EXAM 4 -B3 MATH 261: Elementary Differential Equations MATH 261
FALL 2014 10:00am EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____ (________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____________________________________ EXAM DATE Friday, Nov. 22, 2014 10:00 am

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

____________________________ ______________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 11 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Page 12 contains Laplace transforms you need not memorize. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-11 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

Scores

page	points	score
1	11	
2	5	
3	12	
4	17	
5	12	
6	6	
7	8	
8	4	
9	12	
10	6	
11	7	
12	---	
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
Total	100	

REQUEST FOR REGRADE
Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)
(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.) Date _______ Signature______________________________

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True-false. Laplace transforms.

1. (1 pts) A)True or B)False The definition of the Laplace transform is $\mathscr{L}\{f(t)\}(s) = \int_{t=-\infty}^{t=\infty} f(t)e^{-st}dt$ provided the improper integral exists.

2. (1 pts) A)True or B) False Since the Laplace transform is defined in terms of an improper integral, it involves only one limit process.

3. (1 pts) A)True or B)False The Laplace transform does not exist for some continuous functions on $[0,\infty)$.

4. (1 pts) A)True or B)False The Laplace transform exists for some discontinuous functions.

5. (1 pts) A)True or B)False The function f(t) = 1/(t-4) is piecewise continuous on [0,7].

6. (1 pts) A)True or B)False The function $f(t) = 5e^{4t^2}\cos(t)$ is of exponential order.

7. (1 pts) A)True or B)False The Laplace transform $\mathscr{L}:\mathbf{T}\to\mathbf{F}$ is not a linear operator.

8. (1 pts) A)True or B)False The inverse Laplace transform $\mathscr{L}^{-1}:\mathbf{F}\to\mathbf{T}$ is a linear operator.

9. (1 pts) A)True or B)False The Laplace transform is a one-to-one mapping on the set of continuous functions on $[0,\infty)$ for which the Laplace transform exists.

10. (1 pts) A)True or B)False There is only one continuous function in the null space of $\mathscr{L}$.

11. (1 pts) A)True or B)False The strategy of solving an ODE using Laplace transforms is to transform the problem from the time domain **T** to the (complex) frequency domain **F**, solve the transformed problem using algebra instead of calculus, and then transform the solution back to the time domain **T**.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

12. (5 pts.) The Laplace transform of the function $f(t) = \begin{cases} 2 & 0 \le t \le 5 \\ 0 & t > 5 \end{cases}$

is ____. ___________A B C D E

Hint: Use the definition. Be careful to handle the limit appropriately as discussed in class.

Possible answers this page.

A) $\frac{1}{s}$ E $\frac{-1}{s}$ $\frac{1}{s}e^{-5s}$ $\frac{-1}{s}e^{-5s}$ D) $\frac{1}{s}(1+e^{-5s})$ E) $\frac{1}{s}(1-e^{-5s})$ $\frac{1}{s}(e^{-5s}-1)$

AD) $\frac{5}{s}$ A $\frac{-5}{s}$ $\frac{5}{s}e^{-8s})$ $\frac{-5}{s}e^{-8s}$ BD) $\frac{1}{s}(1+e^{5s})$ E $\frac{2}{s}(1-e^{-5s})$ CD)

CE) $\frac{3}{s}(1-e^{-5s})$ $\frac{4}{s}(1-e^{5s}))$ ABCDE) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE __________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the Laplace transform of the following functions.

13. (4 pts.) $f(t) = 2t - 2t^2$ $\mathcal{L}(f) =$ ____________________________________. ________ A B C D E

14. (4 pts.) $f(t) = 2\,e^{2t} - 2\,e^{-3t}$ $\mathcal{L}(f) =$ __________________________________. ________ A B C D E

15 (4 pts.) $f(t) = 2\sin(2t) - 2\cos(3t)$ $\mathcal{L}(f) =$ ______________________________. _______ A B C D E

Possible answers this page

A) $\frac{2}{s^2}+\frac{2}{s^3}$ $\frac{2}{s^2}-\frac{2}{s^3}$) $\frac{2}{s^2}+\frac{4}{s^3}$ $\frac{2}{s^2}-\frac{4}{s^3}$ $\frac{2}{s^2}+\frac{6}{s^3}$ D) $\frac{2}{s^2}-\frac{6}{s^3}$ $\frac{2}{s^2}+\frac{8}{s^3}$ $\frac{2}{s^2}-\frac{8}{s^3}$ AB)

AE) $\frac{2}{s-2}+\frac{1}{s+3}$ $\frac{2}{s-2}-\frac{1}{s+3}$ $\frac{2}{s-2}+\frac{2}{s+3}$ $\frac{2}{s-2}-\frac{2}{s+3}$ BE)

CD) $\frac{2}{s-2}+\frac{3}{s+3}$ $\frac{2}{s-2}-\frac{3}{s+3}$ $\frac{2}{s-2}+\frac{4}{s+3}$ $\frac{2}{s-2}-\frac{4}{s+3}$ ABC)

ABD) $\frac{4}{s^2+4}+\frac{s}{s^2+9}$ $\frac{4}{s^2+4}-\frac{s}{s^2+9}$ $\frac{4}{s^2+4}+\frac{2s}{s^2+9}$ $\frac{4}{s^2+4}-\frac{2s}{s^2+9}$ ACE

ADE) $\frac{4}{s^2+4}+\frac{3s}{s^2+9}$ $\frac{4}{s^2+4}-\frac{3s}{s^2+9}$ $\frac{4}{s^2+4}+\frac{4s}{s^2+9}$ $\frac{4}{s^2+4}-\frac{4s}{s^2+9}$ CD

ABCD) $-\frac{2}{(s-2)^2}+\frac{3}{(s+3)^2}$ $\frac{2}{s^2+2}+\frac{3s}{s^2+3}$ \BCE) $\frac{2s}{s^2+2}+\frac{3}{s^2+3}$ ABDE)

ACDE) $\mathcal{L}\{f\}$ exists but none of the above is $\mathcal{L}\{f\}$ BCDE) $\mathcal{L}\{f\}$ does not exist.

ABCDE)None of the above.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

DEFINITION. Let $f:X\to Y$. Then f is one-to-one if $\forall\, x_1, x_2\in X$ we have

16.(2 pts.) __________________ _____ A B C D E implies 17(2pt) __________________ _____ A B C D E

THEOREM. Let $T:V\to W$ be a linear operator where V and W are vector spaces over the same field **K**. If the null space N_T contains only the zeo vector, then T is a one-to-one mapping.
Proof. We begin our proof of the theorem by first proving the following lemma:

Lemma. Let $T:V\to W$ be a linear operator, $\vec{v}_1 \in V$, ar $\{\vec{0}\}$ N_T $= T(\vec{v}_1)=\vec{0}$. If $\vec{v}_1=\vec{0}$, then .

Proof of lemma: Let $T:V\to W$ be a linear operator, $\vec{v}_1 \in V$ $\{\vec{0}\}$ $_T$ = $T(\vec{v}_1)=\vec{0}$ id . By the definition

of the null space we have that $N_T = \{\vec{v}$ $\in V$: 18.(2 pt.) ______________________ _____ A B C D E} so that

$T(\vec{v}_1)=\vec{0}$ impl $\vec{v}_1$ s that $\in N_T$. Since N_T contains 19.(1 pt.) ______________________ ______ A B C D E, we have that $\vec{v}_1=\vec{0}$ as was to be proved. QED for lemma.

Having finished the proof of the lemma, we now finish the proof of the theorem. To show that T

is one-to-one, for $\vec{v}_1,\vec{v}_2 \in V$ we assume 20.(1 pt.) ___________________ _______A B C D E and show that

21.(1 pt.) _________________________. _______A B C D E. We use the TATEMENT/REASON format.

STATEMENT	REASON
$T(\vec{v}_1)=T(\vec{v}_2)$	22.(2 pt.) _____________________________ _______ A B C D E
23.(2pt.) ____________________________ __. _______ A B C D E	Vector algebra in W
$T(\vec{v}_1-\vec{v}_2)=\vec{0}$	24.(2 pt.) ___________________________________ A B C D E
$\vec{v}_1-\vec{v}_2=\vec{0}$	25.(2 pt.)_____________________________ . ______ A B C D E
$\vec{v}_1=\vec{v}_2$	Vector algebra in V

Hence T is one-to-one as was to be proved. QED for the theorem.

Possible answers for this page: A) $x_1 = x_2$ B) $x_1 + x_2 = 0$ C) $f(x_1+ x_2) = 0$ D) $f(x_1) = f(x_2)$
E) $f(x_1) + f(x_2) = 0$ AB) Definition of f AC) Hypothesis (or Given) AD) only the zero vector
AE) The lemma proved above BC) T is a one-to-one mapping BD) T is a linear operator
BE)Definition of T CD)Theorems from Calculus CE)Vector algebra in V DE)Vector algebra in W
ABC)only the vector $\vec{v}_1$ AB $\vec{v}_1=\vec{0}$ $\vec{v}_1=\vec{v}_2$ E) $T(\vec{v}_1)=\vec{0}$ ACD) $T(\vec{v}_2)=\vec{0}$ $T(\vec{v})=\vec{0}$ ADE)
BCD) $T(\vec{v}_1)-T(\vec{v}_2)=\vec{0}$ $\vec{v}_1$ BCE $\vec{v}_1$ α $T(\vec{v}_1)=T(\vec{v}_2)$) BDE $T(\vec{v}_1-\vec{v}_2)$ $T(\vec{v}_1+\vec{v}_2)=\vec{0}$)
ABCDE) None of the above.
Total points this page = 17. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions.

26. (4 pts.) F(s) $= \frac{2}{s^3} \quad \frac{2}{s+2}$ $\mathscr{L}^{-1}\{F\}$ = ______________________________. ________ A B C D E

27. (4 pts.) F(s) $= \frac{3s-12}{s^2+9}$ $\mathscr{L}^{-1}\{F\}$ = ______________________________. _________ A B C D E

28. (4 pts.) F(s) $= \frac{2s-4}{s^2-2s+2}$ $\mathscr{L}^{-1}\{F\}$ = _________________________________. ______ A B C D E

Possible answers this page

A) $t^2 + e^{-2t}$ B) $t^2 - e^{-2t}$ C) $t^2 + 2e^{-2t}$ D) $t^2 - 2e^{-2t}$ E) $t^2 + 3e^{-2t}$ AB) $t^2 - 3e^{-2t}$ AC) $t^2 + 4e^{-2t}$
AD) $t^2 - 4e^{-2t}$ AE) cos 3t + (4/3) sin 3t BC) cos 3t − (4/3) sin 3t AD) 2 cos 3t + (4/3) sin 3t
AE) 2 cos 3t − (4/3) sin 3t BC)2 cos t +(4/3)sin 3 t BC)2 cos 3t −(4/3) sin 3t
AD) 3 cos 3t + 4 sin 3t AE) 3 cos 3t −(4/3) sin 3t BD) 4 cos 3t +(4/3) sin 3t
BE) 4 cos 3t − (4/3) sin 3t CD) e^t cos t + $2e^t$ sin t CE) e^t cos t − $2e^t$ sin t
BD) $2e^t$ cos t + e^t sin t BE) 2 e^t cos t − e^t sin t CD) $3e^t$ cos t CE) $4e^t$ cos t + e^t sin t
DE) $4e^t$ cos t − e^t sin t ACDE) $\mathscr{L}^{-1}\{f\}$ exists but none of the above is $\mathscr{L}\{f\}$
BCDE) $\mathscr{L}^{-1}\{f\}$ does not exist. ABCDE)None of the above.

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Consider the IVP: ODE $y'' + 2y' = 0$ IC's: $y(0) = 3$, $y'(0) = 2$

Let $Y = \mathscr{L}\{y(t)\}(s)$.

29. (3 pts.) As discussed in class (attendance is mandatory), taking the Laplace transform of the ODE and using the initial conditions we may obtain the equation:

____. ________A B C D E

Be careful, if you miss this question, you will also miss the next question.

30. (3 pts.) The Laplace transform of the solution to the IVP

is Y = ____. _________ A B C D E

Possible answers this page

A) $s^2Y - 3s - 2 + 2(sY - 3) = 0$ B) $s^2Y - 3s - 2 - Y = 0$ C) $s^2Y - 3s - 2 + 2Y = 0$

D) $s^2Y - 3s - 2 - 2Y = 0$ E) $s^2Y - 3s - 2 + 3Y = 0$ AB) $s^2Y - 3s - 2 - 3Y = 0$

AC) $s^2Y - 3s - 2 + 4Y = 0$ AD) $s^2Y - 3s - 2 - 4Y = 0$ AE) $s^2Y - 3s + 4Y = 0$

BC) $s^2Y - 2 + 4Y = 0$ BD) $s^2Y - 3s - 2 - 4Y = 0$ BE) $s^2Y - 3s - 2 - 4Y = 0$

CD) $\frac{3s+2}{s^2+1}$ $\frac{3s+2}{s^2-1}$ $\frac{3s+2}{s^2+2s}$ DE) $\frac{3s+2}{s^2-2s}$ A $\frac{3s-8}{s^2+2s}$ $\frac{3s+8}{s^2+2s}$ ABD)

ACD) $\frac{3s+2}{s^2+4}$ $\frac{3s+2}{s^2-4}$ $\frac{3s}{s^2+4}$ ADE) $\frac{2}{s^2+4}$ BCD) ABCDE) None of

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ____________

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

31. (4 pts.) Let S = $\{\vec{v}_1, \vec{v}_2, \dots, \vec{v}_n\}$ $\subseteq$V where V is a vector space and (*) be the vector equation $c_1\vec{v}_1+c_2\vec{v}_2+\dots+c_n\vec{v}_n=\vec{0}$. Choose the correct completion of the following:

Definition. The set S is linearly independent

if ____. ________A B C D E

A) (*) has only the solution $c_1 = c_2 = \cdots = c_n = 0$. B) (*) has an infinite number of solutions.
C) (*) has a solution other than the trivial solution. D) (*) has at least two solutions.
E) (*) has no solution. AB) the associated matrix is nonsingular.
AC) the associated matrix is singular. AD) None of the above

32. (4 pts.)Now let S_1 ={ $[x_1(t), y_1(t), z_1(t)]^T$, $[x_2(t), y_2(t), z_2(t)]^T$, ... , $[x_n(t), y_n(t), z_n(t)]^T$ } $\subseteq \vec{\mathscr{A}}(\mathbf{R},\mathbf{R}^3)$ and (**) be the “vector” equation

$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T \quad \forall t \in \mathbf{R}$

Apply the definition above to the space of time varying "vectors" $\vec{\mathscr{A}}(\mathbf{R},\mathbf{R}^3)$. That is, by the definition above the set $S_1 \subseteq \vec{\mathscr{A}}(\mathbf{R},\mathbf{R}^3)$ is linearly independent

if ____. _________ A B C D E

A) (**) has only the solution $c_1 = c_2 = \cdots = c_n = 0$. B) (**) has an infinite number of solutions
C) (**) has a solution other than the trivial solution. D) (**) has at least two solutions.
E) (**) has no solution. AB) the associated matrix is nonsingular.
AC) the associated matrix is singular. AD) None of the above

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

33. (4 pts.) You are to determine Directly Using the Definition (DUD) if the following set of time varying "vectors" are linearly independent. Let S = $\{\vec{x}_1(t),\vec{x}_2(t)\}$ $\mathscr{A}(\mathbf{R},\mathbf{R}^2)$ $\subseteq$ whe

$\vec{x}_1(t)=\begin{bmatrix}3e^t\\4e^t\end{bmatrix}$ $\vec{x}_2(t)=\begin{bmatrix}6e^t\\9e^{-t}\end{bmatrix}$ $\vec{0}=\begin{bmatrix}0\\0\end{bmatrix}$, and $\forall\, t \in \mathbf{R}$. Then S

is ____. ________ A B C D E

A) linearly independent as c_1 $\vec{x}_1(t)$ $\vec{x}_2(t)$ $= [0,0]^T$ $\forall t\in\mathbf{R}$ implies $c_1 = c_2 = 0$.

B) linearly independent as $-2\,\vec{x}_1(t)$ $\vec{x}_2(t)$ $= [0,0]^T$ $\forall t\in\mathbf{R}$.

C) linearly independent as the associated matrix is nonsingular

D) linearly independent as the associated matrix is singular.

E) linearly dependent as $c_1\vec{x}_1(t)$ $\vec{x}_2(t)$ $= [0,0]^T$ $\forall t\in\mathbf{R}$ implies $c_1 = c_2 = 0$.

AB) linearly dependent as $-2\,\vec{x}_1(t)$ $\vec{x}_2(t)$ $= [0,0]^T$ $\forall t\in\mathbf{R}$.

AC) linearly dependent as the associated matrix is nonsingular

AD) linearly dependent as the associated matrix is singular.

AE) neither linearly independent or linearly dependent as the definition does not apply.

ABCDE) None of the above statements are true.

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Using the procedure illustrated in class (attendance is mandatory), find the eigenvalues of

$A = \begin{bmatrix} i & 4 \\ 0 & 3 \end{bmatrix} \in \mathbf{C}^{2x2}$.

34. (4 pts.)) Let $p(\lambda) = \det(A-\lambda I)$. Then $p(\lambda)$ can be factored to obtain

$p(\lambda)$ =____. _______ A B C D E

A) $(i+\lambda)(1+\lambda)$ B) $(i+\lambda)(1-\lambda)$ C) $(i-\lambda)(1+\lambda)$ D) $(i-\lambda)(1-\lambda)$ E) $(i+\lambda)(2+\lambda)$ AB) $(i-\lambda)(2-\lambda)$
AC) $(i-\lambda)(3+\lambda)$ AD) $(i-\lambda)(3-\lambda)$ AE) $(i-\lambda)(4+\lambda)$ BC) $(i-\lambda)(4-\lambda)$ BD) $(2i-\lambda)(1+\lambda)$
BE) $(2i-\lambda)(1-\lambda)$ CD) $(2i+\lambda)(2+\lambda)$ CE) $(2i+\lambda)(2-\lambda)$ DE) $(2i-\lambda)(2+\lambda)$ ABC) $(2i-\lambda)(2-\lambda)$
ABD) $(3i-\lambda)(2+\lambda)$ ABCDE) None of the above.

35. (2 pt.) The degree of $p(\lambda)$ is ___________. ______ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 ABCDE) None of the above.

36. (2 pt.) Counting repeated roots, the number of eigenvalues of A

is _________. ______ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 ABCDE) None of the above

37. (4 pts.) The eigenvalues of A can be written as __ ____________________.________ A B C D E

A) $\lambda_1 = 1, \lambda_2 = i$ B) $\lambda_1 = 1, \lambda_2 = -i$ C) $\lambda_1 = -1, \lambda_2 = i$ D) $\lambda_1 = -1, \lambda_2 = -i$ E) $\lambda_1 = 2, \lambda_2 = i$
AB) $\lambda_1 = 2, \lambda_2 = -i$ AC) $\lambda_1 = 3, \lambda_2 = i$ AD) $\lambda_1 = 4, \lambda_2 = i$ AE) $\lambda_1 = 1, \lambda_2 = 2i$
BC) $\lambda_1 = 1, \lambda_2 = -2i$ BD) $\lambda_1 = -1, \lambda_2 = 2i$ BE) $\lambda_1 = -1, \lambda_2 = -2i$ CD) $\lambda_1 = 2, \lambda_2 = 2i$
CE) $\lambda_1 = 2, \lambda_2 = -2i$ DE) $\lambda_1 = -2, \lambda_2 = 2i$ ABC) $\lambda_1 = -2, \lambda_2 = -2i$ ABCDE) None of the above

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Note that $\lambda_1 = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 2 & 0 \\ 2 & 3 \end{bmatrix}$

38. (4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with λ_1 is B = ________________________. ______A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[1,-1]^T\}$
AB) $\{[1,-2]^T\}$ AC) $\{[1,-3]^T\}$ AD) $\{[1,-4]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$
BD) $\{[2,-1]^T\}$ BE) $\{[3,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$
ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$
ACD) $\lambda = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

39. (2pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, the eigenspace associated with λ_1 is often one dimensional. Hence conventions for selecting eigenvector(s) associated with λ_1 have been developed (by engineers). We say that the eigenvector(s) associated with λ_1

is (are) ____. ______A B C D E

A) $[1,1]^T, [4,4]^T$ B) $[1,1]^T$ C) $\{[1,2]^T\}$ D) $[1,2]^T, [4,8]^T$ E) $[2,1]^T$
AB) $[1,-1]^T$ AC) $[1,-2]^T$ AD) $[1,-3]^T$ AE) $[1,-4]^T$ BC) $[1,-1]^T, [4,4]^T$
BD) $[2,-1]^T$ BE) $[3,-2]^T$ CD) $[1,-2]^T, [4,8]^T$ CE) $[2,1]^T$ DE) $[1,3]^T$
ABC) $[1,-4]^T$ ABD) $[4,-1]^T$ ABE) $[3,-1]^T$
ACD) $\lambda_1 = 2$ is not an eigenvalue of the matrix A
ACE) $\lambda_1 = -1$ is not an eigenvalue of the matrix A
ADE) $\lambda = 3$ is not an eigenvalue of the matrix A ABCDE) None of the above is correct.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

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Answer questions using the instructions on the Exam Cover Sheet. Also, circle your answer.

Let the 2x2 matrix A have the eigenvalue table

Let L:$\mathscr{A}(\mathbf{R},\mathbf{R}^2)\to\mathscr{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}]=\vec{x}'-A\vec{x}$

and let the null space of L be N_L

TABLE

Eigenvalues	Eigenvectors
	$\vec{\xi}_1=\begin{bmatrix}1\\2\end{bmatrix}$
$r_2 = 2$	$\vec{\xi}_2=\begin{bmatrix}2\\1\end{bmatrix}$

40. (2 pt). The dimension of N_L is _______________________________. ___________ A B C D E

A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 ABCDE) None of the above.

41. (3 pts.) A basis for the null space of L is B = _________________________ . ___________ A B C D E

A) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}1\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}1\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ D)

E) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}3\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}3\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{t},\begin{bmatrix}4\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}4\\1\end{bmatrix}e^{2t}\right\}$

AE) $B=\left\{\begin{bmatrix}1\\2\end{bmatrix}e^{-t},\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}\right\}$ $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$ $B=\left\{\begin{bmatrix}1\\-2\end{bmatrix}e^{-t},\begin{bmatrix}2\\1\end{bmatrix}e^{2t}\right\}$

ABCDE) None of the above

42. (2 pts.) The general solution of $\vec{x}'=A\vec{x}$ $\vec{x}(t)$ is = ___________________________. _______ A B C D E

A) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}1\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}1\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$

E) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}3\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}3\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{t}+c_2\begin{bmatrix}4\\1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}4\\1\end{bmatrix}e^{2t}$

AE) $\vec{x}(t)=c_1\begin{bmatrix}1\\2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\-1\end{bmatrix}e^{2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{-2t}$ $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ BD)

BE) $\vec{x}(t)=c_1\begin{bmatrix}1\\-2\end{bmatrix}e^{-t}+c_2\begin{bmatrix}2\\1\end{bmatrix}e^{2t}$ ABCDE)None of the above

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _____________

PRINT NAME ___________________________(___________________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	$s > \lvert a \rvert$
cosh (at)	$\frac{s}{s^2 - a^2}$	$s > \lvert a \rvert$
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
u(t)	$\frac{1}{s}$	$s > 0$
u(t - c)	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	F(s - c)	
f(ct) c > 0	$\frac{1}{c} F(\frac{s}{c})$	
$\delta(t)$	1	
$\delta(t - c)$	e^{-cs}	