EXAM-4 MATH 261: Elementary Differential Equations MATH 261
FALL 2008 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, November 14, 2008

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

________________________________ ________________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 14 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Page 15 contains Laplace transforms you need not memorize. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-14 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

Scores

page	points	score
1	11	
2	5	
3	12	
4	12	
5	12	
6	6	
7	6	
8	4	
9	6	
10	4	
11	8	
12	5	
13	5	
14	6	
15	---	
16		
17		
18		
19		
20		
21		
22		
Total	102	

REQUEST FOR REGRADE

Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature_______________________________

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True-false. Laplace transforms.

1. (1 pts) A)True or B)False By definition, $\mathscr{L}\{f(t)\}(s) = \int_{t=0}^{t=\infty} f(t)e^{-st}dt$ provided the improper integral exists.

2. (1 pts) A)True or B) False Since the Laplace transform is defined in terms of an improper integral, it involves only one limit process.

3. (1 pts) A)True or B)False The Laplace transform exists for all continuous functions on $[0,\infty)$.

4. (1 pts) A)True or B)False The Laplace transform does not exist for any discontinuous functions.

5. (1 pts) A)True or B)False The function f(t) = 1/(t-3) is piecewise continuous on [0,7].

6. (1 pts) A)True or B)False The function $f(t) = e^{4t^2}\cos(t)$ is of exponential order.

7. (1 pts) A)True or B)False The Laplace transform $\mathscr{L}$:**T**→**F** is a linear operator.

8. (1 pts) A)True or B)False The inverse Laplace transform $\mathscr{L}^{-1}$:**F**→**T** is a linear operator.

9. (1 pts) A)True or B)False The Laplace transform is a one-to-one mapping on the set of continuous functions on $[0,\infty)$ for which the Laplace transform exists.

10. (1 pts) A)True or B)False There is at most one continuous functions in the null space of $\mathscr{L}$.

11. (1 pts) A)True or B)False The strategy of solving an ODE using Laplace transforms is to transform the problem from the time domain **T** to the (complex) frequency domain **F**, solve the transformed problem using algebra instead of calculus, and then transform the solution back to the time domain **T**.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

12. (5 pts.) The Laplace transform of the function $f(t) = \begin{cases} 5 & 0 \le t \le 8 \\ 0 & t > 8 \end{cases}$

is ____. _____A B C D E

Hint: Use the definition. Be careful to handle the limit appropriately as discussed in class.

A) $\frac{8}{s}$ B) $\frac{-8}{s}$ C) $\frac{8}{s}e^{-5s}$ D) $\frac{-8}{s}e^{-5s}$ E) $\frac{8}{s}(1+e^{-5s})$ AB) $\frac{8}{s}(1-e^{-5s})$ AC) $\frac{8}{s}(e^{-5s}-1)$

AD) $\frac{5}{s}$ AE) $\frac{-5}{s}$ BC) $\frac{5}{s}e^{-8s}$ BD) $\frac{-5}{s}e^{-8s}$ BE) $\frac{5}{s}(1+e^{-8s})$ CD) $\frac{5}{s}(1-e^{-8s})$

CE) $\frac{5}{s}(e^{-8s}-1)$ ABC) $\frac{5}{s}(1-e^{8s})$ ABD) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the Laplace transform of the following functions.

13. (4 pts.) $f(t) = 2t + 3t^2$ $\mathcal{L}(f)$ = ____. _______ A B C D E

14. (4 pts.) $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ $\mathcal{L}(f)$ = ___________________________________. _______ A B C D E

15 (4 pts.) $f(t) = 2\sin(2t) + 3\cos(3t)$ $\mathcal{L}(f)$ = _____________________________. _______ A B C D E

Possible answers this page

A) $\frac{2}{s}+\frac{3}{s^2}$ B) $\frac{2}{s}-\frac{3}{s^2}$ C) $\frac{2}{s^2}+\frac{3}{s^3}$ D) $\frac{2}{s^2}-\frac{3}{s^3}$ E) $\frac{2}{s^2}+\frac{6}{s^3}$ AB) $\frac{2}{s^2}-\frac{6}{s^3}$

AC) $\frac{2}{s+2}+\frac{3}{s+3}$ AD) $\frac{2}{s-2}+\frac{3}{s+3}$ AE) $\frac{2}{s+2}+\frac{3}{s-3}$ BC) $\frac{2}{s-2}+\frac{3}{s-3}$

BD) $\frac{2}{(s-2)^2}+\frac{3}{(s+3)^2}$ BE) $\frac{2}{s^2+2}+\frac{3s}{s^2+3}$ CD) $\frac{2s}{s^2+2}+\frac{3}{s^2+3}$ CE) $\frac{4}{s^2+4}+\frac{3s}{s^2+9}$

DE) $\frac{2s}{s^2+4}+\frac{9}{s^2+9}$ ABC) $\frac{4}{s^2-4}+\frac{3s}{s^2-9}$ ABD) $\frac{2s}{s^2-4}+\frac{9}{s^2-9}$ ABE) None of the above.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

DEFINITION. Let $f:X\to Y$. Then f is one-to-one if $\forall\, x_1, x_2 \in X$ we have

16.(2 pts.) ____________ ____ A B C D E implies 17(2pt) ________________ ____ A B C D E

THEOREM. Let $T:V\to W$ be a linear operator where V and W are vector spaces over the same field **K**. If the null space N_T is $\{\vec{0}\}$, then T is a one-to-one mapping.

Proof. We begin our proof of the theorem by first proving the following lemma:

Lemma. If $N_T = \{\vec{0}\}$ and $T(\vec{v}_1) = \vec{0}$, then $\vec{v}_1 = \vec{0}$.

Proof of lemma: Let us assume $N_T = \{\vec{0}\}$ and $T(\vec{v}_1) = \vec{0}$. By the definition of the null space we have

that $N_T = \{ \vec{v} \in V$: 18.(1 pt.) _________________ _____ A B C D E} so that $T(\vec{v}_1) = \vec{0}$ implies that

$\vec{v}_1 \in N_T$. Since N_T contains 19.(1 pt.) _________________________________ ______ A B C D E, we have that $\vec{v}_1 = \vec{0}$ as was to be proved. QED for lemma.

Having finished the proof of the lemma, we now finish the proof of the theorem. To show that T

is one-to-one, for $\vec{v}_1, \vec{v}_2 \in V$ we assume 20.(1 pt.) ________________________ _____A B C D E

and show that 21.(1 pt.) _______________________________. _____A B C D E. We use the STATEMENT/REASON format.

STATEMENT	REASON
$T(\vec{v}_1) = T(\vec{v}_2)$	22. (1 pt.) __________________ _____ A B C D E
23. (1pt.) _____________ ___ A B C D E	Vector algebra in W
$T(\vec{v}_1 - \vec{v}_2) = \vec{0}$	24(1 pt.) __________________ _____ A B C D E
$\vec{v}_1 - \vec{v}_2 = \vec{0}$	25(1 pt.) __________________ ______ A B C D E
$\vec{v}_1 = \vec{v}_2$	Vector algebra in V

Hence T is one-to-one as was to be proved. QED for the theorem.

Possible answers for this page: A) $f(x_1 + x_2) = 0$ B) $f(x_1) = f(x_2)$ C) $x_1 = x_2$ D) $x_1 + x_2 = 0$ E) $f(x_1) + f(x_2) = 0$ AB) $T(\vec{v}_1) = \vec{0}$ AC) $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$ AD) $T(\vec{v}) = \vec{0}$ AE) $T(\vec{v}_1) = T(\vec{v}_2)$
BC) Hypothesis (or Given) BD) only the zero vector BE) The lemma proved above
CD) Vector algebra in V CE) Vector algebra in W DE) only the vector $\vec{v}_1$ ABC) $\vec{v}_1 = \vec{v}_2$
ABD) $T(\vec{v}_1 - \vec{v}_2)$ ABE) $T(\vec{v}_1 - \vec{v}_2) = \vec{0}$ BCD) $T(\vec{v}_1 + \vec{v}_2) = \vec{0}$ BCE) $\vec{v}_1 = \vec{0}$ BDE) T is a one-to-one mapping
CDE) T is a linear operator ABCD) Definition of T ABCE) Theorems from Calculus
ACDE) Definition of f BCDE) None of the above.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Compute the inverse Laplace transform of the following functions.

26. (4 pts.) $F(s) = \frac{2}{s} + \frac{3}{s+2}$ $\mathscr{L}^{-1}\{F\}$ = _____________________________. _____ A B C D E

27. (4 pts.) $F(s) = \frac{2s+4}{s^2+9}$ $\mathscr{L}^{-1}\{F\}$ = ________________________________. _____ A B C D E

28. (4 pts.) $F(s) = \frac{2s+3}{s^2-2s+2}$ $\mathscr{L}^{-1}\{F\}$ = ______________________________. _____ A B C D E

A) $2 + 3e^{2t}$ B) $2 + 3e^{-2t}$ C) $2 - 3e^{2t}$ D) $2 - 3e^{-2t}$ E) $2 + e^{-2t}$ AB) $2 \cos 3t + 4 \sin 3t$
AC) $2 \cos 3t - 4 \sin 3t$ AD) $2 \cos 3t + (4/3) \sin 3t$ AE) $3 \cos 3t - (4/3) \sin 3t$
BC) $2 \cos t + 3 \sin t$ BD) $2e^t \cos t + 5e^t \sin t$ BE) $2 e^t \cos t - 5e^t \sin t$
CD) $2e^t \cos t + e^t \sin t$ CE) $2 e^t \cos t - e^t \sin t$ DE) None of the above

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ______________________(________________) ID No. _________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Consider the IVP: ODE $y'' + 4y = 0$ IC's $y(0) = 2$, $y'(0) = 3$ ($y(0) = 3$, $y'(0) = 2$)

Let $Y = \mathscr{L}\{y(t)\}(s)$.

29. (3 pts.) As discussed in class (attendance is mandatory), taking the Laplace transform of the ODE and using the initial conditions we may

obtain ____. ______A B C D E

Be careful, if you miss this question, you will also miss the next question.

A) $s^2Y + 2s + 3 + 4Y = 0$ B) $s^2Y + 2s + 3 - 4Y = 0$ C) $s^2Y - 2s - 3 + 4Y = 0$

D) $s^2Y - 2s - 3 - 4Y = 0$ E) $s^2Y + 3s + 2 + 4Y = 0$ AB) $s^2Y + 3s + 2 - 4Y = 0$

AC) $s^2Y - 3s - 2 + 4Y = 0$ AD) $s^2Y - 3s - 2 - 4Y = 0$ AE) $s^2Y - 2s + 4Y = 0$

BC) $s^2Y - s - 3 + 4Y = 0$ BD) None of the above

30. (3 pts.) The Laplace transform of the solution to the IVP

is Y = ____. _____ A B C D E

A) $\frac{2s+3}{s^2+4}$ B) $\frac{2s+3}{s^2-4}$ C) $-\frac{2s+3}{s^2+4}$ D) $-\frac{2s+3}{s^2-4}$ E) $\frac{3s+2}{s^2+4}$ AB) $\frac{2s+3}{s^2-4}$

AC) $-\frac{3s+2}{s^2+4}$ AD) $-\frac{2s+3}{s^2-4}$ AE) $-\frac{2s}{s^2+4}$ BC) $\frac{2s-3}{s^2+4}$ BD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ____________

PRINT NAME _________________________(______________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Consider the system of ode's $x_1' = 3x_1 + 2x_2$
$x_2' = 2x_1 - 2x_2$

31. (1pt.) Solving the first equation for x_2 we obtain

x_2 = ____. ______A B C D E

A) $(3/2)x_1' + (1/2)x_1$ B) $(3/2)x_1' - (1/2)x_1$ C) $-(3/2)x_1' + (1/2)x_1$ D) $-(3/2)x_1' - (1/2)x_1$
E) $(3/2)x_1' + 2x_1$ AB) $2x_1' - x_1$ AC) $-2x_1' + x_1$ AD) $-2x_1' - x_1$ AE) $2x_1' + 2x_1$
BC) $2x_1' - 2x_1$ BD) $-2x_1' + 2x_1$ BE) $x_1' - 2x_1$ CD) None of the above.

32. (5 pts.) Using the procedure illustrated in class (attendance is mandatory), eliminate x_2 in the system to obtain a single second order ODE in x_1. The ODE obtained by the process discussed

in class is ____. ______A B C D E

A) $x_1'' + 5x_1' + 2x_1 = 0$ B) $x_1'' + 5x_1' - 2x_1 = 0$ C) $x_1'' + 5x_1' + 10x_1 = 0$ D) $x_1'' + 5x_1' - 10x_1 = 0$ E) $x_1'' - 5x_1' + 2x_1 = 0$ AB) $x_1'' - 5x_1' - 2x_1 = 0$ AC) $x_1'' - 5x_1' + 10x_1 = 0$ AD) $x_1'' - 5x_1' - 10x_1 = 0$ AE) $x_1'' + x_1' + 2x_1 = 0$ BC) $x_1'' + x_1' - 2x_1 = 0$ BD) $x_1'' + x_1' + 10x_1 = 0$ BE) $x_1'' + x_1' - 10x_1 = 0$
CD) $x_1'' - x_1' + 2x_1 = 0$ CE) $x_1'' - x_1' - 2x_1 = 0$ DE) $x_1'' - x_1' + 10x_1 = 0$ ABC) $x_1'' - x_1' - 10x_1 = 0$
ABD) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________

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33. (4 pts.) Consider the scalar equation $u'' + 4u' + 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du/dt$). Convert this to a system of two first order equations by letting $u = x$ and $u' = y$. (I.e. obtain two first order scalar equations in x and y. You may think of $u = x$ as the position and $u' = y$ as the velocity of a point particle). Now write this system of two scalar equations in the vector form

$\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix given by

A = ____. ______A B C D E

A) $\begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}$ B) $\begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}$ C) $\begin{bmatrix} 1 & 0 \\ 2 & -4 \end{bmatrix}$ D) $\begin{bmatrix} 1 & 0 \\ -2 & -4 \end{bmatrix}$ E) $\begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}$ AB) $\begin{bmatrix} 1 & 0 \\ 4 & -2 \end{bmatrix}$

AC) $\begin{bmatrix} 1 & 0 \\ 4 & -2 \end{bmatrix}$ AD) $\begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix}$ AE) $\begin{bmatrix} 1 & 0 \\ -4 & -2 \end{bmatrix}$ BC) $\begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}$ BD) $\begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}$ BE) $\begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}$

CD) $\begin{bmatrix} 0 & 1 \\ -2 & -4 \end{bmatrix}$ CE) $\begin{bmatrix} 0 & 1 \\ 4 & 2 \end{bmatrix}$ DE) $\begin{bmatrix} 1 & 0 \\ 4 & -2 \end{bmatrix}$ ABC) $\begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}$ ABD) $\begin{bmatrix} 0 & 1 \\ -4 & 2 \end{bmatrix}$

ABE) $\begin{bmatrix} 0 & 1 \\ -4 & -2 \end{bmatrix}$ BCD) None of the above.

PRINT NAME __________________________(_______________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

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34. (3 pts.) Let S = $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \subseteq V$ where V is a vector space and (*) be the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$. Choose the correct completion of the following:

Definition. The set S is linearly independent

if ____. _____A B C D E

A) (*) has only the solution $c_1 = c_2 = \cdots = c_n = 0$. B) (*) has an infinite number of solutions.
C) (*) has a solution other than the trivial solution. D) (*) has at least two solutions.
E) (*) has no solution. AB) the associated matrix is nonsingular.
AC) the associated matrix is singular. AD) None of the above

35. (3 pts.)Now let $S_1 = \{ [x_1(t), y_1(t), z_1(t)]^T, [x_2(t), y_2(t), z_2(t)]^T, \ldots, [x_n(t), y_n(t), z_n(t)]^T \} \subseteq \vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^3)$ and (**) be the "vector" equation

$$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T \quad \forall t \in \mathbf{R}$$

Apply the definition above to the space of time varying "vectors" $\vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^3)$. That is, by the definition above the set $S_1 \subseteq \vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^3)$ is linearly independent

if ____. ______ A B C D E

A) (**) has only the solution $c_1 = c_2 = \cdots = c_n = 0$. B) (**) has an infinite number of solutions
C) (**) has a solution other than the trivial solution. D) (**) has at least two solutions.
E) (**) has no solution. AB) the associated matrix is nonsingular.
AC) the associated matrix is singular. AD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ___________________________(________________) ID No. ____________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

36. (4 pts.) You are to determine Directly Using the Definition (DUD) if the following set of time varying "vectors" are linearly independent. Let S= $\{\vec{x}_1(t), \vec{x}_2(t)\} \subseteq \vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^2)$ where

$$\vec{x}_1(t) = \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} \text{ and } \vec{x}_2(t) = \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix}.$$ Then S

is ____. _____ A B C D E

A) linearly independent as $c_1\ \vec{x}_1(t) + c_2\ \vec{x}_2(t) = [0,0]^T \quad \forall t \in \mathbf{R}$ implies $c_1 = c_2 = 0$.

B) linearly independent as $-2\ \vec{x}_1(t) + \vec{x}_2(t) = [0,0]^T \quad \forall t \in \mathbf{R}$.

C) linearly independent as the associated matrix is nonsingular.

D) linearly independent as the associated matrix is singular.

E) linearly dependent as $c_1 \vec{x}_1(t) + c_2 \vec{x}_2(t) = [0,0]^T \quad \forall t \in \mathbf{R}$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $-2\ \vec{x}_1(t) + \vec{x}_2(t) = [0,0]^T \quad \forall t \in \mathbf{R}$.

AB) linearly dependent as the associated matrix is nonsingular.

AC) linearly dependent as the associated matrix is singular.

AD) neither linearly independent or linearly dependent as the definition does not apply.

AE) None of the above statements are true.

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Using the procedure illustrated in class (attendance is mandatory) find the eigenvalues of

$A = \begin{bmatrix} i & 4 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

37. (3 pts.) A polynomial $p(\lambda)$ where solving $p(\lambda) = 0$ yields the eigenvalues of A can be written

as $p(\lambda)$ =____. ______ A B C D E

A) $(i+\lambda)(1+\lambda)$ B) $(i+\lambda)(1-\lambda)$ C) $(i-\lambda)(1+\lambda)$ D) $(i-\lambda)(1-\lambda)$ E) $(i+\lambda)(2+\lambda)$ AB) $(i+\lambda)(2-\lambda)$ AC) $(i-\lambda)(2+\lambda)$ AD) $(i-\lambda)(2-\lambda)$ AE) $(2i+\lambda)(1+\lambda)$ BC) $(2i+\lambda)(1-\lambda)$ BD) $(2i-\lambda)(1+\lambda)$ BE) $(2i-\lambda)(1-\lambda)$ CD) $(2i+\lambda)(2+\lambda)$ CE) $(2i+\lambda)(2-\lambda)$ DE) $(2i-\lambda)(2+\lambda)$ ABC) $(2i-\lambda)(2-\lambda)$ ABD) $(3i-\lambda)(2+\lambda)$ ABE) None of the above.

$p(\lambda)$ =

38. (1 pt.) The degree of $p(\lambda)$ is __________. ______ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

39. (1 pt.) Counting repeated roots, the number of eigenvalues of A

is _________. _____ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) None of the above

40. (3 pts.) The eigenvalues of A can be written as ______________________._____ A B C D E

A) $\lambda_1 = 1, \lambda_2 = i$ B) $\lambda_1 = 1, \lambda_2 = -i$ C) $\lambda_1 = -1, \lambda_2 = i$ D) $\lambda_1 = -1, \lambda_2 = -i$ E) $\lambda_1 = 2, \lambda_2 = i$ AB) $\lambda_1 = 2, \lambda_2 = -i$ AC) $\lambda_1 = -2, \lambda_2 = i$ AD) $\lambda_1 = -2, \lambda_2 = -i$ AE) $\lambda_1 = 1, \lambda_2 = 2i$ BC) $\lambda_1 = 1, \lambda_2 = -2i$ BD) $\lambda_1 = -1, \lambda_2 = 2i$ BE) $\lambda_1 = -1, \lambda_2 = -2i$ CD) $\lambda_1 = 2, \lambda_2 = 2i$ CE) $\lambda_1 = 2, \lambda_2 = -2i$ DE) $\lambda_1 = -2, \lambda_2 = 2i$ ABC) $\lambda_1 = -2, \lambda_2 = -2i$ ABD) None of the above

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PRINT NAME _________________________(_________________) ID No. __________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Note that $\lambda_1 = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$

41. (4 pts.) Using the conventions discussed in class (attendance is mandatory), a basis B for

the eigenspace associated with λ_1 is B = ____________________. ___A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$

AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$

BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$

ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$

ACD) $\lambda = 2$ is not an eigenvalue of the matrix A

ACE) $\lambda = -1$ is not an eigenvalue of the matrix A ADE)None of the above is correct.

42. (1pt.) Although there are an infinite number of eigenvectors associated with any eigenvalue, since the eigenspace associated with λ_1 is one dimensional and we have developed conventions for selecting a basis for the eigenspace associated with λ_1, we say that <u>the</u> eigenvector associated

with λ_1 is ______________________________________. ______A B C D E

A) $\{[1,1]^T, [4,4]^T\}$ B) $\{[1,1]^T\}$ C) $\{[1,2]^T\}$ D) $\{[1,2]^T, [4,8]^T\}$ E) $\{[2,1]^T\}$

AB) $\{[1,3]^T\}$ AC) $\{[1,4]^T\}$ AD) $\{[4,1]^T\}$ AE) $\{[3,1]^T\}$ BC) $\{[1,-1]^T, [4,4]^T\}$

BD) $\{[1,-1]^T\}$ BE) $\{[1,-2]^T\}$ CD) $\{[1,-2]^T, [4,8]^T\}$ CE) $\{[2,1]^T\}$ DE) $\{[1,3]^T\}$

ABC) $\{[1,-4]^T\}$ ABD) $\{[4,-1]^T\}$ ABE) $\{[3,-1]^T\}$

ACD) $\lambda_1 = 2$ is not an eigenvalue of the matrix A

ACE) $\lambda_1 = -1$ is not an eigenvalue of the matrix A ADE)None of the above is correct.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

True or false. Eigenvalue Problems for Complex Matrices.
Assume A is an n×n square matrix of possibly complex numbers. Under this hypothesis, determine which of the following is true and which is false.

43. (1 pt.) A)True or B)False A real square matrix may have only real eigenvalues.

44. (1 pt.) A)True or B)False A real square matrix will always have distinct eigenvalues.

45. (1 pt.) A)True or B)False A real symmetric matrix will have only real eigenvalues.

46. (1 pt.) A)True or B)False A Hermitian matrix will always have real eigenvalues.

47. (1 pt.) A)True or B)False A Hermitian matrix will always have distinct eigenvalues.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ____________________________(___________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let the 2x2 matrix A have the eigenvalue table

TABLE

Eigenvalues	Eigenvectors
$r_1 = 1$	$\vec{\xi}_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$
$r_2 = 2$	$\vec{\xi}_2 = \begin{bmatrix} 2 \\ 1 \end{bmatrix}$

Let $L: \mathcal{A}(\mathbf{R},\mathbf{R}^2) \to \mathcal{A}(\mathbf{R},\mathbf{R}^2)$ be defined by $L[\vec{x}] = \vec{x}' - A\vec{x}$

and let the null space of L be N_L

48. (1 pt). The dimension of N_L is _________. ______ A B C D E

A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

49. (3 pts.) A basis for the null space of L is_____________________________ . _____ A B C D E

A) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} \right\}$ B) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t} \right\}$ C) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} \right\}$ D) $B = \left\{ \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t}, \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t} \right\}$

E) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ AB) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ AC) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$ AD) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$ AE) $B = \left\{ \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t} \right\}$ BC) $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t} \right\}$ BD) $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ BE) $B = \left\{ \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t} \right\}$ CD) None of the above

50. (2 pts.) The general solution of $\vec{x}' = A\vec{x}$ is ________________________________. ____ A B C D E

A) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ B) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{2t}$ C) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$ D) $\vec{x}(t) = c_1 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-2t}$

E) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$ AB) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$ AC) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$ AD) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$

AE) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ 2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{2t}$ BC) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{-2t}$ BD) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$

BE) $\vec{x}(t) = c_1 \begin{bmatrix} 1 \\ -2 \end{bmatrix} e^{-t} + c_2 \begin{bmatrix} 2 \\ 1 \end{bmatrix} e^{2t}$ CD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _____________