EXAM-I FALL 2008

Date \_

Signature\_

## MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

PRINT NAME			(		)
Last Name,	First Name	MI	(What you	wish to be	called)
ID #		EXAM DATE _		otember 12. Scores	
I swear and/or affirm that all of the	_		page	points	score
and that I have neither given nor	received any help during	eived any help during the exam.		10	
		<del></del>	2	12	
SIGNATURE		ATE	3	12	
INSTRUCTIONS: Besides this cand problems on this exam. MA	KE SURE YOU HAVI	E ALL THE	4	6	
<b>PAGES.</b> If a page is missing, yo			5	7	
page. Read through the entire exyour hand and I will come to you			3		
exam. Your I.D., this exam, and	a straight edge are all th	at you may have	6	7	
on your desk during the exam. <b>PAPER!</b> Use the back of the ex			7	5	
the staple if you wish. Print your	name on all sheets. Pag	ges 1-12 are Fill-	8	10	
in-the Blank/Multiple Choice or '			9	10	
pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response			10	10	
			11	5	
pages. However, to insure credit and carefully. Your entire solution	t, you should explain you	ır solutions fully	12	5	
answer. SHOW YOUR WORK			13	2	
expressed in your best mathematigiven as deemed appropriate. Pr			14		
computations as time allows. GOOD LUCK!!			15		
			16		
REQUES	T FOR REGRADE		17		
Please regrade the following pro	oblems for the reasons I	ems for the reasons I have indicated:	18		
(e.g., I do not understand what I	I did wrong on page	)	19		
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		in a week of the date the exam is as necessary to explain your reasons.)			
I swear and/or affirm that upon	Tota	al 101			
nothing on this exam except o (Writing or changing anything					

MATH 261 EXAM I Fall 2008 Prof. Moseley Page 1 PRINT NAME \_\_\_\_\_(\_\_\_\_) ID No. Last Name, First Name MI, What you wish to be called For questions 1 and 2 follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Ouestions 2-10 are True/False. As discussed in class, classify the following ODEs as to their order  $(1^{st}, 2^{nd}, 3^{rd}, ..., n^{th})$ 1. (1 pt.) The order of the ODE  $y'' + 2x^5 (y')^2 = \cos x$  is \_\_\_\_\_\_. A B C D E 2. (1 pt.) The order of the ODE  $y^{IV} + e^{3x} y'' = \tan x$  is \_\_\_\_\_\_ A B C D E Possible answers for questions 1 and 2. A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) 8 AE) None of the above **True or False** Circle True or False, but not both. If I cannot read your answer, it is wrong. 3.(1 pt.) A) True or B)False The ODE  $y''' + 2x^5 y y'' = \cos x$  is linear (y as a function of x). 4. (1 pt.) A) True or B) False The ODE  $y^{VI} + e^{3x}y'' = \tan x$  is linear (y as a function of x). 5. (1 pt.) A)True or B)False There is exactly one functions that satisfies the ODE y' + x y = 0. 6. (1 pt.) A)True or B)False To solve the ODE y' + p(x) y = g(x) where p(x) and g(x) are continuous  $\forall x \in \mathbf{R}$ , one uses an integrating factor given by  $\mu = e^{-\int p(x)dx}$ . 7. (1 pt.) A)True or B)False When solving the ODE, y' + p(x) y = g(x), where p(x) and g(x) are continuous  $\forall x \in \mathbb{R}$ , one may not be able to solve for y explicitly as a function of x. 8. (1 pt.) A)True or B)False A direction field will not be of any help in obtaining qualitative information for the IVP: y' = f(x,y),  $y(0) = y_0$ , if the solution cannot be obtained in terms of elementary functions. 9. (1 pt.) A)True or B)False There do not exist techniques to find integrating factors that will convert some first order ODEs which are not exact to ones that are exact. 10. (1 pt.) A)True or B)False The brothers Jakob and Johann Bernoulli did not help to develop methods of solving differential equations or to extend the range of their applications.

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**True or False.** For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

- 1) First order linear (y as a function of x).- Integrating factor =  $\mu = \exp(\int p(x) dx)$
- 2) First order linear (x as a function of y).- Integrating factor =  $\mu = \exp(\int p(y) dy$ )
- 3) Separable.
- 4) Exact Equation (Must be exact in one of the two forms discussed in class).
- 5) Bernoulli, but not linear (y as a function of x). Use the substitution  $v = v^{1-n}$ .
- 6) Bernoulli, but not linear (x as a function of y). Use the substitution  $v = x^{1-n}$ .
- 7) Homogeneous, but not separable. Use the substitution v = y/x or v = x/y.
- 8) None of the above techniques works.

Also recall the following:

- a. In this context, exact means exact as given in either of the forms discussed in class. (Attendance is mandatory.)
- b. Bernoulli is not a correct method of solution if the original equation is linear.
- c. Homogeneous is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong. **DO NOT SOLVE**.

(#) ( 
$$x^2 + 2xy$$
 )  $dx + x^2 dy = 0$ 

- 11. (2 pts.) A)True or B)False (#) is a linear ode (y as a function of x).
- 12. (2 pts.) A)True or B)False (#) is an exact ode.
- 13. (2 pts.) A)True or B)False (#) is a separable ode

(\*) (
$$y^3 + x^2y$$
) dx +  $x^3$  dy = 0

- 14.(2 pts.) A)True or B)False (\*) is a linear ode (y as a function of x).
- 15. (2 pts.) A)True or B)False (\*) is a Bernoulli ode (y as a function of x).
- 16. (2 pts.) A)True or B)False (\*) is a homogeneous ode

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**True or False.** For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

- 1) First order linear (y as a function of x).- Integrating factor =  $\mu = \exp(\int p(x) dx)$
- 2) First order linear (x as a function of y).- Integrating factor =  $\mu = \exp(\int p(y) dy)$
- 3) Separable.
- 4) Exact Equation (Must be exact in one of the two forms discussed in class).
- 5) Bernoulli, but not linear (y as a function of x). Use the substitution  $v = v^{1-n}$ .
- 6) Bernoulli, but not linear (x as a function of y). Use the substitution  $v = x^{1-n}$ .
- 7) Homogeneous, but not separable. Use the substitution v = y/x or v = x/y.
- 8) None of the above techniques works.

Also recall the following:

- a. In this context, exact means exact as given in either of the forms discussed in class. (Attendance is mandatory.)
- b. Bernoulli is not a correct method of solution if the original equation is linear.
- c. Homogeneous is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong.

(#) 
$$(4x + y) dx + (x + 3y) dy = 0$$

- 17. (2 pts.) A)True or B)False (#) is a linear ode (y as a function of x).
- 18. (2 pts.) A)True or B)False (#) is an exact ode.
- 19. (2 pts.) A)True or B)False (#) is a homogeneous ode

(\*) 
$$(3x^2y + 2xy) dx + (x^3 + x^2) dy = 0$$

- 20. (2 pts.) A)True or B)False . (\*) is a linear ode (y as a function of x).
- 21. (2 pts.) A)True or B)False (\*) is a separable ode.
- 22. (2 pts.) A)True or B)False (\*) is an exact ode.

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			in-the Blank/Multiple Che	<u>-</u>
Also, circle your parts.	answer. Be careful.	No part credit. If you	miss one part, it may cau	se you to miss other
	first order linear ODE	$xy' = -y + x \sin(x)$	which we call (*).	
23. (1 pts.) To s	solve (*), you may nee	ed to change (*) to a st	tandard form. The correct	et standard form
for solving (	*) is			_ A B C D E
	$\sin(x)$ AB) $y' + 2y/x =$		$= x \sin(x)  D) y' - y/x = \sin(x)  AD) y' - 2y/x$	
24. (2 pts.) An i	integrating factor for (	(*) is $\mu = $	·	_ A B C D E A)
x B) -x C) 2	$(x D) -2x E) x^{-1} A$ CE) $(e^{-2x} DE)$ Non-	$(B) - x^{-1}  AC) \ 2x^{-1}  A$	$(D) -2x^{-1}$ $(AE) e^{\sin(x)} BC$	$e^{-\sin(x)}$ BD) $e^x$
25. (3 pts.) In se	olving (*) as we did in	class (attendance is m	nandatory), the following	step occurs:
			A	BCDE
E)d(yx <sup>2</sup> )/dx = AE)d(y/x)/dx = s CD)d(y/x <sup>2</sup> )/dx =	$\sin(x)$ AB)d(yx <sup>2</sup> )/dx = $\sin(x)$ BC)d(y/x)/dx = $\sin(x)$ CE)d(y/x <sup>2</sup> )/o	$-\sin(x)  C)d(yx)/dx =$ $= -\sin(x)  AC)d(yx^2)/dx$ $-\sin(x)  BD)d(y/x)/dx$ $dx = -\sin(x)  DE)d(x)$	$x \sin(x)$ D)d(yx)/dx = - x = x sin(x) AD)d(yx <sup>2</sup> )/dx x = x sin(x) BE)d(y/x)/dx	$x \sin(x)$ $dx = -x \sin(x)$ $x = -x \sin(x)$

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer. Be careful. If you miss one part, it may cause you to miss other parts.

An ODE may be considered to be a "vector" equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. To solve a first order linear ODE, we may isolate the unknown function. The isolation of the function (dependent variable) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE of the form L[y] = g(x) where L is of the form L[y] = y' + p(x)y, an integrating factor and the product rule were used to reach the following step:  $\frac{d(ye^x)}{dx} = xe^x$  which we call (\*).

26. (2 pts.) The theorem from calculus that allows you to integrate the left hand side of (\*)

is . ABCDE

- A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem
- D) Product Rule E) Fundamental Theorem of Calculus AB) Chain Rule
- AC) Integration by Parts AD) Partial Fractions AE) None of the above.

27. (5 pts.) The solution (or family of solutions) to the ODE (\*) may be written

A) 
$$y = x + 1 + c e^x$$
 B)  $y = -x + 1 + c e^x$  C)  $y = x - 1 + c e^x$  D)  $y = -x - 1 + c e^x$ 

E) 
$$y = x + 1 + c e^{-x}$$
 AB)  $y = -x + 1 + c e^{-x}$  AC)  $y = x - 1 + c e^{-x}$  AD)  $y = -x - 1 + c e^{-x}$ 

AE) 
$$y = 2x + 2 + ce^x$$
 BC)  $y = -2x + 2 + ce^x$  BD)  $y = 2x - 2 + ce^x$  BE)  $y = -2x - 2 + ce^x$ 

CD) 
$$y = 2x + 2 + ce^{-x}$$
 CE)  $y = -2x + 2 + ce^{-x}$  DE)  $y = 2x - 2 + ce^{-x}$  ABC)  $y = -2x - 2 + ce^{-x}$ 

ABD) None of the above solutions or families of solutions is correct.

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28. (3 pts.) Suppose that the general solution of the ODE y' = f(x,y) is  $y = x + ce^x$ , Solve the IVP. ODE y' = f(x,y) IC y(1) = 2

The value of the function you found as the solution to the IVP at x = 0 is

 $y_{|x=0} = =$  \_\_\_\_\_\_. A B C D E

29. (4 pts.) Solve the IVP ODE dy/dx = x/y IC  $y(0) = \sqrt{3}$ The value of the function you found as the solution to the IVP at x = 1 is

 $y_{|x=1} =$  \_\_\_\_\_\_. A B C D E

Possible answers this page.

A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) -1 AD) -2 AE) -3 BC) -4 BD) -5 BE) e CD)  $e^2$  CE)  $e^3$  DE)  $e^4$  ABC)  $e^{-1}$  ABD)  $e^{-2}$  ABE)  $e^{-3}$  BCD)  $e^{-4}$  BCE)None of the above. Possible points this page = 7. POINTS EARNED THIS PAGE = \_\_\_\_\_

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE:  $(2x + y^2) dx + (2xy+2y) dy = 0$ , call this ODE (\*).

30. (5 pts.) The solution of (\*) may be written as

Be careful with your computations as there will be no part credit for an incorrect answer.

A) 
$$\psi(x,y) = x^2 + xy^2 + y^2$$

B) 
$$\psi(x,y) = x^2 + xy^2 + y^2 + C$$

C) 
$$\psi(x,y) = x^2 + 2xy^2 + y^2 + C$$

A) 
$$\psi(x,y) = x^2 + xy^2 + y^2$$
 B)  $\psi(x,y) = x^2 + xy^2 + y^2 + C$  C)  $\psi(x,y) = x^2 + 2xy^2 + y^2 + C$  D)  $\psi(x,y) = x^2 + 2xy^2 + y^2 + C$  E)  $x^2 + 2xy^2 + y^2 = C$  AB)  $x^2 + xy^2 + y^2 = C$  AB) None of the above.

E) 
$$x^2 + 2xy^2 + y^2 = C$$

AB) 
$$x^2 + xy^2 + y^2 = C$$

AC) 
$$x^2 + x^2 y^2 + y^2 = C$$

AD) 
$$x^2 + 2x^2y^2 + y^2 = 0$$

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE  $dy/dx = e^{y/x} + (y/x)$ . Call this ODE (\*).

31. (1 pt). The appropriate classification for (\*) is \_\_\_\_\_\_\_. A B C D E A) Exact Equation B) Bernoulli (y as a function of x) C) Bernoulli (x as a function of y).

- D) Homogeneous E) None of the above techniques works.

32. (2 pts.) An appropriate substitution (change of variable) to convert (\*) to a new solvable

ODE, call it (\*\*), is v = \_\_\_\_\_

ODE, call it (\*\*\*), is v =\_\_\_\_\_\_\_. A B C D E A) 1/y B)  $1/y^2$  C)  $1/y^3$  D)  $y^2$  E)  $y^3$  AB)  $\sqrt{y}$  AC) y/x AD) None of the above.

33. (2 pts.) As  $v = \underline{\hspace{1cm}}$ ,  $y = \underline{\hspace{1cm}}$ , and  $dy/dx = \underline{\hspace{1cm}}$ , the correct term for dy/dx (in terms of x and v) for this substitution is

- AC)  $v + x \frac{dv}{dx}$  AD)  $-v + x \frac{dv}{dx}$  AE) None of the above.

34. (3 pts.) The new ODE (\*\*) that is derived may be written as

A) 
$$x \frac{dv}{dx} = e^{v}$$
 B)  $x \frac{dv}{dx} = -e^{v}$  C)  $x \frac{dv}{dx} = e^{v} - 2v$  D)  $x \frac{dv}{dx} = -e^{v} - 2v$  E)  $x \frac{dv}{dx} = e^{v} + v$  AB)

 $x\frac{dv}{dv} = -e^v + v$  AC)  $\frac{dv}{dv} = e^v - 3v$  AD)  $x\frac{dv}{dx} = -e^v - 3v$  AE) None of the above.

35. (2 pts.) The correct classification of the new ODE (\*\*) that you derived

(Do not solve this equation.)

- A) First order linear (v as a function of x), B) First order linear (x as a function of v)
- C) Separable. D) Exact E) None of the above.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE  $dy/dx = 2xy + 4y^3$ . Call this ODE (\*).

36. (1 pt) The appropriate classification for (\*) is \_\_\_\_\_\_. \_\_\_\_. \_\_\_\_A B C D E A) Exact B) Bernoulli (y as a function of x) C) Bernoulli (x as a function of y).

- D) Homogeneous E) None of the above techniques works.

37. (2 pts.) An appropriate substitution (change of variable) to convert (\*) to a new solvable

ODE, call it (\*\*), is v =\_\_\_\_\_\_. A B C D E A) 1/y B)  $1/y^2$  C)  $1/y^3$  D)  $y^2$  E)  $y^3$  AB)  $\sqrt{y}$  AC) y/x AD) None of the above.

38. (3 pts.)As  $v = \underline{\hspace{1cm}}$ ,  $y = \underline{\hspace{1cm}}$ , and  $dy/dx = \underline{\hspace{1cm}}$ , the correct term for dy/dx in terms of x and v for this substitution is

AC)  $v + x \frac{dv}{dx}$  AD)  $-v + x \frac{dv}{dx}$  AE)None of the above.

39. (3 pts.) The new ODE (\*\*) that is derived is \_\_\_\_\_\_. \_\_\_\_A B C D E

- A) dv/dx + 4x v = 8 B) dv/dx + 4x v = -8 C) dv/dx 4x v = 8 D) dv/dx 4x v = -8
- E)  $\frac{dv}{dx} + 4v = 8x$  AB)  $\frac{dv}{dx} + 4v = -8x$  AC)  $\frac{dv}{dx} 4v = 8x$  AD)  $\frac{dv}{dx} 4v = -8x$ AE) None of the above.

40. (1 pts.) The correct classification of the new ODE (\*\*) that you derived

- A) First order linear (v as a function of x), B) First order linear (x as a function of v)
- C) Separable. D) Exact Equation E) None of the above.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Suppose that the ODE dy/dx = f(x,y), call it (\*), is not linear or separable, but that it can be solved using the substitution (change of variable), v = y/x. Suppose further that this substitution results in the derived ODE  $v + x \frac{dv}{dv} = v^2$ . Call this ODE (\*\*).

41. (2 pts.) The correct classification of (\*\*) is \_\_\_\_\_\_\_. A B C D E First order linear (v as a function of x) B) First order linear (x as a function of v) A)

D) Exact E) None of the above. C) Separable

42. (2 pts.) (\*\*) may be rewritten as \_\_\_\_\_\_. A B C D E A)  $\frac{dv}{v^2+v} = \frac{dx}{x}$  B)  $\frac{dv}{v^2+v} = -\frac{dx}{x}$  C)  $\frac{dv}{v^2-v} = \frac{dx}{x}$  D)  $\frac{dv}{v^2-v} = -\frac{dx}{x}$  E)None of the above.

43. (4 pts.) The solution of (\*\*) may be written as v =\_\_\_\_\_. \_A B C D E A)  $\frac{1}{1-cx}$  B)  $\frac{-1}{1-cx}$  C)  $\frac{x}{1-cx}$  D)  $\frac{-x}{1-cx}$  E)  $\frac{x^2}{1-cx}$  AB)  $\frac{-x^2}{1-cx}$  AC)None of the above.

44. (2 pts.) The solution of (\*) may be written as y =\_\_\_\_\_. A B C D E

A)  $\frac{x}{1-cx}$  B)  $\frac{-x}{1-cx}$  C)  $\frac{x^2}{1-cx}$  D)  $\frac{-x^2}{1-cx}$  E)  $\frac{x^3}{1-cx}$  AB)  $\frac{-x^3}{1-cx}$  AC)None of the above.

MATH 261 EXAM 1 Fall 2008 Prof. Moseley Page 11 \_\_\_\_\_) ID No. \_\_\_\_\_ PRINT NAME \_\_\_\_\_ Last Name, First Name MI What you wish to be called addition, circle your answer in the list. 45. (5 pts.) The direction field for the ODE y' = (3-y)/2 is given below. On this direction field are five curves labeled 1, 2, 3, 4, and 5 that were correctly or incorrectly drawn using the direction field. Consider the initial value problem (IVP): IVP ODE y' = (3-y)/2IC y(0) = 1The curve or curves that is the solution to this IVP is \_\_\_\_\_\_. \_\_\_A B C D E (Hint: Do not solve the IVP.) A) 1 B) 2 E) 5 F) 1 and 2 C) 3 D) 4 AB) 2 and 3 AC). 3 and 4 AD) 4 and 5 AE) 1, 2, and 3 BC) 2, 3, and 4 BD) 3, 4, and 5

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	structions on the Exan our answer in the list.	n Cover Sheet for Fill-	in-the Blank/Multiple Choic	e questions. In
MATHEMAT	ICAL MODELING.	Consider the following	g applied math problem:	
offers resista	•		n rest at time $t = 0$ in a medicity of the object where the	
	0 0	eneral model you deve n. <b>DO NOT SOLVE!</b>	eloped on the previous page	to obtain
50. (2 pts.)The n v(t) as a funct		the system whose sol	ution yields the velocity	
is			A B	CDE
A) $5\dot{v} = 160 + 3v^3$	$v(0) = 0 \qquad \mathbf{B})  5\dot{\mathbf{v}} = 1$	$60 - 3v^3$ C) $5\dot{v} = 160 - 3v^3$	$10kv^3$ D) $10\dot{v} = 320 - 3v^3$ $v(0)$	) = 0

E)  $5\dot{v} = -160 + 3v^3$  v(0) = 0 AB)  $5\dot{v} = -160 - 3v^3$  v(0) = 3 AC) None of the above