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| Last Name, | First Name |

ID \# $\qquad$

## EXAM DATE

$\qquad$ Scores
I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

## SIGNATURE

DATE
INSTRUCTIONS: Besides this cover page, there are 12 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL THE PAGES. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. NO CALCULATORS! NO SCRATCH PAPER! Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-12 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, particularly for regrades you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. SHOW YOUR WORK! Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

## REQUEST FOR REGRADE

Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page $\qquad$ .)
(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have written nothing on this exam except on this REGRADE FORM.

| page | points | score |
| :---: | :---: | :---: |
| 1 | 10 |  |
| 2 | 12 |  |
| 3 | 12 |  |
| 4 | 8 |  |
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| 22 |  |  |
| Total | 100 |  | (Writing or changing anything is considered to be cheating.)

Date
Signature
$\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI, What you wish to be called
For questions 1 and 2 follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Questions 3-10 are True/False.

As discussed in class, classify the following ODEs as to their order $\left(1^{\text {st }}, 2^{\text {nd }}, 3^{\text {rd }}, \ldots, \mathrm{n}^{\text {th }}\right)$

1. (1 pt.) The order of the ODE $y^{\prime \prime}+2 x^{5}\left(y^{\prime}\right)^{4}=\cos x$ is $\qquad$ . $\qquad$ A B C D E
2. (1 pt.) The order of the ODE $y^{\text {viI }}+e^{3 x} y^{\prime \prime}=\tan x$ is $\qquad$ . $\qquad$ A B C D E Possible answers for questions 1 and 2.
A) 1
B) 2
C) 3
D) 4
E) 5
AB) 6
AC) 7 AD) 8
AE) None of the above

True or False Circle True or False, but not both. If I cannot read your answer, it is wrong.
3. (1 pt.) A) True or B)False The ODE $y^{\prime \prime \prime}+2 x^{5} y y^{\prime \prime}=\cos x$ is linear ( y as a function of x ).
4. (1 pt.) A) True or B)False The ODE $y^{v 1}+e^{3 x} y^{\prime \prime}=\tan x$ is linear ( $y$ as a function of $x$ ).
5. (1 pt.) A)True or B)False It is not the case that there are exactly two functions that satisfies the ODE $y^{\prime}+\mathrm{xy}=0$.
6. (1 pt.) A)True or B)False To solve the ODE $y^{\prime}+p(x) y=g(x)$ where $p(x)$ and $g(x)$ are continuous $\forall \mathbf{x} \in \mathbf{R}$, one uses an integrating factor given by $\mu=\int p(x) d x$
7. (1 pt.) A)True or B)False When solving the ODE, $y^{\prime}+p(x) y=g(x)$, where $p(x)$ and $g(x) \in A(R, R)$, one is always able to solve for $y$ explicitly as a function of x .
8. (1 pt.) A)True or B)False A direction field helps in obtaining qualitative information for the IVP: $y^{\prime}=f(x, y), y(0)=y_{0}$, even if the solution cannot be obtained in terms of elementary functions.
9. (1 pt.) A)True or B)False There exist techniques to find integrating factors that will convert some first order ODEs which are not exact to ones that are exact.
10. (1 pt.) A)True or B)False The brothers Jakob and Johann Bernoulli did much to develop methods of solving differential equations and to extend the range of their applications.
$\qquad$

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True or False. For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear (y as a function of $x$ ).- Integrating factor $=\mu=\exp \left(\int p(x) d x\right)$
2) First order linear ( $x$ as a function of $y$ ). - Integrating factor $=\mu=\exp \left(\int p(y) d y\right)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear ( $y$ as a function of $x$ ). Use the substitution $v=y^{1-n}$.
6) Bernoulli, but not linear ( $x$ as a function of $y$ ). Use the substitution $v=x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v=y / x$ or $v=x / y$.
8) None of the above techniques works.

Also recall the following discussed in class (Attendance is mandatory):
a. In this context, exact means exact as given in either of the forms discussed in class.
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong. DO NOT SOLVE.
(\#) $\left(3 x^{2} y+2 x y\right) d x+\left(x^{3}+x^{2}\right) d y=0$
11. (2 pts.) A)True or B)False
$(\#)$ is a linear ode ( $y$ as a function of $x$ ).
12. (2 pts.) A)True or B)False
(\#) is an exact ode.
13. (2 pts.) A)True or B)False
(\#) is a separable ode
(*) $\quad\left(x^{2}+2 x y\right) d x+x^{2} d y=0$
14.(2 pts.) A)True or B)False $\quad\left(^{*}\right)$ is a linear ode ( y as a function of x ).
15. (2 pts.) A)True or B)False $\quad\left(^{*}\right)$ is an exact ode
16. (2 pts.) A)True or B)False $\quad\left(^{*}\right)$ is a separable ode
$\qquad$

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True or False. For the given first order ODEs, determine if the statements below are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear (y as a function of $x$ ).- Integrating factor $=\mu=\exp \left(\int p(x) d x\right.$ )
2) First order linear ( $x$ as a function of $y$ ). - Integrating factor $=\mu=\exp \left(\int p(y) d y\right)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear ( $y$ as a function of $x$ ). Use the substitution $v=y^{1-n}$.
6) Bernoulli, but not linear ( $x$ as a function of $y$ ). Use the substitution $v=x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v=y / x$ or $v=x / y$.
8) None of the above techniques works.

Also recall the following discussed in class (Attendance is mandatory):
a. In this context, exact means exact as given in either of the forms discussed in class.
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous is not a correct method of solution if the originalequation is separable.

Circle True or False, but not both. If I cannot read your answer, it is wrong.

$$
\left(2 y^{3}+x^{2} y\right) d x+3 x^{3} d y=0
$$

17. (2 pts.) A)True or B)False (\#) is a linear ode (y as a function of $x$ ).
18. (2 pts.) A)True or B)False (\#) is a Bernoulli ode (y as a function of $x$ ).
19. (2 pts.) A)True or B)False (\#) is a homogeneous ode
(*) $\quad(4 x+y) d x+(x+3 y) d y=0$
20. (2 pts.) A)True or B)False . $\left(^{*}\right.$ ) is a linear ode ( y as a function of x ).
21. (2 pts.) A)True or B)False .(*) is a separable ode
22. (2 pts.) A)True or B)False $\quad\left(^{*}\right)$ is an exact ode.
$\qquad$

PRINT NAME $\qquad$ ( $\qquad$ ) ID No. $\qquad$
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer. Be careful. No part credit. If you miss one part, it may cause you to miss other parts.

An ODE may be considered to be a vector equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. Sometimes we can solve an ODE by isolating the unknown function (dependent variable). This isolation solves for all of the (infinite number of) unknowns simultaneously. On the back of the previous sheet you are to obtain a partial solution to the ODE $\mathrm{xy}^{\prime}=-\mathrm{y}+\sin (\mathrm{x})$ which we call $(*)$. Then answer the questions below.
23. ( 1 pts .) To solve $\left(^{*}\right)$, you may need to change $\left(^{*}\right)$ to a standard form. The correct standard form
for solving $(*)$ is $\qquad$ . A B C D E
A) $x y^{\prime}+y=x^{-2} \sin (x) \quad$ B) $x y^{\prime}-y=x^{-2} \sin (x) \quad$ C) $x y^{\prime}+2 y+x^{-3} \sin (x) \quad$ D) $x y^{\prime}-2 y=x^{-3} \sin (x)$
E) $\left.\left.x y^{\prime}+3 y=x^{-4} \sin (x) A B\right) x y^{\prime}-3 y=x^{-4} \sin (x) A C\right) x y^{\prime}+4 y=x^{-5} \sin (x)$

AD) $\left.\left.\left.x y^{\prime}-4 y=x^{-5} \sin (x) A E\right) y^{\prime}+y / x=x^{-1} \sin (x) B C\right) y^{\prime}-y / x=x^{-1} \sin (x) B D\right) y^{\prime}+2 y / x=x^{-2} \sin (x)$
BE) $\left.\left.\left.y^{\prime}-2 y / x=x^{-2} \sin (x) C D\right) y^{\prime}+3 y / x=x^{-3} \sin (x) \quad C E\right) y^{\prime}-3 y / x=x^{-3} \sin (x) D E\right) y^{\prime}+4 y / x=x^{-4} \sin (x)$
$\left.A B C) y^{\prime}-4 y / x=x x^{4} \sin (x) \quad A B C D E\right)$ None of the above
24. ( 3 pts.) An integrating factor for $(*)$ is $\mu=$ $\qquad$ . A B CDE
A) $x^{-1}$
B) $-x^{-1}$ C) $x \quad$ D) $-x$
E) $2 x$
$\left.\left.-2 x \quad A C) x^{2} A D\right)-x^{2} A E\right) x^{3} B$
(C) $-x^{3}$ BD) $x^{4}$
BE) $-x^{4}$ CD) $e^{x}$ CE) $e^{-x}$ ABCDE) None of the above
25. ( 4 pts.) In solving $\left({ }^{*}\right)$ as we did in class (attendance is mandatory), the following step occurs:
A) $\left.\left.d(y / x) / d x=\sin (x) \quad B) d\left(y / x^{2}\right) / d x=\sin (x) \quad C\right) d\left(y / x^{3}\right) / d x=\sin (x) \quad D\right) d\left(y / x^{4}\right) / d x=\sin (x)$ E) $\left.\left.d(x y) / d x=\sin (x) A B) d\left(x^{2} y\right) / d x=\sin (x) A C\right) d\left(x^{3} y\right) / d x=\sin (x) \quad A D\right) d\left(x^{4} y\right) / d x=\sin (x)$

AE) $\left.\left.d(y / x) / d x=\left[\begin{array}{lll}\sin (x)\end{array}\right] / x \quad B C\right) d(y / x) / d x=-[\sin (x)] / x \quad B D\right) d(y / x) / d x=x \sin (x)$
$B E) d(y / x) / d x=-x \sin (x) \quad C D) d(y / x) / d x=\sin (x) \quad C E) d(y / x) / d x=-\sin (x)$
ABCDE)None of the above steps ever appears in any solution of (*).
$\qquad$
$\qquad$ ( $\qquad$ ) ID No. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also circle the correct answer. Be careful. If you miss one part, it may cause you to miss other parts. This problem is a continuation of the problem on the previous page, but with different functions. An ODE may be considered to be a vector equation with the infinite number of unknowns being the values of the function for each value of the independent variable in the function's domain. Sometimes we can solve an ODE by isolating the unknown function (dependent variable). This isolation solves for all of the (infinite number of) unknowns simultaneously. Let (*) be an ODE of the form $\mathrm{L}[\mathrm{y}]=\mathrm{g}(\mathrm{x})$ where L is of the form $L[y]=y^{\prime}+p(x) y$. In solving $\left(^{*}\right)$, the following step was reached: $\frac{d\left(y e^{x}\right)}{d x}=-x e^{x}$
. We call this ODE (
On the back of the previous sheet, solve $\left(^{*}\right)$ and $\left({ }^{* *}\right)$ and answer the following questions. 26. ( 2 pts.) The theorem from calculus that allows you to integrate the left hand side of (**)
is $\qquad$ . $\qquad$ A B C D E
A) Intermediate Value Theorem B) Mean Value Theorem C) Rolle's Theorem
D) Product Rule
E) Fundamental Theorem of Calculus

AB) Chain Rule
AC) Integration by Parts AD) Partial Fractions ABCDE)None of the above.
27. (5 pts.) The solution (or family of solutions) to the $\operatorname{ODE}\left({ }^{*}\right)$ may be written
as $y=$ $\qquad$ . A B C D E
$\begin{array}{lllll}\text { A) } x+1+c e^{x} & \text { B) } x-1+c e^{x} & \text { C) }-x+1+e^{-x} & \text { D) } x-1+c^{-x} & \text { E) } x / 2+1 / 4+c e^{2 x} \\ \text { AB) } x / 2-1 / 4\end{array} \bar{c} e^{2 x}$ $\begin{array}{lll}\text { AC) } x / 2+1 / 4+\mathrm{ce}^{-2 x} & \text { AD) } \mathrm{x} / 2-1 / 4+\mathrm{ce}^{-2 x} & \text { AE) } \mathrm{x} / 3+1 / 9+\mathrm{ce}^{3 x} \\ \text { BC) } \mathrm{x} / 3-1 / 9+\mathrm{ce}^{3 x}\end{array}$ D) $x / 3+1 / 9+\mathrm{ce}^{-3 x}$ BE) $x / 3-1 / 9+\mathrm{ce}^{-3 x} \quad$ CD) $\mathrm{x} / 4+1 /(16)+\mathrm{ce}^{4 \mathrm{x}}$ CE) $\mathrm{x} / 4-1 /(16)+\mathrm{ce}^{4 \mathrm{x}}$ DE) $\left.x / 4+1 /(16)+\mathrm{ce}^{-4 \mathrm{x}} \mathrm{ABC}\right) \mathrm{x} / 4-1 /(16)+\mathrm{ce}^{-4 \mathrm{x}} \quad$ ABCDE)None of the above is correct.
$\qquad$
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$\qquad$ ) ID No. $\qquad$
Last Name, First Name MI, What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions 28. (4 pts.) Suppose that the general solution of the ODE $y^{\prime}=f(x, y)$ is $y=2+c \cos x$

Solve the IVP. ODE $y^{\prime}=f(x, y) \quad$ IC $\quad y(0)=2$
At $x=\pi$, the value of the function you found as the solution to the IVP is

$$
{ }^{\mathrm{y}} \mid \mathrm{x}=\pi^{=}
$$

$\qquad$
$\qquad$
29. (5 pts.) Solve the IVP ODE $d y / d x=x / y$ IC $y(0)=2$

At $x=2$, the value of the function you found as the solution to the IVP is

$$
\left.y\right|_{x=2}=\quad \ldots \text { A B C D E }
$$

Possible answers this page.
A) 0
B) 1 C) 2
D) 3 E) 4 AB)
AC) 6 AD) 7 AE) 8 BC) 9 BD$)-1$ BE) -2
CD) $-3 \mathrm{CE})-4 \quad \mathrm{DE})-5 \quad \mathrm{ABC})-6 \quad \mathrm{ABD})-7 \quad \mathrm{ABE})-8 \quad \mathrm{BCD})-9 \quad \mathrm{BCE}) \pi / 2 \quad \mathrm{BDE}) \pi / 3$
$\left.\begin{array}{llllll}\mathrm{CDE}) & \pi / 4 & \mathrm{ABCD}) & \pi & \mathrm{ABCE}) & 3 \pi / 2\end{array} \mathrm{ABDE}\right) \sqrt{2} \quad \mathrm{ACL} \sqrt{5} \quad \mathrm{~B} 2 \sqrt{2}$ )
ABCDE ) None of the above.
Possible points this page $=9$. POINTS EARNED THIS PAGE $=$ $\qquad$
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE: $\quad\left(2 x-y^{2}\right) d x+(-2 x y+2 y) d y=0$, call this ODE $(*)$.
30. (1 pt.) A) True or B)False The ODE (*) is exact.
31. (2 pts.) A potential function for $\left({ }^{*}\right)$ is $\qquad$ . $\qquad$ ABCDE
32. (2 pts.) The solution of $\left(^{*}\right)$ may be written implicitly as
as $\qquad$ . $\qquad$ ABCDE
Be careful with your computations as there will be no part credit for an incorrect answer.

Possible answers this page
A) $\psi(x, y)=x^{2}+x y^{2}+y^{2}$
B) $\psi(x, y)=x^{2}+x y^{2}+y^{2}+c$
C) $\psi(x, y)=x^{2}-x y^{2}+y^{2}$
D) $\psi(x, y)=x^{2}-x y^{2}+2 y^{2}+c$
E)A) $\psi(x, y)=x^{2}+x y^{2}+3 y^{2}+c$
AB) $\psi(x, y)=x^{2}+2 x y^{2}+4 y^{2}+c$
AC) $\psi(x, y)=x^{2}-2 x y^{2}+y^{2}$
AD) $\psi(x, y)=x^{2}-2 x y^{2}+y^{2}+c$
AE) $x^{2}+x y^{2}+y^{2}=c$
BC) $x^{2}-2 x y^{2}+y^{2}=c$
BD) $x^{2}+2 x^{2} y^{2}+y^{2}=c$
BE) $x^{2}-2 x^{2} y^{2}+2 y^{2}=c$
CD) $x^{2}-x y^{2}+y^{2}=c$
CE) $x^{2}-x y^{2}+2 y^{2}=c$
DE) $x^{2}-x y^{2}+3 y^{2}=c$

ABC) $x^{2}-x^{2} y^{2}+4 y^{2}=c \quad A B D$ ) No technique that we have learned can be used to solve this ODE. ABCDE) None of the above.
Possible points this page $=5$.
TOTAL POINTS EARNED $\qquad$

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer.

Consider the ODE $d y / d x=e^{y / x}+(y / x)+1$. Call this ODE $\left(^{*}\right)$. On the back of the previous sheet provide a particle solution to $\left(^{*}\right)$ and answer the questions below.
33. ( 1 pt ). The appropriate classification for $\left(^{*}\right.$ ) is $\qquad$ . $\qquad$ ABCDE
A) First order linear (y as a function of $x$ ), B) First order linear ( $x$ as a function of $y$ )
C) Bernoulli ( y as a function of x ) D) Bernoulli ( x as a function of y ).
E) Homogeneous ABCDE ) None of the above techniques works.
34. (2 pts.) An appropriate substitution (change of variable) to convert (*) to a new solvable

ODE, call it $\left({ }^{* *}\right)$, is $\mathrm{v}=$ $\qquad$ . $\qquad$ A B C DE
A) $1 / y$
B) $1 / y^{2}$
C) $1 / y^{3}$
D) $y^{2}$
E) $y^{3}$ AB) $\sqrt{y}$
AC) $y / x$
$\mathrm{ABCDE})$ None of the above.
35. (2 pts.) We have dy/dx = $\qquad$ . $\qquad$ A B C D E
A) $\frac{1}{2} v^{-\frac{1}{2}} \frac{d v}{d x}$
$-\frac{1}{2} \mathrm{v}^{-\frac{1}{2}} \frac{\mathrm{dv}}{\mathrm{dx}} \quad \frac{1}{2} \mathrm{v}^{-\frac{3}{2}} \frac{\mathrm{dv}}{\mathrm{dx}}$
$\mathrm{v}+\mathrm{x} \frac{\mathrm{dv}}{\mathrm{dx}}$
$-v+x \frac{d v}{d x}$
$v-x \frac{d v}{d x}$
E)
AC) $-v-x \frac{d v}{d x} \quad-v+x^{2} \frac{d v}{d x}$
ABCDE) None of the above.
36. (3 pts.) The new $\operatorname{ODE}\left({ }^{* *}\right)$ that is derived may be written as
A) $x \frac{d v}{d x}=e^{v} \frac{d v}{d x}$
$\left.=-\frac{d v}{d x} C\right) x \quad=e^{v} \frac{d v}{d x} v$
D) $x$
. $\qquad$ ABCDE
$\left.A B) x \frac{d v}{d x}=e^{v}+2 A\left(\frac{d v}{d x} \quad=e^{v}+3 \frac{d v}{d x}\right)\right) x \quad=e^{v}+4$ ABCDE $)$ None of the above.
37. ( 2 pts .) The correct classification of the new $\operatorname{ODE}\left({ }^{* *}\right)$ that you derived
is (do not solve this equation.) $\qquad$ . $\qquad$ A B C D E
A) First order linear ( $v$ as a function of $x$ ), B) First order linear ( $x$ as a function of $v$ )
C) Separable. D) Exact ABCDE) None of the above.
$\qquad$

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answers.

Suppose that the ODE $d y / d x=f(x, y)$, call it $\left({ }^{*}\right)$, is not linear, separable, or exact, but that it can be solved using the substitution (change of variable), $v=y^{-2}$. Suppose further that this substitution results in the derived ODE $-(1 / 2) \mathrm{v}^{-(3 / 2)}(\mathrm{dv} / \mathrm{dx})+2 \mathrm{v}^{-(1 / 2)}=\left(\mathrm{v}^{-(1 / 2)}\right)^{3}$. Call this ODE $(* *)$. On the back of the previous sheet, solve $\left({ }^{* *}\right)$ and then $\left({ }^{*}\right)$ and then answer the following questions.
38. (3 pts.) ${ }^{* *}$ ) may be rewritten as $\qquad$ . $\qquad$ A B C D E
A) $d v / d x+v=x \quad B) d v / d x+v=-x \quad C) d v / d x-v=x \quad D) d v / d x-2 v=-2 \quad E) d v / d x-2 v=-4$
$A B) d v / d x-4 v=-2 \quad A C) d v / d x-4 v=-2 \quad A D) d v / d x-4 v=-4 x \quad$ ABCDE $)$ None of the above.
39. (1 pts.) A correct classification of $\left({ }^{* *}\right)$ is $\qquad$ . $\qquad$ A B C D E
A) Linear ( $v$ as a function of $x$ )
B) Linear ( $x$ as a function of $v$ )
C) Bernoulli
D) Exact
E) Homogeneous
ABCDE) None of the above.
40. ( 5 pts .) The solution of $\left({ }^{* *}\right)$ may be written as $\mathrm{v}=$ $\qquad$ . $\qquad$ A B C D E
A) $x+(1 / 2)+c e^{4 x}$
B) $x-(1 / 2)+c e^{-4 x}$
C) $-x+(1 / 2)+c e^{4 x}$
D) $\frac{1}{2}+c e^{4=}$
E) $1+\mathrm{ce}^{4 x}$ AB) $2+\mathrm{ce}^{-4 \mathrm{x}}$ AC) $3+\mathrm{ce}^{-4 \mathrm{x}}$
$\left.\left.\mathrm{AD}) 4+\mathrm{ce}^{-4 \mathrm{x}} \quad \mathrm{AE}\right)-\mathrm{x}-(1 / 2)+\mathrm{ce}^{-4 x} \quad \mathrm{ABCDE}\right)$ None of the above.
41. (3 pts.) The solution set for (*) may be written implicitly
as $\qquad$ . $\qquad$ A B CDE
A) $y^{2}=\left(x+(1 / 2)+c e^{2 x}\right)^{-1}$
B) $y^{2}=\left(x-(1 / 2)+c e^{-2 x}\right)^{-1}$
C) $y^{2}=\left(-x+(1 / 2)+c e^{2 x}\right)^{-1}$
D $y^{2}=\left(\frac{1}{2}+c e^{4 x}\right)^{-1}$
E) $y^{2}=\left(1+c e^{2 x}\right)^{-1}$
$\mathrm{AB}) \mathrm{y}^{2}=\left(2+\mathrm{ce}^{-2 x}\right)^{-1}$
AC) $y^{2}=\left(3+c e^{-2 x}\right)^{-1}$
AD) $y^{2}=\left(4+c e^{-2 x}\right)^{-1}$
AE) $y^{2}=\left(-x-(1 / 2)+c e^{-2 x}\right)^{-1} \quad$ ABCDE $)$ None of the above.
$\qquad$

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer in the list.
42. ( 5 pts.) The direction field for the ODE $y^{\prime}=(3-y) / 2$ is given below. On this direction field are seven curves labeled $1,2,3,4,5,6$, and 7 that were correctly or incorrectly drawn using the direction field. Consider the initial value problem (IVP):

IVP $\begin{array}{cc}\text { ODE } & y^{\prime}=(3-y) / 2 \\ \text { IC } & y(0)=-2\end{array}$
The curve or curves that is the solution to this IVP is $\qquad$ . $\qquad$ ABCDE (Hint: Do not solve the IVP.)
A) 1
B) 2
C) 3
D) 4
E) 5
AB) 6
AC) 7
AD) 3 and 4
AE) 4 and 5
BC) 1,2 , and 3
BD) 2,3 , and 4
BE) 3 , 4 , and 5
CD) $1,2,3,4$, and 5
ABCDE) None of the above


FIGURE 1 Direction field for $y^{\prime}-2 x y=x$.
$\qquad$
$\qquad$ ( $\qquad$ ) ID NO. $\qquad$
Last Name, First Name MI What you wish to be called
Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answers in the lists.

MATHEMATICAL MODELING. As done in class (attendance is mandatory), on the back of the previous sheet, you are to develop a general mathematical model for a Mixing (Tank) problem. Let T be a tank which initially has $\mathrm{S}_{0}$ lbs of salt dissolved in $\mathrm{W}_{0}$ gallons of water. Suppose brine at a concentration of $\mathrm{C}_{0}$ lbs of salt per gallon is entering the tank at the rate of $\mathrm{r}_{0}$ gal. $/ \mathrm{min}$. and the well stirred mixture leaves the tank at the same rate.
43. $(2 \mathrm{pt})$ The fundamental physical law used to develop the ODE in the model
is $\qquad$ .

A B C D E
A)Conservation of mass B)Conservation of energy C)Conservation of time D)Ohm's law
E)Newton's second law (Conservation of momentum) AB)Bernoulli's Law

AC)Kirchoff's voltage law AE)Kirchoff's current law ABCDE)None of the above.
44. (1 pt.) Recall that there are two state variables for this system: 1. W, the water in the tank and 2. S, the salt in the tank. Since the well stirred mixture flows out at the same rate, the amount of water W
in the tank is $\mathrm{W}=$ $\qquad$ A B C D E
A) T
B) $\mathrm{S}_{0} \quad$ C) $\mathrm{W}_{0}$
D) $\mathrm{W}_{0}+\mathrm{t}$
E) $\mathrm{W}_{0}+\mathrm{t}^{2}$
AB) $\mathrm{C}_{0}$ AC) $\mathrm{r}_{0} \quad$ ABCDE)None of the above.
45. ( 3 pts .) This allows for a reduced mathematical model as developed in class (attendance is mandatory). The reduced model for this system is $-\frac{\mathbf{d S}}{\mathbf{d t}}=\mathbf{r}_{0} \mathbf{c}_{0}-\mathbf{r}_{0} \frac{\mathbf{S}}{\mathbf{W}_{0}} \quad \mathbf{S}(0)=\mathbf{S}_{0}$
A) $\left.\frac{\mathrm{dw}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}-\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}+\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}} \mathrm{D}\right)$

AB) $\frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}-\mathrm{r}_{0} \mathrm{~S} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}-\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}}, \mathrm{~S}(0)=\mathrm{S}_{0} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}, \mathrm{~S}(0)=\mathrm{S}_{0}$
AE) $\frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}}, \mathrm{~S}(0)=\mathrm{S}_{0} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}+\mathrm{r}_{0} \frac{\mathrm{~S}}{\mathrm{~W}_{0}}, \mathrm{~S}(0)=\mathrm{S}_{0} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\mathrm{r}_{0} \mathrm{c}_{0}-\mathrm{r}_{0} \mathrm{~S}, \mathrm{~S}(0)=\mathrm{S}_{0}$
ABCDE)None of the above.
46. (1 pt.) The units for the ODE in the model you selected above
are $\qquad$ . $\qquad$ A B C D E
A) Feet
B) Seconds
C) Feet per second
D) Feet per second squared
E) Pounds
$A B$ ) Slugs $\quad A C)$ Slug feet $A D)$ pounds per minute $A B C D E)$ None of the above.
$\qquad$

## PRINT NAME

$\qquad$ ( $\qquad$ ) ID NO. $\qquad$
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. In addition, circle your answer in the list.

MATHEMATICAL MODELING. Let T be a tank which initially has 10 lbs of salt dissolved in 100 gals of water. If brine at a concentration of $1 / 4 \mathrm{lb}$ of salt per gallon is entering the tank at the rate of 3 gals $/ \mathrm{min}$, and the well stirred mixture leaves the tank at the same rate. Determine the amount of salt in the tank after 30 min . What is the maximum amount of salt which accumulates in the tank.

Apply the data given above to the general reduced model you developed on the previous page to obtain a specific reduced model for this problem. DO NOT SOLVE!
47. (3 pts.) The mathematical model for the system whose solution yields the velocity $\mathrm{v}(\mathrm{t})$ as a function of time
is $\qquad$ . $\qquad$ A B C D E
A) $\frac{\mathrm{dw}}{\mathrm{dt}}=\frac{3}{4} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}-3 \frac{\mathrm{~S}}{100} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=3 \frac{\mathrm{~S}}{100} \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}+3 \frac{\mathrm{~S}}{100}$
E)

AB) $\frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}-3 \mathrm{~S}$

$$
\frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}-3 \frac{\mathrm{~S}}{100}, \mathrm{~S}(0)=10
$$

$$
\frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}, \mathrm{~S}(0)=10
$$

$\mathrm{AE}) \frac{\mathrm{dS}}{\mathrm{dt}}=3 \frac{\mathrm{~S}}{100}, \mathrm{~S}(0)=10 \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{4}+3 \frac{\mathrm{~S}}{100}, \mathrm{~S}(0)=10 \quad \frac{\mathrm{dS}}{\mathrm{dt}}=\frac{3}{40}-3 \mathrm{~S}, \mathrm{~S}(0)=10$
ABCDE)None of the above.
$\qquad$

