`EXAM-I MATH 261: Elementary Differential Equations MATH 261
SPRING 2005I EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(______________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, February 4, 2005

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

________________________________ ________________
SIGNATURE DATE

INSTRUCTIONS

1. Besides this cover page, there are 12 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS**! **NO SCRATCH PAPER**! Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-8 are multiple choice. Expect no part credit on these pages. Pages 9-12 are free response. Explain your solutions fully and carefully. Your <u>entire</u> solution will be graded, not just your final answer. **SHOW YOUR WORK**! Every thought you have should be expressed in your best mathematics. Partial credit will be given as deemed appropriate. Proof-read your solutions and check your computations as time allows. **GOOD LUCK!!!!!!!!!!!**

	Scores	
page	points	score
1	10	
2	12	
3	12	
4	7	
5	7	
6	5	
7	12	
8	9	
9	5	
10	14	
11	4	
12	3	
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17		
18		
19		
20		
21		
22		
Total	100	

REQUEST FOR REGRADE

Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing **anything** is considered to be cheating.)

Date _______ Signature________________________________

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(2 pts.) Multiple Choice. As discussed in class, **classify** the following ODEs as to their order ($1^{st}, 2^{nd}, 3^{rd}, ..., n^{th}$). (If I cannot read your answer, it is wrong.)

1. The order of the ODE $y'' + 2x^5 (y')^2 = \cos x$ is A 1, B 2, C 3, D 4, E 5, AB 6, AC 7.

2. The order of the ODE $y^{IV} + e^{3x} y'' = \tan x$ is A 1, B 2, C 3, D 4, E 5, AB 6, AC 7.

(8 pts.) **True or False** Circle True or False, but not both. (If I cannot read your answer, it is **WRONG**.)

True or False 3. The ODE $y''' + 2x^5 (y'')^2 = \cos x$ is nonlinear.

True or False 4. The ODE $y^{VI} + e^{3x} y'' = \tan x$ is nonlinear.

True or False 5. There is exactly one function that satisfies the ODE $y' + x y = 0$.

True or False 6. To solve the ODE $y' + p(x) y = g(x)$ where $p(x)$ and $g(x)$ are continuous $\forall x \in R$, one uses the integrating factor given by $\mu = e^{\int p(x)dx}$.

True or False 7. When solving the ODE, $y' + p(x) y = g(x)$, where $p(x)$ and $g(x)$ are continuous $\forall x \in R$, one can always obtain y explicitly as a function of x.

True or False 8. A direction field is of help in obtaining qualitative information for the IVP: $y' = f(x,y)$, $y(0) = y_0$, even if the solution cannot be obtained in terms of elementary functions.

True or False 9. There do not exist techniques to find integrating factors that will convert some first order ODEs which are not exact to ones that are exact.

True or False 10. The brothers Jakob and Johann Bernoulli did nothing to develop methods of solving differential equations and to extend the range of their applications.

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(12 pts.) **True or False.** For the given first order ODEs, determine if the following statements are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear (y as a function of x).- Integrating factor = $\mu = \exp(\int p(x)\,dx)$
2) First order linear (x as a function of y).- Integrating factor = $\mu = \exp(\int p(y)\,dy)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear (y as a function of x). Use the substitution $v = y^{1-n}$.
6) Bernoulli, but not linear (x as a function of y). Use the substitution $v = x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v = y/x$ or $v = x/y$.
8) None of the above techniques works.

Also recall the following:
a. In this context, exact means exact as given (in either of the forms discussed in class).
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous (use the substitution v=y/x) is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is **WRONG**.

DO NOT SOLVE.

(#) $(x^2 + 2xy)\,dx + x^2\,dy = 0$

True or False 11. (#) is a linear ode (y as a function of x).

True or False 12. (#) is an exact ode (in either of the two forms discussed in class).

True or False 13. (#) is a homogeneous ode and can be solved using the substitution v=y/x.

(*) $(y^3 + x^2 y)\,dx + x^3\,dy = 0$

True or False 14. (*) is a linear ode (y as a function of x).

True or False 15. (*) is a Bernoulli ode (y as a function of x).

True or False 16. (*) is a homogeneous ode and can be solved using the substitution v=y/x.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE _______

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(12 pts.) **True or False.** For the given first order ODEs, determine if the following statements are true or false. The statements relate to possible methods of solution. Recall from class that the possible methods are:

1) First order linear (y as a function of x).- Integrating factor = $\mu = \exp(\int p(x)\, dx)$
2) First order linear (x as a function of y).- Integrating factor = $\mu = \exp(\int p(y)\, dy)$
3) Separable.
4) Exact Equation (Must be exact in one of the two forms discussed in class).
5) Bernoulli, but not linear (y as a function of x). Use the substitution $v = y^{1-n}$.
6) Bernoulli, but not linear (x as a function of y). Use the substitution $v = x^{1-n}$.
7) Homogeneous, but not separable. Use the substitution $v = y/x$ or $v = x/y$.
8) None of the above techniques works.

Also recall the following:

a. In this context, exact means exact as given (in either of the forms discussed in class).
b. Bernoulli is not a correct method of solution if the original equation is linear.
c. Homogeneous (use the substitution $v=y/x$) is not a correct method of solution if the original equation is separable.

Circle True or False, but not both. If I cannot read your answer, it is **WRONG**.

(*) $(3x^2y + 2xy)\, dx + (x^3 + x^2)\, dy = 0$

True or False 17. (*) is a linear ode (y as a function of x).

True or False 18. (*) is a separable ode.

True or False 19. (*) is an exact ode (in either of the two forms discussed in class).

(#) $(4x + y)\, dx + (x + 3y)\, dy = 0$

True or False 20. (#) is an exact ode (in either of the two forms discussed in class).

True or False 21. (#) is a Bernoulli ode (y as a function of x).

True or False 22. (#) is a homogeneous ode and can be solved using the substitution $v=y/x$.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE _______

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23. (1 pts.) To solve the first order linear Ordinary Differential Equation (ODE) **$y' = 2y + \cos(x)$,** we first put it in the standard form (for solving first order linear ODE's) (Circle the correct (first order linear) standard form for this ODE's. Be careful. No part credit for this problem. Hence if you miss this part, it may cause you to miss all parts):

A. $y' = 2y + \cos(x)$ (It's already in the appropriate form for solving a first order linear ODE)

B. $y' + 2y + \cos(x) = 0$, C. $y' - 2y = \cos(x)$, D. $y' + 2y = \cos(x)$

E. $y' - 2y - \cos(x) = 0$ AB. None of the above

24. (3 pts.) To solve the first order linear Ordinary Differential Equation (ODE) given above, we must find an integrating factor. An integrating factor μ for the linear ODE given above is (Circle the correct integrating factor. Be careful. No part credit for this problem):

A. $\mu = 2x$, B. $\mu = \cos(x)$, C. $\mu = e^{-2x}$,

D. $\mu = e^{2x}$ E. $\mu = e^{-\cos(x)}$, AB. $\mu = e^{\cos(x)}$

AC. $\mu = e^{x}$, AD. $\mu = e^{-x}$, AE. None of the above

25. (3 pts.) In solving the linear Ordinary Differential Equation (ODE) given above, which of the following steps occurs (Circle the step that is correct. Be careful. No part credit for this problem):

A. $\dfrac{d(ye^{-2x})}{dx} = e^{-2x}\sin(x)$, B. $\dfrac{d(ye^{-2x})}{dx} = e^{-2x}\cos(x)$, C. $\dfrac{d(ye^{2x})}{dx} = e^{2x}\sin(x)$

D. $\dfrac{d(ye^{2x})}{dx} = e^{2x}\cos(x)$, E. $\dfrac{d(ye^{\cos(x)})}{dx} = x^2 e^{\cos(x)}$, AB. $\dfrac{d(ye^{-2x})}{dx} = xe^{-2x}$,

AC. $\dfrac{d(ye^{2x})}{dx} = x\cos(x)$, AD. None of the above steps ever appears in any solution of this problem.

Possible points this page = 7. TOTAL POINTS EARNED _________

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To solve the first order linear ODE, we isolate the unknown function on the left side of the equation. Recall that an ODE is really a “vector” equation with the infinite number of unknown variables being the values of the function for each value of the independent variable in the function’s domain. The isolation of the dependent variable (or function) solves for all of the (infinite number of) unknowns simultaneously. In solving a particular first order linear ODE, an integrating factor and the product rule were used to reach the following step: $\frac{d(ye^x)}{dx} = xe^x$.

26. (2 pts.) Circle the theorem from calculus that allows you to integrate the **Left Hand Side** of this equation: A. Intermediate Value Theorem, B. Mean Value Theorem C. Rolle's Theorem,

D. Fundamental Theorem of Calculus, E. Chain Rule, AB. Product Rule,

AC. Integration by Parts. AD. Partial Fractions. AE. None of the above.

27. (5 pts.) Now complete the solution process to obtain y as a function of x. Circle the correct solution or family of solutions. A. $y = x + 1 + ce^x$, B. $y = -x + 1 + ce^x$, C. $y = x - 1 + ce^x$,

D. $y = x + 1 + ce^{-x}$, E. $y = -x + 1 + ce^{-x}$, AB. $y = x - 1 + ce^{-x}$, AC. $y = -x - 1 - ce^{-x}$,

AD. $y = x + 1 + e^x + c$, AE. $y = -x + 1 + e^x + c$ BC. $y = x - 1 + e^x + c$ BD. $y = x + 1 + e^{-x} + c$
BE. $y = -x + 1 + e^{-x} + c$ CD. $y = x - 1 + e^{-x} + c$ CE. $y = x + 1 - e^{-x} + c$

DE. None of the above families of solutions is correct.

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28. (5 pts.) Circle the correct solution to the following exact differential equation. Recall that your entire solution will be graded, not just your final answer. Show your work. No credit will be given if you do not explain how you obtained your solution. However, be careful with your computations as there will be **no part credit** for an incorrect answer to this problem.

ODE $(2x + y^2)\,dx + (2xy)\,dy = 0.$

A. $\psi(x,y) = x^2 + xy^2$

B. $\psi(x,y) = x^2 + xy^2 + C$

C. $\psi(x,y) = x^2 + 2xy^2 + C$

D. $x^2 + 2xy^2 = C$

E. $x^2 + xy^2 = C$

AB. None of the above.

Possible points this page = 5. TOTAL POINTS EARNED _________

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29. (2 pt) The ODE $dy/dx = 2xy + 4y^3$ is not linear or separable. Choose the appropriate classification:
A) Exact Equation (Must be exact in one of the two forms discussed in class).
B) Bernoulli, but not linear (y as a function of x). Use the substitution $v = y^{1-n}$.
C) Bernoulli, but not linear (x as a function of y). Use the substitution $v = x^{1-n}$.
D) Homogeneous, but not separable. Use the substitution $v = y/x$ or $v = x/y$.
E) None of the above techniques works.

30. (2 pts.) Circle an appropriate substitution (change of variable) to convert the ODE
$dy/dx = 2xy + 4y^3$ to one that you can solve: A) $v = 1/y$ B) $v = 1/y^2$ C) $v = y^2$
D) $v = y/x$ E) $v = 1/y^3$ AB) $v = y^3$ AC) $v = \sqrt{y}$ AD) None of the above.

31. (3 pts.)Next, circle the correct term for dy/dx in terms of x and v.
A) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{1}{2}}\frac{dv}{dx}$ B) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx}$ C) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dv}{dx}$ D) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}$
E) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dy}{dx}$ AB) $\frac{dy}{dx} = -\frac{1}{2}v^{-\frac{3}{2}}\frac{dy}{dx}$ AC)None of the above.

32. (3 pts.) Next, circle the new ODE that is derived.
A) $\frac{dv}{dx} + 4x\,v = 8$ B) $\frac{dv}{dx} + 2x\,v = 4$ C) $\frac{dv}{dx} + 4x\,v = 8$ D) $\frac{dv}{dx} - 4x\,v = 8$ E) $\frac{dv}{dx} + 4x\,v = -8$
AB) $\frac{dv}{dx} - 4x\,v = -8$ AC) None of the above.

33. (2 pts.) Next, circle the correct classification of the new ODE that you derived.

A) First order linear (v as a function of x), B) First order linear (v as a function of x) C) Separable.
D) Exact Equation E) None of the above.

Possible points this page = 12. TOTAL POINTS EARNED _________

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Suppose that the ODE dy/dx = f(x,y) is not linear or separable, but that it can be solved using the substitution (change of variable), v = y/x. Suppose further that this substitution results in the derived ODE

$$v + x\frac{dv}{dx} = v^2.$$

34. (2 pts.) Circle the correct classification of the new derived ODE.
A) First order linear (v as a function of x), B) First order linear (x as a function of v) C) Separable.
D) Exact Equation E) None of the above.

35. (5 pts.) Circle the solution of the derived ODE .
A) $v = \frac{1}{1-cx}$, B) $v = \frac{x}{1-cx}$, C) $v = \frac{x^2}{1-cx}$, D) $v = \frac{1}{x-c}$, E) $v = \frac{x}{x-c}$, AB) $v = \frac{x^2}{x-c}$, AC)None of the above.

36. (2 pts.)Circle the solution of the original ODE. A) $y = \frac{1}{1-cx}$, B) $y = \frac{x}{1-cx}$, C) $y = \frac{x^2}{1-cx}$,
D) $y = \frac{1}{x-c}$, E) $y = \frac{x}{x-c}$, AB) $y = \frac{x^2}{x-c}$, AC) None of the above.

Possible points this page = 9. TOTAL POINTS EARNED _________

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(4 pts.) The direction field for the ODE $y' = (3-y)/2$ is given below. On this direction field sketch the solution to the IVP given below. (Hint: Do not solve the IVP.)

IVP	ODE	$y' = (3-y)/2$
	IC	$y(0) = 2$

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE __________

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(14 pts.) Solve: $y' = \dfrac{y^2 + 2xy}{x^2}$. Recall the rules concerning implicit solutions discussed in class. Put your final answer in the box, but recall that your entire solution will be graded, not just your final answer. **SHOW YOUR WORK**. No credit will be given if you do not explain how you obtained your final answer. If possible, obtain y as a function of x.

Final Answer

Possible points this page = 14. TOTAL POINTS EARNED _________

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(4 pts.) MATHEMATICAL MODELING. As per our class discussion (attendance is mandatory), develop a general mathematical model for an object of mass m in slugs that is travelling downward in a medium that offers resistance equal to three times the cube of the velocity of the object where the velocity is measured in feet per second. Assume an initial velocity of v_0 in feet per second. Begin by making a ①**list** of the variables and parameters you will use (you may add to the list as you proceed) starting with a mass of m slugs and the magnitude of the acceleration due to gravity g. Assume a coordinate system in which the x-axis points down (i.e. x becomes more positive as you move down) and make a ②**sketch**. Now list any ③**assumptions** you need and ④**physical laws** you wish to use. Now write down an ⑤**Initial Value Problem** (IVP) that provides a general model for this general physical situation; that is the IVP that can be solved to obtain the velocity v of the point particle. **DO NOT SOLVE THE IVP.**

Possible points this page = 4. POINTS EARNED THIS PAGE = ________

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(3 pts.) MATHEMATICAL MODELING. Consider the following applied math problem:

An object (point particle) of mass 10 slugs is dropped from rest at time t = 0 in a medium that offers resistance equal to three times the cube of the velocity of the object where the velocity is measured in feet per second.

Apply the data given above to the general model you developed on the previous page to find the ⑥**specific model** for the problem given above. Also, give the ⑦**units** of the <u>differential equation</u>. **<u>DO NOT SOLVE THE IVP</u>.**

Possible points this page = 3. POINTS EARNED THIS PAGE = ________