EXAM 4 -A1 FALL 2009

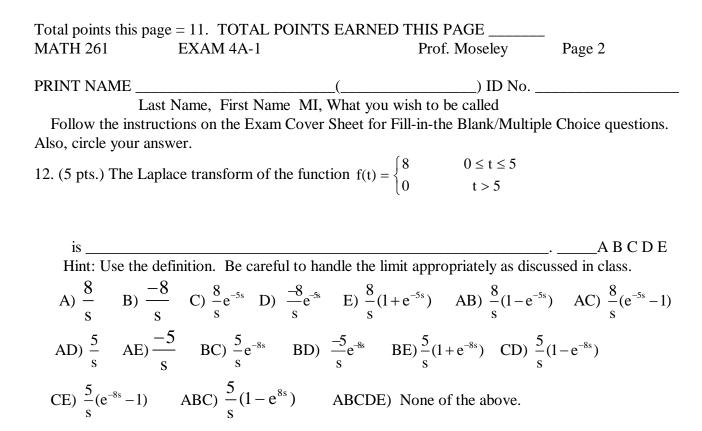
## MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

PRINT NAME			(		)
Last Name,	First Name	MI	(What you	wish to be	e called)
ID#	EXA	M DATE	Friday, Nov.	19, 2009	11:30
<u>am</u>			•		
I swear and/or affirm that all of th own and that I have neither given	<u> </u>	•	Date		Signature
SIGNATUDE	DATE		page	Scores points	score
SIGNATURE	DATE		1	11	1
INSTRUCTIONS: Besides this cover page, there are 11 pages of questions and problems on this exam. MAKE SURE YOU HAVE ALL		-	2	5	
<b>THE PAGES</b> . If a page is missing	ng, you will receive a grade of z	ero for	3	12	
that page. Page 12 contains Lapla Read through the entire exam. If			4	17	
hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have		5	12		
on your desk during the exam. $N$	O CALCULATORS! NO SC	RATCH		6	
<b>PAPER!</b> Use the back of the exam sheets if necess the staple if you wish. Print your name on all sheet	•	•	7	8	
in-the Blank/Multiple Choice or True/False. Expect no part credit on		8	4		
these pages. For each Fill-in-the I your answer in the blank provided			9	12	
given and write the corresponding	•		10	6	
blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. <b>SHOW YOUR WORK!</b> Every thought you have		11	7		
		12			
should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check	13				
your computations as time allows.	<del>-</del>	ia cneck	14		
REQUEST	Γ FOR REGRADE		15		
Please regrade the following pro	blems for the reasons I have inc	licated:	16		
(e.g., I do not understand what I	did wrong on page)		17		
			18	-	
(Regrades should be requested w			19		
returned. Attach additional shee I swear and/or affirm that upon t	• • •				
nothing on this exam except on	n this REGRADE FORM. (Wri		21		
changing anything is considered	to be cheating.)		22		

Total	100	
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Prof. Moseley **MATH 261** EXAM 4A-1 Page 1 PRINT NAME \_\_\_\_\_ \_\_\_\_\_) ID No. \_\_\_\_\_ Last Name, First Name MI, What you wish to be called True-false. Laplace transforms. 1. (1 pts) A)True or B)False The definition of the Laplace transform is  $\mathcal{L}\{f(t)\}(s) = \int_{0}^{t=\infty} f(t)e^{-t}dt$ provided the improper integral exists. 2. (1 pts) A)True or B) False Since the Laplace transform is defined in terms of an improper integral, it involves two limit processes. 3. (1 pts) A)True or B)False The Laplace transform exists for some continuous functions on  $[0,\infty)$ . 4. (1 pts) A)True or B)False The Laplace transform does exists for some discontinuous functions. 5. (1 pts) A)True or B)False The function f(t) = 1/(t-3) is not piecewise continuous on [0,7]. 6. (1 pts) A)True or B)False The function  $f(t) = e^{4t^2} \cos(t)$  is not of exponential order. 7. (1 pts) A)True or B)False The Laplace transform  $\mathcal{L}: \mathbf{T} \to \mathbf{F}$  is not a linear operator. The inverse Laplace transform  $\mathcal{Q}^{-1}$ :  $\mathbf{F} \rightarrow \mathbf{T}$  is not a linear operator. 8. (1 pts) A)True or B)False 9. (1 pts) A)True or B)False The Laplace transform is not a one-to-one mapping on the set of continuous functions on  $[0,\infty)$  for which the Laplace transform exists. 10. (1 pts) A)True or B)False There is more than one continuous function in the null space of 𝔾. 11. (1 pts) A)True or B)False The strategy of solving an ODE using Laplace transforms is not to transform the problem from the time domain **T** to the (complex) frequency domain **F**, solve the transformed problem using algebra instead of calculus, and then transform the solution back to the time domain T.



Total points this page = 5. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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PRINT NAME \_\_\_\_\_\_\_ (\_\_\_\_\_\_\_) ID No. \_\_\_\_\_\_

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Compute the Laplace transform of the following functions.

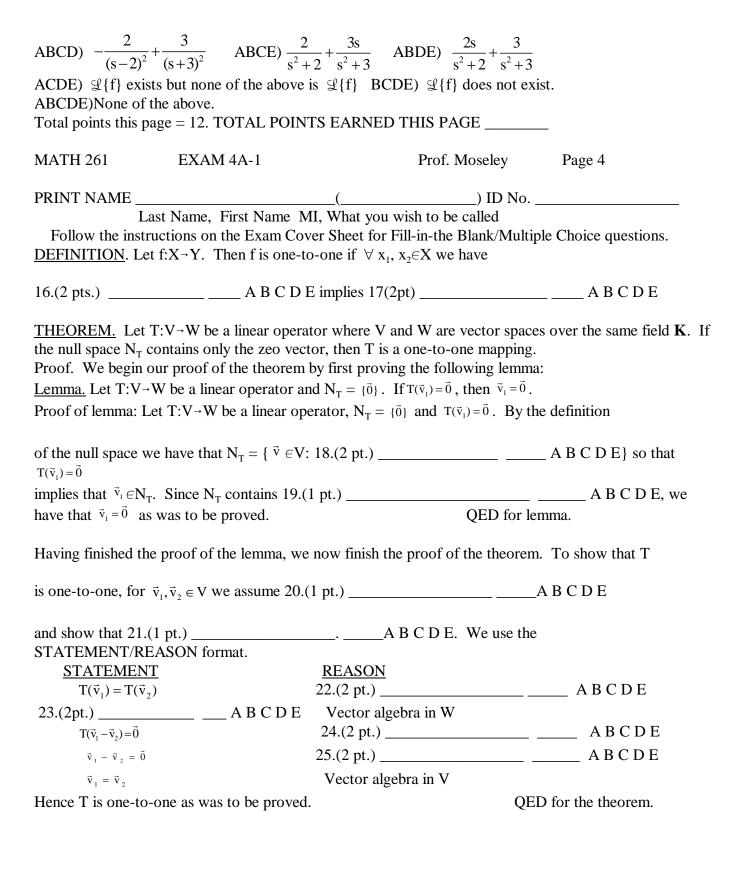
13. (4 pts.) 
$$f(t) = 2t + 3t^2$$
  $\mathcal{L}(f) =$ \_\_\_\_\_\_ A B C D E

14. (4 pts.) 
$$f(t) = 2 e^{2t} - 3 e^{-3t}$$
  $g(f) =$ \_\_\_\_\_\_\_ A B C D E

15 (4 pts.) 
$$f(t) = 2 \sin(2t) - 3 \cos(3t)$$
  $\mathcal{L}(f) =$ \_\_\_\_\_\_ A B C D E

Possible answers this page

A) 
$$\frac{2}{s} + \frac{3}{s^2}$$
 B)  $\frac{2}{s} - \frac{3}{s^2}$  C)  $-\frac{2}{s} + \frac{3}{s^2}$  D)  $-\frac{2}{s} - \frac{3}{s^2}$  E)  $\frac{2}{s^2} + \frac{6}{s^3}$  AB)  $\frac{2}{s^2} - \frac{6}{s^3}$  AC)  $-\frac{2}{s^2} + \frac{6}{s^3}$  AD)  $-\frac{2}{s^2} - \frac{6}{s^3}$  AE)  $\frac{2}{s+2} + \frac{3}{s+3}$  BC)  $\frac{2}{s+2} - \frac{3}{s+3}$  BD)  $-\frac{2}{s+2} + \frac{3}{s+3}$  BE)  $-\frac{2}{s+2} - \frac{3}{s+3}$  CD)  $\frac{2}{s-2} + \frac{3}{s+3}$  ABC)  $-\frac{2}{s-2} - \frac{3}{s+3}$  ABC)  $-\frac{2}{s-2} - \frac{3}{s+3}$  ABC)  $-\frac{2}{s-2} - \frac{3}{s+3}$  ABC)  $-\frac{4}{s^2+4} + \frac{3s}{s^2+9}$  ACE)  $-\frac{4}{s^2+4} - \frac{3s}{s^2+9}$  ACE)  $-\frac{4}{s^2+4} - \frac{3s}{s^2+9}$  ACE)  $-\frac{4}{s^2+4} - \frac{3s}{s^2-9}$  BCD)  $-\frac{4}{s^2-4} - \frac{3s}{s^2-9}$  BDE)  $-\frac{4}{s^2-4} + \frac{3s}{s^2-9}$  CDE)  $-\frac{4}{s^2-4} - \frac{3s}{s^2-9}$ 



Possible answers for this page: A) $x_1 = x_2$  B) $x_1 + x_2 = 0$  C)  $f(x_1 + x_2) = 0$  D)  $f(x_1) = f(x_2)$  E)  $f(x_1) + f(x_2) = 0$  AB) $T(\vec{v}_1) = \vec{0}$  AC)  $T(\alpha \vec{v}_1) = \alpha T(\vec{v}_1)$  AD)  $T(\vec{v}_1) = \vec{0}$  AE)  $T(\vec{v}_1) = T(\vec{v}_2)$  BC) Hypothesis (or Given) BD) only the zero vector BE) The lemma proved above

CD) Vector algebra in V CE) Vector algebra in W DE) only the vector  $\vec{v}_1$  ABC)  $\vec{v}_1 = \vec{v}_2$ 

ABD)  $T(\vec{v}_1 - \vec{v}_2)$  ABE)  $T(\vec{v}_1 + \vec{v}_2) = \vec{0}$  BCD)  $T(\vec{v}_2) = \vec{0}$  BCE)  $\vec{v}_1 = \vec{0}$  BDE) T is a one-to-one mapping CDE)

T is a linear operator ABCD) Definition of T ABCE) Theorems from Calculus

ACDE) Definition of f ABCDE) None of the above.

Total points this page = 17. TOTAL POINTS EARNED THIS PAGE \_\_\_\_\_

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PRINT NAME \_\_\_\_\_ (\_\_\_\_\_) ID No. \_\_\_\_\_\_ Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Compute the inverse Laplace transform of the following functions.

26. (4 pts.)  $F(s) = \frac{2}{s} - \frac{3}{s+2}$   $\mathcal{Q}^{-1}\{F\} =$ \_\_\_\_\_\_ A B C D E

27. (4 pts.)  $F(s) = \frac{2s-4}{s^2+9}$   $\mathcal{L}^{-1}{F} = \underline{\hspace{1cm}}$  A B C D E

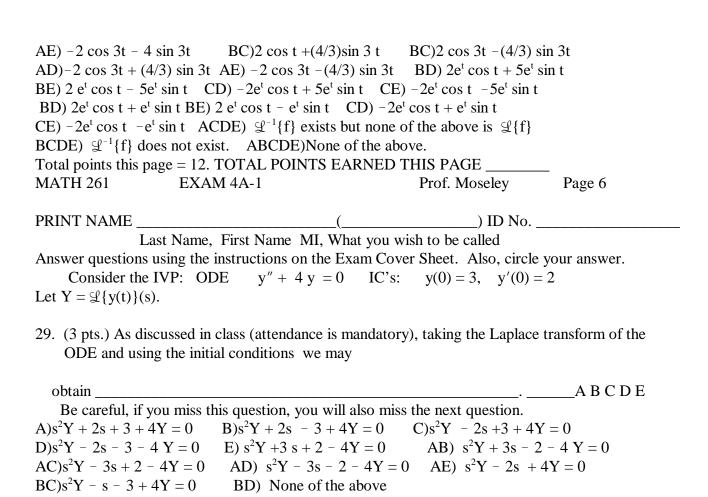
28. (4 pts.)  $F(s) = \frac{2s-3}{s^2-2s+2}$   $\mathcal{Q}^{-1}\{F\} =$  A B C D E

Possible answers this page

A) 
$$2 + 3e^{2t}$$
 B)  $2 - 3e^{2t}$  C)  $-2 + 3e^{2t}$  D)  $-2 - 3e^{2t}$  E)  $2 + 3e^{-2t}$  AB)  $2 - 3e^{-2t}$  AC)  $-2 + 3e^{-2t}$  AD)  $-2 - 3e^{-2t}$  AE)2 cos 3t + 4 sin 3t BC)2 cos 3t - 4 sin 3t AD)  $-2 \cos 3t + 4 \sin 3t$ 

BC)2 cos 3t 
$$-4 \sin 3t$$

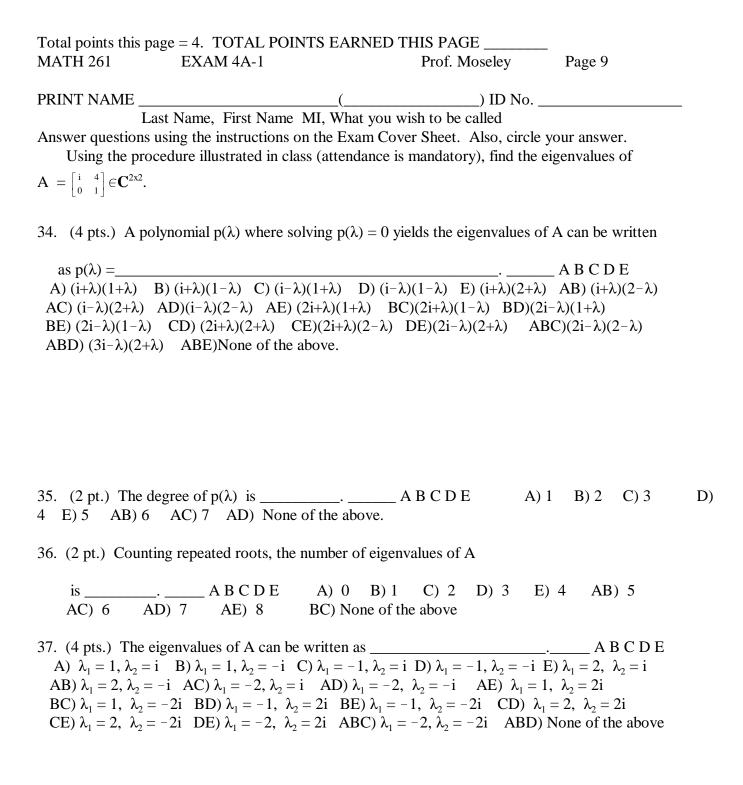
AB) 
$$2 - 3e^{-2t}$$
 AC)  $-2 + 3e^{-2t}$ 



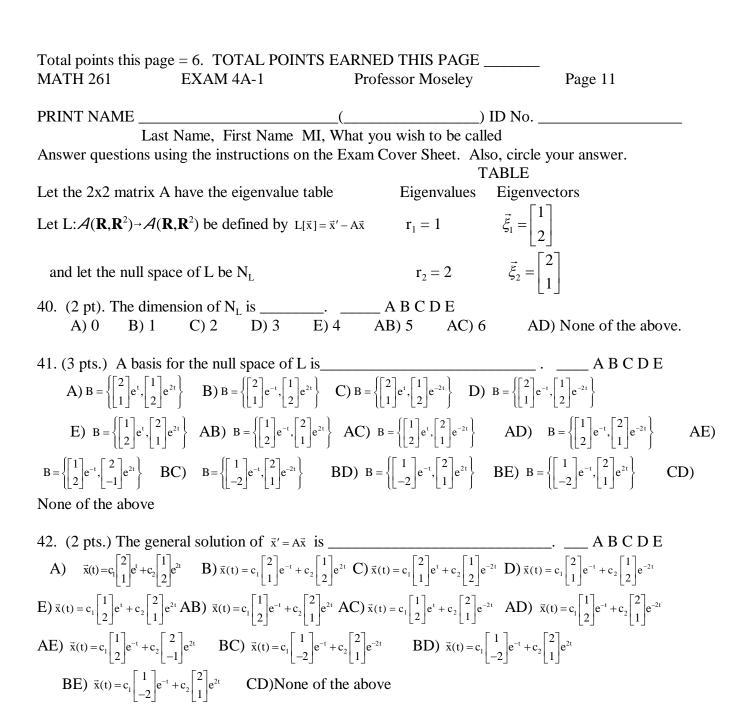
30. (3 pts.) The Laplace transform of the solution to the IVP

Total points this p	age = 6. TOTAL POI	NTS EARNED	THIS PAGE	
MATH 261	EXAM 4A-1		Prof. Moseley	Page7
	ast Name, First Name			
Answer questions	using the instructions of	on the Exam Co	ver Sheet. Also, circle	your answer.
equation $c_1 \vec{v}_1$	$S = \{\vec{v}_1, \vec{v}_2,, \vec{v}_n\} \subseteq V$ $+c_2\vec{v}_2 + + c_n\vec{v}_n = \vec{0}.$	Choose the corr		
<u>Definition</u> . Th	ne set S is linearly indep	endent		
C) (*) has a so E) (*) has no	the solution $c_1 = c_2 = c_3$ lution other than the trisolution. AB) the assoluted matrix is singular.	ivial solution. ociated matrix is	D) (*) has at least two s nonsingular.	
$\subseteq \mathcal{A}(\mathbf{R}, \mathbf{R}^3)$ $c_1 [x_1(t), y_1(t)]$ Apply the def	let $S_1 = \{ [x_1(t), y_1(t), z \text{ and } (**) \text{ be the "vector } , z_1(t)]^T + c_2 [x_2(t), y_2(t)]^T + c_3 [x_2(t), y_3(t)]^T + c_4 [x_1(t), y_3(t)]^T + c_5 [x_2(t), y_3(t)]^T + c_5 [x_3(t), y_$	r" equation ), $z_2(t)]^T + \cdots +$ ace of time vary	$c_n [x_n(t), y_n(t), z_n(t)]^T$ ing "vectors" $\mathcal{A}(\mathbf{R}, \mathbf{R}^3)$	$\mathbf{r} = [0, 0, 0]^{\mathrm{T}} \ \forall \mathbf{t} \in \mathbf{R}$
C) (**) has a s E) (**) has no	ly the solution $c_1 = c_2 = 0$ solution other than the solution. AB) the a sated matrix is singular.	trivial solution. associated matri	D) (**) has at lea x is nonsingular.	A B C D E number of solutions st two solutions.

Total points this page = 8. TOTAL POINTS EAF MATH 261 EXAM 4A-1	RNED THIS PAGE Prof. Moseley	
PRINT NAME(	at you wish to be called	
33. (4 pts.) You are to determine Directly Using time varying "vectors" are linearly independ $\vec{x}_1(t) = \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} \text{ and } \vec{x}_2(t) = \begin{bmatrix} 6e^t \\ 9e^{-t} \end{bmatrix}. \text{ Then S}$		•
is	$= [0,0]^{T}  \forall t \in \mathbf{R} \text{ implies } \mathbf{c}_{1} = \mathbf{c}_{1}$ $= [0,0]^{T}  \forall t \in \mathbf{R} \text{ implies } \mathbf{c}_{1} = \mathbf{c}_{1}$ $= [0,0]^{T}  \forall t \in \mathbf{R}.$ $[0,0]^{T}  \forall t \in \mathbf{R}.$ x is nonsingular. x is nonsingular. x is singular.	
AD) neither linearly independent or linearly dep ABCDE) None of the above statements are true	pendent as the definition does	s not apply.



Total points this page MATH 261	e = 12. TOTAL POINTS E EXAM 4A-1		AGE Moseley	
Last	Name, First Name MI, W ns on the Exam Cover Shee wer.	hat you wish to be	called	
Note that $\lambda_1 = 2$	is an eigenvalue of the mate			a basis B for
AB) $\{[1,3]^T\}$ BD) $\{[1,-1]^T\}$ ABC) $\{[1,-4]^T\}$ ACD) $\lambda = 2$ is not ACE) $\lambda = -1$ is not ADE) $\lambda = 3$ is not 39. (1pt.) Although the eigenspace as	sociated with $\lambda_1$ is $B = \underline{}$ $B) \{[1,1]^T\}$ C) $\{[1,4]^T\}$ AD) $BE) \{[1,-2]^T\}$ CD) $\{[1,-ABD) \{[4,-1]^T\}$ an eigenvalue of the matrix of an eigenvalue of the matrix an eigenvalue of the matrix here are an infinite number sociated with $\lambda_1$ is often on sociated with $\lambda_1$ have been dissociated with $\lambda_1$	$\{[4,1]^T\}$ AE) $\{[-2]^T, [4,8]^T\}$ CE ABE) $\{[3,-1]^T\}$ X A rix A X A ABCDE) No of eigenvectors assume dimensional. He	(3,1] <sup>T</sup> } (E) {[2,1] <sup>T</sup> }  one of the abounce of the abounce convention	BC) {[1,-1] <sup>T</sup> , [4,4] <sup>T</sup> } DE) {[1,3] <sup>T</sup> }  ove is correct.  any eigenvalue, ons for selecting
BD) $[1,-1]^T$ BE ABC) $[1,-4]^T$ ACD) $\lambda_1 = 2$ is no ACE) $\lambda_1 = -1$ is no	B) $[1,1]^T$ C) $\{[1,2]^T\}$ AC) $[1,4]^T$ AD) $[4,1]^T$ B) $[1,-2]^T$ CD) $[1,-2]^T$ , $[4]^T$ ABD of an eigenvalue of the matrix of an eigenvalue of the matrix	l,8] <sup>™</sup> CE) [2,1] <sup>™</sup> E) [3,−1] <sup>™</sup> rix A rix A	_A B C D E  [-,8] <sup>T</sup> E) [2  BC) [1,-1]  DE) [1,3  one of the abo	] <sup>1</sup>



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