EXAM-4 | MATH 261: Elementary Differential Equations | MATH 261
FALL 2006 | EXAMINATION COVER PAGE | Professor Moseley

PRINT NAME____ (_________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, November 17, 2006

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

________________________________ _________________
SIGNATURE DATE

INSTRUCTIONS

1. Besides this cover page, there are 14 pages of questions and problems on this exam. Page 15 contains Laplace transforms you need not memorize. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-14 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. GOOD LUCK!!

REQUEST FOR REGRADE
Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)
(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)
Date _______ Signature_______________________________

Scores

page	points	score
1	11	
2	5	
3	12	
4	12	
5	6	
6	5	
7	6	
8	5	
9	6	
10	4	
11	4	
12	5	
13	14	
14	5	
15	---	
16		
17		
18		
19		
20		
21		
22		
Total	102	

PRINT NAME ________________________(________________) ID No. _________________
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True-false. Laplace transforms.

1. (1 pts) A)True or B)False By definition, $\mathcal{L}\{f(t)\}(s) = \int_{t=0}^{t=\infty} f(s)e^{-st}dt$ provided the improper integral exists.

2. (1 pts) A)True or B) False Since the Laplace transform is defined in terms of an improper integral, it involves two limit processes.

3. (1 pts) A)True or B)False The Laplace transform exists for all continuous functions on $[0,\infty)$.

4. (1 pts) A)True or B)False The Laplace transform does not exist for any discontinuous function.

5. (1 pts) A)True or B)False The function f(t) = 1/(t-3) is piecewise continuous on [0,7].

6. (1 pts) A)True or B)False The function $f(t) = e^{4t^2}\cos(t)$ is of exponential order.

7. (1 pts) A)True or B)False The Laplace transform $\mathcal{L}$:**T**→**F** is a linear operator.

8. (1 pts) A)True or B)False The inverse Laplace transform $\mathcal{L}^{-1}$:**F**→**T** is not a linear operator.

9. (1 pts) A)True or B)False The Laplace transform is a one-to-one mapping on the set of continuous functions on $[0,\infty)$ for which the Laplace transform exists.

10. (1 pts) A)True or B)False There at least two continuous functions in the null space of $\mathcal{L}$.

11. (1 pts) A)True or B)False The strategy of solving an ODE using Laplace transforms is to transform the problem from the time domain **T** to the (complex) frequency domain **F**, solve the transformed problem using algebra instead of calculus, and then transform the solution back to the time domain **T**.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME _________________________(_________________) ID No. __________________

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For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

12. (5 pts.) The Laplace transform of the function $f(t) = \begin{cases} 8 & 0 \le t \le 5 \\ 0 & t > 5 \end{cases}$

is ____. _____A B C D E

Hint: Use the definition. Be careful to handle the limit appropriately as discussed in class.

A) $\frac{8}{s}$ B) $\frac{8}{s}e^{-5s}$ C) $\frac{8}{s}(1+e^{-5s})$ D) $\frac{8}{s}(1-e^{-5s})$ E) $\frac{s}{8}(1-e^{-5s})$

AB) $\frac{5}{s}$ AC) $\frac{5}{s}(1-e^{-8s})$ AD) $\frac{5}{s}(1+e^{-8s})$ AE) $\frac{5}{s}(1-e^{-8s})$ AE) $\frac{5}{s}(1-e^{8s})$

BC) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _________

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters.

Compute the Laplace transform of the following functions.

13. (4 pts.) $f(t) = 2 + 3t$ $\mathcal{L}(f) =$ _____________________________________. _______ A B C D E

14. (4 pts.) $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ $\mathcal{L}(f) =$ ________________________________. _______ A B C D E

15 (4 pts.) $f(t) = 2\sin(2t) + 3\cos(3t)$ $\mathcal{L}(f) =$ __________________________. _______ A B C D E

A) $\frac{2}{s}+\frac{3}{s^2}$ B) $\frac{2}{s^2}+\frac{3}{s^3}$ C) $\frac{2}{s^2}+\frac{3/2}{s^3}$ D) $\frac{2}{s^2}+\frac{1}{s^3}$ E) $\frac{1}{s}+\frac{1}{s^2}$ AB) $2+\frac{3}{s}$

AC) $\frac{2}{s-2}+\frac{3}{s+3}$ AD) $\frac{2}{s+2}+\frac{3}{s-3}$ AE) $\frac{2}{s-3}+\frac{3}{s+2}$ BC) $\frac{2}{(s-3)^2}+\frac{3}{(s+3)^3}$

BD) $\frac{2}{s-4}+\frac{4}{s+2}$ BE) $\frac{2}{s^2+1}+\frac{3s}{s^2+4}$ CD. $\frac{2s}{s^2+4}+\frac{3}{s+9}$ CE) $\frac{2}{s^2-1}+\frac{3}{s^2-4}$

DE) $\frac{2}{s^2+4}+\frac{3}{s^2+9}$ ABC) $\frac{2}{s^2+4}+\frac{2s}{s^2+9}$ ABD) $\frac{2s}{s^2+4}+\frac{3}{s^2+9}$ ABE) None of the above.

Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters.

Compute the inverse Laplace transform of the following functions.

16. (4 pts.) $F(s) = \frac{2}{s} + \frac{3}{s+2}$ $\mathcal{L}^{-1}\{F\}$ = _____________________________. _____ A B C D E

17. (4 pts.) $F(s) = \frac{2s+4}{s^2+9}$ $\mathcal{L}^{-1}\{F\}$ = ________________________________. _____ A B C D E

18 (4 pts.) $F(s) = \frac{2s+3}{s^2 - 2s + 2}$ $\mathcal{L}^{-1}\{F\}$ = ______________________________. ____ A B C D E

A) $2 + 2e^{-2t}$ B) $3 + 2e^{-2t}$ C) $2 + 2e^{-3t}$ D) $2 + 3e^{-2t}$ E) $2 + e^{-2t}$ AB) $2\cos 3t + (4/3)\sin 3t$
AC) $2\cos 3t + 4\sin 3t$ AD) $2\cos 2t + (4/3)\sin 2t$ AE) $3\cos 3t + (4/3)\sin 3t$
BC) $2\cos 3t + 3\sin 3t$ BD) $2e^t\cos 3t + 5e^t\sin 3t$ BE) $2e^t\cos t + 5e^t\sin t$
CD) $2e^t$

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ____________

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

Consider the IVP: ODE $y'' + 4y = 0$ IC's $y(0) = 2$, $y'(0) = 0$

19. (3 pts.) Letting $Y = \mathcal{L}\{y(t)\}(s)$, the Laplace transform of the ODE using the IC's

is ____. ______A B C D E

Be careful, if you miss this question, you will also miss the next question.

A) $s^2Y - 2s - 1 + 4Y = 0$ B) $s^2Y - 2s + 4Y = 0$ C) $s^2Y - 2 + 4Y = 0$
D) $s^2Y - s - 1 + 4Y = 0$ E) $s^2Y + s + 1 - 4Y = 0$ AB) $s^2Y + s + 1 + 4Y = 0$
AC) $s^2Y - 2s - 2 + 4Y = 0$ AD) $s^2Y + 2s + 2 + 4Y = 0$ AE) $s^2Y - 2s + 4Y = 0$
BC) $s^2Y - s - 1 + 4Y = 0$ BD) None of the above

20. (3 pts.) The Laplace transform of the solution to the IVP

is ____. _____ A B C D E

A) $Y = \dfrac{2s+1}{s^2+4}$ B) $Y = \dfrac{2s}{s^2+4}$ C) $Y = \dfrac{-2}{s^2+4}$ D) $Y = \dfrac{s+1}{s^2+4}$,
E) $Y = \dfrac{-s-1}{s^2+4}$ AB) $Y = \dfrac{2s+2}{s^2+4}$ AC) $Y = \dfrac{2s+1}{s^2+4}$ AD) $Y = \dfrac{2s+1}{s^2+4}$,
AE) $Y = \dfrac{2s+1}{s^2+4}$ BC. $Y = \dfrac{2s+1}{s^2+4}$ BD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ____________

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True or false. Eigenvalue Problems for Complex Matrices.
Assume A is an n×n square matrix of possibly complex numbers. Under this hypothesis, determine which of the following is true and which is false.

21. (1 pt.) A)True or B)False A real square matrix will have only real eigenvalues.

22. (1 pt.) A)True or B)False A real square matrix will have only distinct eigenvalues.

23. (1 pt.) A)True or B)False A real symmetric matrix will have only real eigenvalues.

24. (1 pt.) A)True or B)False A Hermitian matrix will have only real eigenvalues.

25. (1 pt.) A)True or B)False A Hermitian matrix will have only distinct eigenvalues.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

Consider the matrix $A = \begin{bmatrix} i & 3 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

26. (1 pt.) The degree of the polynomial where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is __________. ______ A B C D E A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

27. (2 pt.) The polynomial $p(\lambda)$ where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is ______________. ______ A B C D E A) $p(\lambda) = (i-\lambda)(1-\lambda)$ B) $p(\lambda) = (i+\lambda)(1+\lambda)$ C) $p(\lambda) = (3-\lambda)(i-\lambda)$ D) $p(\lambda) = (i-\lambda)(1+\lambda)$ E) $p(\lambda) = (3+\lambda)(1-\lambda)$ AB) $p(\lambda) = (i-\lambda)(3-\lambda)$ AC) $p(\lambda) = (i-\lambda)(3+\lambda)$ AD) None of the above.

28. (1 pt.) Counting repeated roots, the number of eigenvalues that the matrix A has is _________. _____ A B C D E A) 0 B) 1 C) 2 D) 3 E) 4 AB) 5 AC) 6 AD) 7 AE) 8 BC) None of the above

29. (2 pts.) The eigenvalues of A are ______________________________._____ A B C D E
A) $\lambda_1 = 2,\ \lambda_2 = i$ B) $\lambda_1 = 1,\ \lambda_2 = 2i$ C) $\lambda_1 = 1,\ \lambda_2 = i$ D) $\lambda_1 = 3,\ \lambda_2 = i$ E) $\lambda_1 = 1,\ \lambda_2 = 3i$
AB) $\lambda_1 = 2,\ \lambda_2 = 2i$ AC) $\lambda_1 = 2,\ \lambda_2 = 3i$ AD) $\lambda_1 = -1,\ \lambda_2 = -i$ AE) $\lambda_1 = -2,\ \lambda_2 = i$
BC) $\lambda_1 = -1,\ \lambda_2 = -2i$ BD) $\lambda_1 = 2,\ \lambda_2 = -i$ BE) $\lambda_1 = -2,\ \lambda_2 = -2i$ CD) None of the above

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

30. (5 pts.) $\lambda = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$. Using the conventions discussed in class (attendance is mandatory), a basis B for the eigenspace associated with this eigenvalue is ____. _____A B C D E
Such vectors are usually called the eigenvectors associated with the eigenvalue $\lambda = 2$. Recall that basis sets are not unique so that conventions discussed in class are mandatory.

A) $B = \{[1,1]^T, [4,4]^T\}$ B) $B = \{[1,1]^T\}$ C) $B = \{[1,2]^T\}$ D) $B = \{[1,2]^T, [4,8]^T\}$
E) $B = \{[2,1]^T\}$ AB) $B = \{[1,3]^T\}$ AC) $B = \{[1,4]^T\}$ AD) $B = \{[4,1]^T\}$
AE) $B = \{[3,1]^T\}$ BC) $B = \varnothing$ BD) None of the above.

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31. (3 pts.) Choose the correct completion of the following definition.

Definition. The set S = $\{\vec{v}_1, \vec{v}_2, \ldots, \vec{v}_n\} \subseteq V$ where V is a vector space is linearly

independent if ____. _____A B C D E

A) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has only the solution $c_1 = c_2 = \cdots = c_n = 0$.

B) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has an infinite number of solutions.

C) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has a solution other than the trivial solution.

D) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has at least two solutions.

E) the vector equation $c_1\vec{v}_1 + c_2\vec{v}_2 + \ldots + c_n\vec{v}_n = \vec{0}$ has no solution.

AB) the associated matrix is nonsingular. AC) the associated matrix is singular.

32. (3 pts.)Now apply this definition to the space of time varying "vectors" $\vec{A}(\boldsymbol{R}, \boldsymbol{R}^3)$. That is, By the definition above the set S ={ $[x_1(t), y_1(t), z_1(t)]^T$, $[x_2(t), y_2(t), z_2(t)]^T$, ... , $[x_n(t), y_n(t), z_n(t)]^T$ } $\subseteq \vec{A}(\boldsymbol{R}, \boldsymbol{R}^3)$ is linearly independent

if ____. ______ A B C D E

A) the vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has only the trivial solution $c_1 = c_2 = \cdots = c_n = 0$.

B) the vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has an infinite number of solutions

C) the vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has a solution other than the trivial solution.

D) the vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has at least two solutions.

E) the vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has no solution.

AB) the associated matrix is nonsingular. AC) the associated matrix is singular

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ________

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For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

33. (4 pts.) You are to determine Directly Using the Definition (DUD) if the following set of time varying "vectors" are linearly independent. Let S= $\{\vec{x}_1,\vec{x}_2\} \subseteq \vec{A}(\boldsymbol{R},\boldsymbol{R}^2)$ where $\vec{x}_1 = \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix}$ and $\vec{x}_2 = \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix}$. Then S

is ____. _____ A B C D E

A) linearly independent as $c_1 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + c_2 \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t\in\mathbf{R}$ implies $c_1 = 0$ and $c_2 = 0$.

B) linearly independent as $-2 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t\in\mathbf{R}$.

C) linearly dependent as $c_1 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + c_2 \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t\in\mathbf{R}$ implies $c_1 = 0$ and $c_2 = 0$.

D) linearly dependent as $-2 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t\in\mathbf{R}$.

E) neither linearly independent or linearly dependent as the definition does not apply.

AB) None of the above statements are true.

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For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

34. (4 pts.) Consider the scalar equation $u'' + 4u' - 2u = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du/dt$). Convert this to a system of two first order equations by letting $u = x$ and $u' = y$. (I.e. obtain two first order scalar equations in x and y. You may think of $u = x$ as the position and $u' = y$ as the velocity of a point particle). Now write this system of two scalar equations in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. This system

is____. ______A B C D E

A) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ B) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ C) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

D) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ E) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AB) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

AC) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AD) $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$ AE) None of the above.

Total points this page = 4. TOTAL POINTS EARNED THIS PAGE _______

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For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

35. (5 pts.) Using the procedure illustrated in class (attendance is mandatory), eliminate x_2 in the following system to obtain a single second order ODE in x_1.

$x_1' = 3x_1 - 2x_2$
$x_2' = 2x_1 - 2x_2$

The ODE obtained by the process discussed in class

is ____. ______A B C D E

Possible answers
A) $x_1'' + x_1' + 2x_1 = 0$ B) $x_1'' + x_1' - 2x_1 = 0$ C) $x_1'' - x_1' + 2x_1 = 0$ D) $x_1'' - x_1' - 2x_1 = 0$ E) $x_1'' + 2x_1' + x_1 = 0$ AB) $x_1'' + 2x_1' - x_1 = 0$ AC) $x_1'' - 2x_1' + x_1 = 0$
AD) $x_1'' - 2x_1' - x_1 = 0$ AE) $x_1'' + 2x_1' + 2x_1 = 0$ BC) $x_1'' + 2x_1' - 2x_1 = 0$ BD) $x_1'' - 2x_1' + 2x_1 = 0$ BE) $x_1'' - x_1' - 2x_1 = 0$ CD) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______

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Fill in the blank. Then circle the letter or letters that corresponds to your answer from the list below.

DEFINITION. Let $f:X\to Y$. Then f is one-to-one if $\forall\, x_1, x_2\in X$ we have

36.(1 pts.) _____________ ____ A B C D E implies 37(1pt) _________________ ____ A B C D E

THEOREM. Let $T:V\to W$ be a linear operator where V and W are vector spaces over the same field **K**. If the null space N_T is $\{\vec{0}\}$, then T is a one-to-one mapping.

Proof. We begin our proof of the theorem by first proving the following lemma:

Lemma. If $N_T = \{\vec{0}\}$ and $T(\vec{v}_1)=\vec{0}$, then $\vec{v}_1=\vec{0}$.

Proof of lemma: Let us assume $N_T = \{\vec{0}\}$ and $T(\vec{v}_1)=\vec{0}$. By the definition of the null space we have that $N_T = \{\, \vec{v} \in V$: 38.(2 pts.) _________________ _____ } A B C D E so that

39.(1 pts.) _______________________________ ______ A B C D E implies that $\vec{v}_1 \in N_T$. Since N_T contains 40.(1 pts.) _________________________________ _______ A B C D E, we have that

41 (1pt.) ___________________________________ _______ A B C D E as was to be proved.

QED for lemma.

Having finished the proof of the lemma, we now finish the proof of the theorem. To show that T is one-to-one, for $\vec{v}_1,\vec{v}_2 \in V$ we assume 42.(1 pts.) ___________________________ _____A B C D E and show that 43.(1 pts.) _______________________________ _____A B C D E. We use the STATEMENT/REASON format.

STATEMENT	REASON
$T(\vec{v}_1)=T(\vec{v}_2)$	44. (1 pt.) ___________________ _____ A B C D E
45. (1pt.) ______________ ___ A B C D E	Vector algebra in W
$T(\vec{v}_1-\vec{v}_2)=\vec{0}$	46(1 pt.) ___________________ _____ A B C D E
$\vec{v}_1-\vec{v}_2=\vec{0}$	47(1 pt.) ___________________ ______ A B C D E
$\vec{v}_1=\vec{v}_2$	Vector algebra in V

Hence T is one-to-one as was to be proved.

QED for the theorem.

Possible answers.

A) $f(x_1+ x_2) =0$ B) $f(x_1) = f(x_2)$ C) $x_1 = x_2$ D) $x_1 + x_2 = 0$ E) $f(x_1) + f(x_2) =0$
AB) $T(\vec{v}_1) = \vec{0}$ AC) $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$ AD) $T(\vec{v}) = \vec{0}$ AE) only the zero vector
BC) Hypothesis (or Given) BD) $T(\vec{v}_1)=T(\vec{v}_2)$ BE) The lemma proved above
CD) Vector algebra in V CE) Vector algebra in W DE) only the vector $\vec{v}_1$ ABC) $\vec{v}_1=\vec{v}_2$
ABD) $T(\vec{v}_1-\vec{v}_2)$ ABE) $T(\vec{v}_1-\vec{v}_2)=\vec{0}$ BCD) $\vec{v}_1-\vec{v}_2=\vec{0}$ BCE) $\vec{v}_1=\vec{0}$ BDE) T is a one-to-one mapping CDE) T is a linear operator ABCD) Definition of T ABCE) Theorems from Calculus
ACDE) Definition of f BCDE) None of the above.

Total points this page = 14. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME __________________________(__________________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

For this question write your answer in the blank provided. Next find your answer from the list of possible answers listed and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

48. (5 pts.) The solution set of the Boundary Value Problem (BVP)

BVP
- ODE $y'' + y = 0$.
- BC's $y(0) = 0, \quad y(\pi) = 0$

is S = ____. _____A B C D E

A. $\emptyset$ (i.e. the BVP has no solution) B. {sin x } C. {cos x }
D. $\{ c \sin x : c\in\mathbf{R}\}$ E. $\{ c \cos x : c\in\mathbf{R}\}$ AB. {tan x } AC. $\{ c \tan x : c\in\mathbf{R}\}$,
AD. None of the above sets is the solution set for this boundary value problem.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE _______