where we look for solutions of the heat equation and $\bar{D}(\ell)$ contains $D(\ell)$ and its boundary so that $\bar{D}(\ell)$ Hence $\bar{D}(\ell)$ is a closed set. Next we let $\mathscr{A}_{fss}(\bar{D}(\ell)$,**R**) be the set of functio $\bar{D}(\ell)$ in $\mathscr{F}($,**R**) whose restrictio $D(\ell)$ is analytic, whose restriction to $\bar{D}(\ell)$ is continuous, and whose restriction to $[0,\ell]\times\{0\}$ $PC^1_{fss}([0,\ell],\mathbf{R})$ with the additional condition that the function be continuous at all points in $[0,\ell]\times\{0\}$ where its restriction to $[0,\ell]\times\{0\}$ is continuous. Now let the operator $L_{Bfss(\ell,\alpha^2)}$ be d $L_{Bfss(\ell,\alpha^2)}$ $\bar{D}(\ell)$ $:\mathscr{A}_{fss}($,**R**) where $L_{Bfss(\ell,\alpha^2)}$ $[u] = u_t - \alpha^2 u_{xx}$. T $\bar{D}$ s $\mathscr{A}_{fss}($,**R**) is the Σ set where we look for solutions to the pde in $D(\ell)$. These functions are "nice" at the boundary of $D(\ell)$. Now let $N_{L_{Bfssz(\ell,\alpha^2)}}$ $= \{\bar{D}(\ell)$ t$)\in\mathscr{A}_f L_{Bfssz(\ell,\alpha^2)}$,**R**): $[u] = 0\}$ be the $L_{Bfss(\ell,\alpha^2)}$ e of the operator $u(x,t) \in N_{L_{Bfss(\ell,\alpha^2)}}$ then it satisfies the the pde $u_t - \alpha^2 u_{xx} = 0$ and is "nice" on the boundary of $D(\ell)$.

Now let $\mathscr{A}_{fssz}(\bar{D}$,**R**) = {u(x,t)∈ $\mathscr{A}_{\bar{D}}($,**R**)): $u(0,t) = 0$ and $u(\ell,t) = 0$ for $t > 0\}$. Then $\bar{D}(\ell)_z($,**R**) is the Σ set where we look for solutions of the pde in $D(\ell)$ that also satisfy the zero boundary conditions. Since they are in $\mathscr{A}_{fss}(\bar{D}(\ell)$,**R**), these functions are all "nice" on the boundary of $D(\ell)$, particularly on $[0,\ell]\times\{0\}$ where $t = 0$. Now let the operator $L_{Bfssz(\ell,\alpha^2)}$ be d $L_{Bfssz(\ell,\alpha^2)}$ $\bar{D}(\ell)$ $:\mathscr{A}_{fssz}($,**R**) → $L_{Bfss(\ell,\alpha^2)}$ where, as let $L_{Bfssz(\ell,\alpha^2)}$ $[u] = u_t - \alpha^2 u_{xx}$. A $L_{Bfssz(\ell,\alpha^2)}$ map $\bar{D}(\ell)$ unctions in $\mathscr{A}_{fssz}($,**R**) to the same fu operator $L_{Bfss(\ell,\alpha^2)}$ $\bar{D}(\ell)$ $\mathscr{A}_{fss}($ $\bar{D}(\ell)$,**R**) → $\mathscr{A}($,**R**) defined by this same formula , the BC's are now incorpo the domain $\mathscr{A}_{fssz}($ $\bar{D}(\ell)$,**R**) $L_{Bfssz(\ell,\alpha^2)}$; that is, functions in t $L_{Bfssz(\ell,\alpha^2)}$ ain of also satisfy boundary conditions. Recall that the definition of an operator (like that of a function) includes its domain and codomain and not just the formula that tells you where the element is mapped. Now let $N_{L_{Bfssz(\ell,\alpha^2)}}$ $= \{\bar{D}(\ell)$ t$)\in\mathscr{A}_f L_{Bfssz(\ell,\alpha^2)}$,**R**): $[u] = 0\}$ be the $L_{Bfssz(\ell,\alpha^2)}$ e of the operator $u(x,t) \in N_{L_{Bfssz(\ell,\alpha^2)}}$ then it satisfies the the zero boundary conditions as well as the pde $u_t = \alpha^2 u_{xx}$ and is "nic on the boundary of $D(\ell)$. Thus if we let $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$,**I** $L_{Bfssz(\ell,\alpha^2)}$ $[u] = 0;\ell, \alpha^2)$ be the problem defi

PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell,\ \ t > 0$
BC $u(0,t) = 0,\ \ u(\ell,t) = 0,$ $t > 0$

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then the set of solutions for $\text{Prob}_{HC}(\mathscr{A}_{fssz}($ $\bar{D}(\ell)$ $L_{Hszz(\ell,\alpha^2)}$ $[u] = 0; \ell,\alpha^2)$) is just the null space of the ope $L_{Hszz(\ell,\alpha^2)}$. Again, the null $L_{Hszz(\ell,\alpha^2)}$ f is just tl $\bar{D}$ functions in $\mathscr{A}_{fssz}($,**R**) that satisfy the the pde (and are nice on the boundary). Also, because of the definition of $\mathscr{A}_{fss}(\bar{D}$,**R**), if u(x, $N_{L_{Bfssz(\ell,\alpha^2)}}$ then its restriction to $[0,\ell]\times\{0\}$ is in $PC^1_{fss}([0,\ell],\mathbf{R})$. This will allow us to satisfy the initial condition for th Heat Conduction Problem which we denote by $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}$,**R** $L_{Bfssz(\ell,\alpha^2)}[u] = 0$, $u(x,0) = u_0(x)$;

PDE	$u_t = \alpha^2 u_{xx}$	$0 < x < \ell, \quad t > 0$
BC	$u(0,t) = 0, \quad u(\ell,t) = 0,$	$t > 0$
IC	$u(x,0) = u_0(x)$	$0 < x < \ell$

where $u_0(x) \in PC^1_{fss}([0,\ell],\mathbf{R})$.

Note that a Fourier Sine Series in $PC^1_{fss}([0,\ell],\mathbf{R})$ gives the state of the rod (i.e., the temperature at each point on the rod in the interval $[0,\ell]$). Thus the temperature u(x,t) is a function of two variables, time and the location on the rod. In your engineering courses, you may study heat conduction in two or three physical dimensions as well as time. Again, for the rod in one physical dimension, the dimension of the state space is countably infinite as although there are an uncountalbly infinite number of temperatures in the interval $[0,\ell]$, for our formulation of the heat conductiuon problem, every temperatute can be represented by a fourier sine series so that the dimension of the state space is only countably infinite.. Recall that a vector space is an algebraic, not a geometric construct. Often, the term dimension is replaced by the term "degrees of freedom". Again, although there are a uncountably infinite number of temperatures on the rod, with our formulation, we only allow a countably infinite number of temperatures and hence only a countably infinite number of degrees of freedom.

9. Now recall the definition of $\text{Prob}_{HC}(\mathscr{A}_{fssz}(\bar{D}$,**R** $L_{Bfssz(\ell,\alpha^2)}[u] = 0$, $u(x,0) = u_0(x)$; ℓ,α^2) as given abo claim that if $u_0(x) \in PC^1_{fss}([0,\ell],\mathbf{R};\ell)$, then the solution of $\text{Prob}_{HC}(\mathscr{A}_{fss}(\bar{D}$,**R**) $L_{Hszz(\ell,\alpha^2)}$ u] = 0, $u(x,0) = u_0(x)$ $N_{L_{Bfssz(\ell,\alpha^2)}}$. It is the function i $N_{L_{Bfssz(\ell,\alpha^2)}}$ that has $u_0(x)$ as its initial condition.

. Next we let $\mathscr{A}_{ffss}(\bar{D}(\ell)$,**R**) be the set of functions i $\bar{D}(\ell)$,**R**) whose restriction to $D(\ell)$ is analytic, whose restriction to $\bar{D}(\ell)$ is continuous, and whose restriction to $[0,\ell]\times\{0\}$ is no $PC^1_{ffss}([0,\ell],\mathbf{R})$ with the addi condition that the function be continuous at all points in $[0,\ell]\times\{0\}$ where its restriction to $[0,\ell]\times\{0\}$ is continuous. We now claim that if $u_0(x) \in PC^1_{ffss}([0,\ell],\mathbf{R};\ell)$, then the solution of $\text{Prob}_{HC}(\mathscr{A}_{ffss}(\bar{D}(\ell)$, $L_{Hszz(\ell,\alpha^2)}$ $[u] = 0$, $u(x,0) = u_0(x)$;ℓ, α^2) is in

$N_{L_{Bffs(\ell,\alpha^2)}}$ $=\{u$ $\bar{D}(\ell) \in \mathscr{A}_{ffss}$ $L_{Hszz(\ell,\alpha^2)}$,**R**): $B_{N_{L_{Bffs(\ell,\alpha^2)}}} = \{e^{-(\alpha n\pi/\ell)^2 t}\sin(\frac{n\pi}{\ell}x) : n \in \mathbf{N}\}$

for $N_{L_{Bffs(\ell,\alpha^2)}}$ and a Schauder b $N_{L_{Bffs(\ell,\alpha^2)}}$.

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SUMMARY OF NOTATION FOR FOURIER SERIES AND HEAT CONDUCTION PROBLEM

Notation	Definition
$PC^1_5(\mathbf{R},\mathbf{R};\ell)$	{f∈𝒜(**R**,**R**): f is periodic of period 2ℓ, f and f′ are piecewise continuous on [−ℓ,ℓ], and f(x) =(f(x+) + f(x−)/2 at points of discontinuity}
$PC^1_{5ft}(\mathbf{R},\mathbf{R};\ell)$	The s $PC^1_5(\mathbf{R},\mathbf{R};\ell)$ for which the Fourier series is finite.
$PC^1_{5o}(\mathbf{R},\mathbf{R};\ell)$	The s $PC^1_5(\mathbf{R},\mathbf{R};\ell)$ containing only odd functions
$PC^1_{5o}([0,\ell],\mathbf{R})$	The set of $PC^1_{5o}(\mathbf{R},\mathbf{R};\ell)$ with their domains restricted to [0,ℓ].
$PC^1_{fso}([0,\ell],\mathbf{R})$	The $PC^1_{5o}([0,\ell],\mathbf{R};\ell)$ for which the Fourier sine series is finite.
$(f,g) = \int_{-\ell}^{\ell} f(x)g(x)dx$	In $PC^1_5(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{5ft}(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{5o}(\mathbf{R},\mathbf{R};\ell)$ $PC^1_{5o}([0,\ell],\mathbf{R})$ $PC^1_{fso}([0,\ell],\mathbf{R})$
$B_{5(\ell)} = \{1/2\}\cup\{\cos(\frac{n\pi}{\ell}): n\in\mathbf{N}\}\cup\{\sin(\frac{n\pi}{\ell}): n\in\mathbf{N}\}$	$PC^1_5(\mathbf{R},\mathbf{R};\ell)$ .n orthogonal Schauder bas and an orthogonal Hamel basis for $PC^1_{5ft}(\mathbf{R},\mathbf{R};\ell)$.
$B_{fso(\ell)} = \{\sin(\frac{k\pi}{\ell}): k\in\mathbf{N}\}$	An orthogo $PC^1_{5o}([0,\ell],\mathbf{R};\ell)$ basis for and an ortl Hamel basis of $PC^1_{fso}([0,\ell],\mathbf{R};\ell)$.
D(ℓ) = (0,ℓ)×(0,∞)	
$\bar{D}(\ell)$ = [0,ℓ]×[0,∞).	
$\mathscr{A}_{fss}(\bar{D}(\ell),\mathbf{R})$	The set of functions in $\bar{D}(\ell)$(,**R**) whose restriction to D(ℓ) is analytic, whose restriction t [0,ℓ]×(0,∞) is continuous, and whose restriction to [0,ℓ]×{0} is in $PC^1_{5o}([0,\ell],\mathbf{R})$ with the additional condition that the function be continuous at all points in [0,ℓ]×{0} where its restriction to [0,ℓ]×{0} is continuous.
$\mathscr{A}_{fssz}(\bar{D}$,**R**)	= {u(x,t)∈𝒜 $\bar{D}$(,**R**)): u(0,t) = 0 and u(ℓ,t) = 0 for t > 0}
$Prob_{HC}(\mathscr{A}_{fssz}(\bar{D}$,**R** $\mathbf{L}_{Bfssz(\ell,\alpha^2)}[u] = 0$, $u(x,0) = u_0(x)$; ℓ,α²)	Heat conduction problem defined by BVP: PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$; BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$; IC $u(x,0) = u_0(x)$ $0 < x < \ell$
$\mathbf{L}_{Bfssz(\ell,\alpha^2)}$	$\bar{D}(\ell)$:𝒜$_{fssz}$(,**R**) → 𝒜(D(ℓ),**R** $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$ned by $[u] = u_t - \alpha^2 u_{xx}$.
$\mathbf{L}_{Bfss(\ell,\alpha^2)}$	$\bar{D}(\ell)$ 𝒜$_{fss}$(D(ℓ) ,**R**) → 𝒜(,$\mathbf{L}_{Bfss(\ell,\alpha^2)}$ fined by $[u] = u_t - \alpha^2 u_{xx}$,
$Prob_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$,**R** $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$[u] = 0; α²,ℓ)	Heat conduction problem defined by PDE $u_t = \alpha^2 u_{xx}$ $0 < x < \ell$, $t > 0$; BC $u(0,t) = 0$, $u(\ell,t) = 0$, $t > 0$.
$N_{L_{Bfs(\alpha^2,\ell)}}$	={u $\bar{D}(\ell)$ ∈ 𝒜$_{fss}$ $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$,**R**): [u] = 0} The n $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$ of the operator. Also the “general”solution of $Prob_{HC}(\mathscr{A}_{fssz}(\bar{D}(\ell)$ $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$ [u] =)
$N_{L_{Bffs(\alpha^2,\ell)}}$	={u $\bar{D}(\ell)$ ∈ 𝒜$_{ffss}$ $\mathbf{L}_{Bfssz(\ell,\alpha^2)}$,**R**): [u] = 0}
$B_{N_{L_{Bfs(\ell,\alpha^2)}}} = \{e^{-(\alpha n\pi/\ell)^2 t}\sin(n\pi/\ell): n\in\mathbf{N}\}$	$N_{L_{Bffs(\alpha^2,\ell)}}$ is a Hamel basis for $N_{L_{Bfs(\alpha^2,\ell)}}$ and a Schauder