$\qquad$ EXAM DATE Friday, September 12, 2008 Scores
I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

## SIGNATURE

DATE
INSTRUCTIONS: Besides this cover page, there are 13 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! NO SCRATCH PAPER! Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-12 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

## REQUEST FOR REGRADE

Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page $\qquad$ .)
(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have written nothing on this exam except on this REGRADE FORM.

| page | points | score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 12 |  |
| 3 | 12 |  |
| 4 | 6 |  |
| 5 | 7 |  |
| 6 | 7 |  |
| 7 | 5 |  |
| 8 | 10 |  |
| 9 | 10 |  |
| 10 | 10 |  |
| 11 | 5 |  |
| 12 | 5 |  |
| 13 | 2 |  |
| 14 |  |  |
| 15 |  |  |
| 16 |  |  |
| 17 |  |  |
| 18 |  |  |
| 19 |  |  |
| 20 |  |  |
| 21 |  |  |
| 22 |  |  |
| Total | 101 |  | (Writing or changing anything is considered to be cheating.)

Date $\qquad$ Signature
$\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI, What you wish to be called
For questions 1 and 2 follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Questions 2-10 are True/False.

As discussed in class, classify the following ODEs as to their order $\left(1^{\text {st }}, 2^{\text {nd }}, 33^{\text {rd }}, \ldots, \mathrm{n}^{\text {th }}\right)$

1. (1 pt.) The order of the ODE $y^{\prime \prime}+2 x^{5}\left(y^{\prime}\right)^{2}=\cos x$ is $\qquad$ . $\qquad$ A B C D E
2. (1 pt.) The order of the ODE $y^{\text {IV }}+e^{3 x} y^{\prime \prime}=\tan x$ is $\qquad$
$\qquad$ A B C D E

Possible answers for questions 1 and 2.
A) 1
B) 2
C) 3
D) 4
E) 5
AB) 6
AC) 7
AD) 8
AE) None of the above

True or False Circle True or False, but not both. If I cannot read your answer, it is wrong.
3.(1 pt.) A) True or B)False The ODE $y^{\prime \prime \prime}+2 x^{5} y y^{\prime \prime}=\cos x$ is linear $(y$ as a function of $x)$.
4. (1 pt.) A) True or B)False The ODE $y^{V I}+e^{3 x} y^{\prime \prime}=\tan x$ is linear ( $y$ as a function of $x$ ).
5. (1 pt.) A)True or B)False There is exactly one functions that satisfies the ODE $y^{\prime}+x y=0$.
6. (1 pt.) A)True or B)False To solve the ODE $y^{\prime}+p(x) y=g(x)$ where $p(x)$ and $g(x)$ are continuous $\forall x \in \mathbf{R}$, one uses an integrating factor given by $\mu=\mathrm{e}^{-\int \mathrm{p}(\mathrm{x}) \mathrm{dx}}$.
7. (1 pt.) A)True or B)False When solving the ODE, $y^{\prime}+p(x) y=g(x)$, where $p(x)$ and $g(x)$ are continuous $\forall \mathrm{x} \in \mathbf{R}$, one may not be able to solve for y explicitly as a function of $x$.
8. (1 pt.) A)True or B)False A direction field will not be of any help in obtaining qualitative information for the IVP: $y^{\prime}=f(x, y), y(0)=y_{0}$, if the solution cannot be obtained in terms of elementary functions.
9. (1 pt.) A)True or B)False There do not exist techniques to find integrating factors that will convert some first order ODEs which are not exact to ones that are exact.
10. (1 pt.) A)True or B)False The brothers Jakob and Johann Bernoulli did not help to develop methods of solving differential equations or to extend the range of their applications.
$\qquad$

PRINT NAME $\qquad$ ) ID No. $\qquad$ Last Name, First Name MI What you wish to be called

True or False. For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear ( $y$ as a function of $x$ ).- Integrating factor $=\mu=\exp \left(\int p(x) d x\right)$
2) First order linear ( $x$ as a function of $y$ ).- Integrating factor $=\mu=\exp \left(\int p(y) d y\right)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear ( $y$ as a function of $x$ ). Use the substitution $v=y^{1-n}$.
6) Bernoulli, but not linear ( $x$ as a function of $y$ ). Use the substitution $v=x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v=y / x$ or $v=x / y$.
8) None of the above techniques works.

Also recall the following:
a. In this context, exact means exact as given in either of the forms discussed in class.
(Attendance is mandatory.)
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong. DO NOT SOLVE.
(\#) $\left(x^{2}+2 x y\right) d x+x^{2} d y=0$
11. (2 pts.) A)True or B)False
(\#) is a linear ode ( $y$ as a function of $x$ ).
12. (2 pts.) A)True or B)False
(\#) is an exact ode.
13. (2 pts.) A)True or B)False
(\#) is a separable ode
(*) $\left(y^{3}+x^{2} y\right) d x+x^{3} d y=0$
14. $\left(2\right.$ pts.) A)True or B)False $\quad\left(^{*}\right)$ is a linear ode ( y as a function of x ).
15. (2 pts.) A)True or B)False $\quad\left(^{*}\right.$ ) is a Bernoulli ode ( $y$ as a function of $x$ ).
16. (2 pts.) A)True or B)False (*) is a homogeneous ode
$\qquad$

PRINT NAME $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI What you wish to be called
True or False. For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear ( $y$ as a function of $x$ ).- Integrating factor $=\mu=\exp \left(\int p(x) d x\right)$
2) First order linear ( $x$ as a function of $y$ ).- Integrating factor $=\mu=\exp \left(\int p(y) d y\right)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear (y as a function of $x$ ). Use the substitution $v=y^{1-n}$.
6) Bernoulli, but not linear ( $x$ as a function of $y$ ). Use the substitution $v=x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v=y / x$ or $v=x / y$.
8) None of the above techniques works.

Also recall the following:
a. In this context, exact means exact as given in either of the forms discussed in class. (Attendance is mandatory.)
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous is not a correct method of solution if the originalequation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong.
(\#) $\quad(4 x+y) d x+(x+3 y) d y=0$
17. (2 pts.) A)True or B)False (\#) is a linear ode (y as a function of $x$ ).
18. (2 pts.) A)True or B)False (\#) is an exact ode.
19. (2 pts.) A)True or B)False (\#) is a homogeneous ode
(*) $\left(3 x^{2} y+2 x y\right) d x+\left(x^{3}+x^{2}\right) d y=0$
20. (2 pts.) A)True or B)False . (*) is a linear ode (y as a function of $x$ ).
21. (2 pts.) A)True or B)False (*) is a separable ode.
22. (2 pts.) A)True or B)False (*) is an exact ode.

Total points this page $=12$. TOTAL POINTS EARNED THIS PAGE $\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

Consider the first order linear ODE $\mathrm{xy}^{\prime}=-\mathrm{y}+\mathrm{x} \sin (\mathrm{x})$ which we call (*).
23. ( 1 pts .) To solve $\left(^{*}\right)$, you may need to change $\left(^{*}\right)$ to a standard form. The correct standard form for solving (*) is $\qquad$ . A B C DE
A) $y^{\prime}+y / x=\sin (x)$
B) $y^{\prime}+y / x=-\sin (x)$
C) $y^{\prime}-y / x=x \sin (x)$
D) $y^{\prime}-y / x=-x \sin (x)$
E) $y^{\prime}+2 y / x=\sin (x)$

AB) $y^{\prime}+2 y / x=x \sin (x)$
AC) $y^{\prime}-2 y / x=\sin (x)$
AD) $y^{\prime}-2 y / x=-\sin (x)$
AE) None of the above
24. ( 2 pts.) An integrating factor for $(*)$ is $\mu=$ $\qquad$ . $\qquad$ A B CDE A)
$\begin{array}{llllllll}x & B) & -x & \text { C) } 2 x & \text { D) }-2 x & \text { E) } x^{-1} & \text { AB) }-x^{-1} & \text { AC) } 2 x^{-1}\end{array}$ AD) $-2 x^{-1} \quad$ AE) $e^{\sin (x)}$ BC) $e^{-\sin (x)} \quad$ BD) $e^{x}$ BE) $e^{-x}$ CD) $e^{2 x} \quad$ CE) $e^{-2 x}$ DE) None of the above
25. ( 3 pts.) In solving $\left(^{*}\right.$ ) as we did in class (attendance is mandatory), the following step occurs:

[^0]$\qquad$

PRINT NAME $\qquad$ ( ) ID No. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer. Be careful. If you miss one part, it may cause you to miss other parts.

An ODE may be considered to be a "vector" equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE of the form $L[y]=g(x)$ where $L$ is of the form $L[y]=y^{\prime}+p(x) y$, an integrating factor and the product rule were used to reach the following step: $\frac{d\left(y e^{x}\right)}{d x}=x e^{x}$ which we call $(*)$.
26. (2 pts.) The theorem from calculus that allows you to integrate the left hand side of $\left({ }^{*}\right)$
is $\qquad$ . A B C D E
A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem
D) Product Rule
E) Fundamental Theorem of Calculus
AB) Chain Rule AC) Integration by Parts AD) Partial Fractions AE) None of the above.
27. (5 pts.) The solution (or family of solutions) to the ODE (*) may be written
as . $\qquad$ ABCDE
A) $y=x+1+c e^{x}$
B) $y=-x+1+c e^{x}$
C) $y=x-1+c e^{x}$
D) $y=-x-1+c e^{x}$ $\begin{array}{llll}\text { E) } y=x+1+c e^{-x} & \text { AB) } y=-x+1+c e^{-x} & \text { AC) } y=x-1+c e^{-x} & \text { AD) } y=-x-1+c e^{-x}\end{array}$
$\begin{array}{lll}\text { AE) } y=2 x+2+c^{x} & \text { BC) } y=-2 x+2+e^{x} & \text { BD) } y=2 x-2+c^{x}\end{array}$ BE) $y=-2 x-2+c e^{x}$
CD) $y=2 x+2+$ ce $^{-x}$ CE) $y=-2 x+2+$ ce $^{-x} \quad$ DE) $y=2 x-2+c^{-x} \quad$ ABC) $y=-2 x-2+c e^{-x}$
$A B D)$ None of the above solutions or families of solutions is correct.
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI, What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions
28. (3 pts.) Suppose that the general solution of the ODE $y^{\prime}=f(x, y)$ is $y=x+e^{x}$, Solve the IVP. ODE $\quad y^{\prime}=f(x, y) \quad$ IC $\quad y(1)=2$

The value of the function you found as the solution to the IVP at $x=0$ is

$$
\mathrm{y}_{\mid \mathrm{x}=0}==
$$

$\qquad$ . ABCDE
29. (4 pts.) Solve the IVP ODE $d y / d x=x / y \quad$ IC $\quad y(0)=\sqrt{3}$

The value of the function you found as the solution to the IVP at $x=1$ is
$\qquad$

Possible answers this page.
A) 0 B) 1 C) 2 D) 3 E$) 4 \mathrm{AB}) 5 \mathrm{AC})-1 \mathrm{AD})-2 \mathrm{AE})-3 \mathrm{BC})-4 \mathrm{BD})-5 \mathrm{BE}) \mathrm{e}$ CD) $e^{2}$ CE) $e^{3}$ DE) $e^{4}$ ABC) $e^{-1} \quad$ ABD) $e^{-2} \quad$ ABE) $e^{-3} \quad$ BCD) $e^{-4} \quad$ BCE)None of the above. Possible points this page $=7$. POINTS EARNED THIS PAGE $=$ $\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name. First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE: $\left(2 x+y^{2}\right) d x+(2 x y+2 y) d y=0$, call this $\operatorname{ODE}(*)$.
30. (5 pts.) The solution of (*) may be written as
$\qquad$
$\qquad$ ABCDE
Be careful with your computations as there will be no part credit for an incorrect answer.
A) $\psi(x, y)=x^{2}+x y^{2}+y^{2}$
B) $\psi(x, y)=x^{2}+x y^{2}+y^{2}+C$
C) $\psi(x, y)=x^{2}+2 x y^{2}+y^{2}+C$
D) $\psi(x, y)=x^{2}+2 x y^{2}+y^{2}+C$
E) $x^{2}+2 x y^{2}+y^{2}=C$
AB) $x^{2}+x y^{2}+y^{2}=C$
AC) $x^{2}+x^{2} y^{2}+y^{2}=C$
AD) $x^{2}+2 x^{2} y^{2}+y^{2}=C$
AE) None of the above.
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE $d y / d x=e^{y / x}+(y / x)$. Call this $\operatorname{ODE}(*)$.
31. ( 1 pt ). The appropriate classification for $(*)$ is $\qquad$ . $\qquad$ A B C D E
A) Exact Equation
B) Bernoulli (y as a function of $x$ )
C) Bernoulli ( $x$ as a function of $y$ ).
D) Homogeneous
E) None of the above techniques works.
32. (2 pts.) An appropriate substitution (change of variable) to convert $(*)$ to a new solvable

ODE, call it $\left({ }^{* *}\right)$, is $v=$ $\qquad$ . $\qquad$ ABCDE
A) $1 / y$
B) $1 / y^{2}$
C) $1 / y^{3}$
D) $y^{2}$
E) $\left.y^{3} \quad A B\right) \sqrt{y}$ AC) $y / x$
$\mathrm{AD})$ None of the above.
33. (2 pts.) As $\mathrm{v}=$ $\qquad$ $\mathrm{y}=$ $\qquad$ , and $d y / d x=$ $\qquad$ , the correct term for dy/dx (in terms of $x$ and $v$ ) for this substitution is $d y / d x=$ $\qquad$ . ABCDE
A) $\frac{1}{2} v^{-\frac{1}{2}} \frac{d v}{d x}$
B) $-\frac{1}{2} v^{-\frac{1}{2}} \frac{d v}{d x}$
C) $\frac{1}{2} v^{-\frac{3}{2}} \frac{d v}{d x}$
D) $-\frac{1}{2} v^{-\frac{3}{2}} \frac{d v}{d x}$
E) $-\frac{3}{2} v^{-\frac{3}{2}} \frac{\mathrm{dv}}{\mathrm{dx}}$
AB) $-\frac{3}{2} v^{-\frac{3}{2}} \frac{d y}{d x}$
AC) $v+x \frac{d v}{d x}$
AD) $-v+x \frac{d v}{d x}$
AE) None of the above.
34. (3 pts.) The new $\operatorname{ODE}\left({ }^{* *}\right)$ that is derived may be written as

$$
\ldots
$$ .___A B C D E

A) $x \frac{d v}{d x}=e^{v}$
B) $x \frac{d v}{d x}=-e^{v}$
$\begin{array}{lll}\text { C) } x \frac{d v}{d x}=e^{v}-2 v & \text { D) } x \frac{d v}{d x}=-e^{v}-2 v\end{array}$
E) $x \frac{d v}{d x}=e^{v}+v$

AB)
$\left.\left.\left.x \frac{d v}{d x}=-e^{v}+v \quad A C\right) \frac{d v}{d x}=e^{v}-3 v \quad A D\right) x \frac{d v}{d x}=-e^{v}-3 v \quad A E\right)$ None of the above.
35. ( 2 pts.) The correct classification of the new $\operatorname{ODE}\left({ }^{* *}\right)$ that you derived is $\qquad$ . $\qquad$ A B C D E (Do not solve this equation.)
A) First order linear ( $v$ as a function of $x$ ), B) First order linear ( $x$ as a function of $v$ )
C) Separable. D) Exact E) None of the above.
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name. First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE $\mathrm{dy} / \mathrm{dx}=2 \mathrm{xy}+4 \mathrm{y}^{3}$. Call this $\operatorname{ODE}\left({ }^{*}\right)$.
36. ( 1 pt ) The appropriate classification for $(*)$ is $\qquad$ . $\qquad$ A B C D E
A) Exact $\quad$ B) Bernoulli ( $y$ as a function of $x$ ) C) Bernoulli ( $x$ as a function of $y$ ).
D) Homogeneous $\quad$ E) None of the above techniques works.
37. (2 pts.) An appropriate substitution (change of variable) to convert (*) to a new solvable

ODE, call it $\left({ }^{* *}\right)$, is $v=$ $\qquad$ . $\qquad$ ABCDE
D) $y^{2}$
E) $\left.y^{3} A B\right) \sqrt{y}$
A) $1 / y$
B) $1 / y^{2}$
C) $1 / y^{3}$
AC) $y / x$
$A D)$ None of the above.
38. $(3 \mathrm{pts}$.$) As \mathrm{v}=$ $\qquad$ , $\mathrm{y}=$ $\qquad$ , and dy/dx = $\qquad$ , the correct term for $\mathrm{dy} / \mathrm{dx}$ in terms of x and v for this substitution is
$\mathrm{dy} / \mathrm{dx}=$ $\qquad$
$\qquad$
A) $\frac{1}{2} v^{-\frac{1}{2}} \frac{\mathrm{dv}}{\mathrm{dx}}$
B) $-\frac{1}{2} v^{-\frac{1}{2}} \frac{d v}{d x}$
C) $\frac{1}{2} v^{-\frac{3}{2}} \frac{d v}{d x}$
D) $-\frac{1}{2} v^{-\frac{3}{2}} \frac{d v}{d x}$
E) $-\frac{3}{2} v^{-\frac{3}{2}} \frac{\mathrm{dv}}{\mathrm{dx}}$
AB) $-\frac{3}{2} v^{-\frac{3}{2}} \frac{d y}{d x}$

AC) $v+x \frac{d v}{d x} \quad$ AD) $-v+x \frac{d v}{d x} \quad$ AE)None of the above.
39. (3 pts.) The new $\operatorname{ODE}\left({ }^{* *}\right)$ that is derived is $\qquad$ . $\qquad$ A B C D E
$\begin{array}{llll}\text { A) } d v / d x+4 x v=8 & \text { B) } d v / d x+4 x v=-8 & \text { C) } d v / d x-4 x v=8 & \text { D) } d v / d x-4 x v=-8\end{array}$
E) $d v / d x+4 v=8 x \quad A B) d v / d x+4 v=-8 x \quad A C) d v / d x-4 v=8 x \quad A D) d v / d x-4 v=-8 x$ AE) None of the above.
40. (1 pts.) The correct classification of the new $\operatorname{ODE}\left({ }^{* *}\right)$ that you derived is $\qquad$ . $\qquad$ ABCDE
A) First order linear ( $v$ as a function of $x$ ), B) First order linear ( $x$ as a function of $v$ )
C) Separable. D) Exact Equation E) None of the above.
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name. First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Suppose that the ODE dy/dx $=f(x, y)$, call it $\left(^{*}\right)$, is not linear or separable, but that it can be solved using the substitution (change of variable), $v=y / x$. Suppose further that this substitution results in the derived ODE $v+x \frac{d v}{d x}=v^{2}$. Call this $\operatorname{ODE}\left({ }^{(* *)}\right.$.
41. (2 pts.) The correct classification of $\left({ }^{* *}\right)$ is $\qquad$ . A B C D E A)
First order linear ( $v$ as a function of $x$ ) $\quad$ B) First order linear ( $x$ as a function of $v$ )
C) Separable
D) Exact
E) None of the above.
42. (2 pts.) (**) may be rewritten as $\qquad$ . $\qquad$ A B C D E
A) $\frac{d v}{v^{2}+v}=\frac{d}{x}$
B) $\frac{d v}{v^{2}+v}=-\frac{d x}{x}$
C) $\frac{d v}{v^{2}-v}=\frac{d x}{x}$
D) $\frac{d v}{v^{2}-v}=-\frac{d x}{x}$
E)None of the above.
43. (4 pts.) The solution of $\left({ }^{* *}\right)$ may be written as $v=$ $\qquad$ . $\qquad$ ABCDE
A) $\frac{1}{1-c x}$
B) $\frac{-1}{1-c x}$
C) $\frac{x}{1-c x}$
D) $\frac{-x}{1-c x}$ E) $\frac{x^{2}}{1-c x}$
AB) $\frac{-x^{2}}{1-c x}$
AC)None of the above.
44. (2 pts.) The solution of $\left({ }^{*}\right)$ may be written as $y=$ $\qquad$ . $\qquad$ ABCDE
A) $\frac{x}{1-c x}$
B) $\frac{-x}{1-c x}$
C) $\frac{x^{2}}{1-c x}$
D) $\frac{-x^{2}}{1-c x}$ E) $\frac{x^{3}}{1-c x}$
$\begin{array}{ll}A B) \frac{-x^{3}}{1-c x} & A C) \text { None of the above. }\end{array}$
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer in the list.
45. ( 5 pts .) The direction field for the $\mathrm{ODE} \mathrm{y}^{\prime}=(3-\mathrm{y}) / 2$ is given below. On this direction field are five curves labeled $1,2,3,4$, and 5 that were correctly or incorrectly drawn using the direction field. Consider the initial value problem (IVP):

$$
\begin{array}{cc}
\text { IVP } \begin{array}{c}
\text { ODE } \\
\text { IC }
\end{array} y^{\prime}=(3-y) / 2 \\
y(0)=1
\end{array}
$$

The curve or curves that is the solution to this IVP is $\qquad$ ABCDE (Hint: Do not solve the IVP.)
A) 1
B) 2
C) 3
D) 4
E) 5
F) 1 and 2
AB) 2 and 3
AC). 3 and 4
AD) 4 and 5
AE) 1, 2, and 3
BC) 2,3 , and 4
BD) 3,4 , and 5
BE) 1, 2, 3, 4, and 5
CD) None of the above
$\qquad$
$\qquad$ ) ID NO. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer in the list.

MATHEMATICAL MODELING. As done in class (attendance is mandatory), you are to develop a general mathematical model for a point mass traveling down in a fluid. Take positive distance to be down. Suppose a mass m has weight $\mathrm{W}=\mathrm{mg}$ where g is the acceleration due to gravity and initial downward velocity $\mathrm{v}_{0} \geq 0$. Suppose it is traveling down in a fluid that offers resistance proportional to the cube of the velocity of the object where the velocity $v(t)$ is measured in feet per second. Assume that the proportionality constant is $\mathrm{k}>0$.
46. (1 pt) The fundamental physical law used to develop the ODE in the model
is $\qquad$ . $\qquad$ A B C D E A)Conservation of mass B)Conservation of energy C)Ohm's law D)Kirchoff's voltage law E)Kirchoff's current law AB)Newton's second law (Conserv. of momentum) AC)None of the above.
47. (2 pts.) The mathematical model for the particle in a fluid system whose solution yields the downward velocity $\mathrm{v}(\mathrm{t})$ of the particle as a function of time
is $\qquad$ . $\qquad$ A B C D E
$\begin{array}{llll}\text { A) } m \dot{v}=m g+k v^{3} & \text { B) } m \dot{v}=m g-k v^{3} & \text { C) } m \dot{v}=-m g+k v^{3} & \text { D) } m \dot{v}=-m g-k v^{3}\end{array} \quad$ E) $m \dot{v}=m g+k v^{3} \quad v(0)=v_{0} \geq 0$
$\begin{array}{llll}A B) ~ \\ A v \\ =m g-k v^{3} & v(0)=v_{0} \geq 0 & \text { AC) } m \dot{v}=-m g+k v^{3} \quad v(0)=v_{0} \geq 0 & \text { AD) } m \dot{v}=m g-k v^{3} \quad v(0)=v_{0} \geq 0\end{array}$
$\begin{array}{llll}\text { AE) } m \dot{v}=m g-k v^{2} \quad v(0)=v_{0} \geq 0 & \left.\text { BC) } m \dot{v}=m g-v^{3} \quad v(0)=v_{0} \geq 0 \quad B D\right) N o n e ~ o f ~ t h e ~ a b o v e . ~\end{array}$
48. (1 pt.) The units for the ODE in the model you selected in question 47 above
are $\qquad$ .__A B C D E
A) Feet
B) Seconds
C) feet per second
D) feet per second squared
E) Pounds
AB) Slugs AC) Slug feet AD) None of the above.
49. (1 pt.) A)True or B)False If the particle is dropped, the model given in question 47 above can be solved to obtain a general formula for v without additional data.
$\qquad$
$\qquad$ ( $\qquad$ ) ID NO. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer in the list.

MATHEMATICAL MODELING. Consider the following applied math problem:

An object (point particle) of mass 5 slugs is dropped from rest at time $t=0$ in a medium that offers resistance equal to three times the cube of the velocity of the object where the velocity is measured in feet per second.

Apply the data given above to the general model you developed on the previous page to obtain the specific model for this problem. DO NOT SOLVE!
50. ( 2 pts.) The mathematical model for the system whose solution yields the velocity $\mathrm{v}(\mathrm{t})$ as a function of time
is $\qquad$ . $\qquad$ A B C D E
A) $5 \dot{v}=160+3 v^{3}$
$\mathrm{v}(0)=0$
B) $5 \dot{v}=160-3 v^{3}$
C) $5 \dot{v}=160-10 \mathrm{kv}^{3}$
D) $10 \dot{v}=320-3 v^{3} \quad v(0)=0$
E) $5 \dot{v}=-160+3 v^{3} \quad v(0)=0$
AB) $5 \dot{v}=-160-3 v^{3} \quad v(0)=3$
AC) None of the above
$\qquad$


[^0]:    $\mathrm{A}) \mathrm{d}(\mathrm{yx}) / \mathrm{dx}=\sin (\mathrm{x}) \quad \mathrm{B}) \mathrm{d}(\mathrm{yx}) / \mathrm{dx}=-\sin (\mathrm{x}) \quad \mathrm{C}) \mathrm{d}(\mathrm{yx}) / \mathrm{dx}=\mathrm{x} \sin (\mathrm{x}) \quad \mathrm{D}) \mathrm{d}(\mathrm{yx}) / \mathrm{dx}=-\mathrm{x} \sin (\mathrm{x})$
    E) $\left.\left.\left.d\left(y^{2}\right) / d x=\sin (x) A B\right) d\left(y x^{2}\right) / d x=-\sin (x) A C\right) d\left(y x^{2}\right) / d x=x \sin (x) \quad A D\right) d\left(y x^{2}\right) / d x=-x \sin (x)$
    $A E) d(y / x) / d x=\sin (x) B C) d(y / x) / d x=-\sin (x) B D) d(y / x) / d x=x \sin (x) B E) d(y / x) / d x=-x \sin (x)$
    $\left.\left.C D) d\left(y / x^{2}\right) / d x=\sin (x) \quad C E\right) d\left(y / x^{2}\right) / d x=-\sin (x) \quad D E\right) d\left(y / x^{2}\right) / d x=x \sin (x)$
    $\left.A B C) d\left(y / x^{2}\right) / d x=-x \sin (x) B C\right)$ None of the above steps ever appears in any solution of $\left({ }^{*}\right)$.

