EXAM-4 MATH 261: Elementary Differential Equations MATH 261
FALL 2005 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____ (_______________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, November 18, 2005

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

___________________________________ _________________
SIGNATURE DATE

INSTRUCTIONS

1. Besides this cover page, there are 14 pages of questions and problems on this exam. Page 15 contains Laplace transforms you need not memorize. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-14 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your <u>entire</u> solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. G **Proofread your solutions** and **check your computations** as time allows. OOD LUCK!!

Scores

page	points	score
1	11	
2	5	
3	12	
4	12	
5	6	
6	5	
7	6	
8	5	
9	6	
10	4	
11	4	
12	5	
13	14	
14	5	
15	---	
16		
17		
18		
19		
20		
21		
22		
Total	100	

REQUEST FOR REGRADE

Please regrade the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature_________________________________

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(11 pts) True-false. Laplace transforms.

True or False 1. By definition, $\mathcal{L}\{f(t)\}(s) = \int_{t=0}^{t=\infty} f(s)e^{-st}dt$ provided the improper integral exists.

True or False 2. Since the Laplace transform is defined in terms of an improper integral, it involves only one limit processes.

True or False 3. The Laplace transform exists for all continuous functions on $[0,\infty)$.

True or False 4. The Laplace transform does not exist for any discontinuous functions.

True or False 5. The function f(t) = 1/(t-3) is piecewise continuous on [3,7].

True or False 6. The function $f(t) = e^{4t}\cos(t)$ is of exponential order.

True or False 7. The Laplace transform $\mathcal{L}$:**T**→**F** is a linear operator.

True or False 8. The inverse Laplace transform $\mathcal{L}^{-1}$:**F**→**T** is not a linear operator.

True or False 9. The Laplace transform is a one-to-one mapping on the set of continuous functions on $[0,\infty)$ for which the Laplace transform exists.

True or False 10. There is more that one continuous function in the null space of $\mathcal{L}$..

True or False 11. The stategy of solving an ODE using Laplace transforms is not to transfrom the problem into the (complex) frequency domain, solve the transformed problem using algebra instead of calculus, and then transform the solution back to the time domaiin.

Total points this page = 11. TOTAL POINTS EARNED THIS PAGE _______

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12. (5 pts.) Compute the Laplace transform of the function

$$f(t) = \begin{cases} 8 & 0 \le t \le 5 \\ 0 & t > 5 \end{cases}$$

Hint: Use the definition. Be careful to handle the limit appropriately as discussed in class.

Circle the correct answer below.

A. $\frac{8}{s}$, B. $\frac{8}{s}e^{-5s}$, C. $\frac{8}{s}(1+e^{-5s})$, D. $\frac{8}{s}(1-e^{-5s})$, E. $\frac{s}{8}(1-e^{-5s})$,

AB. $\frac{5}{s}$, AC. $\frac{5}{s}(1-e^{-8s})$, AD. $\frac{5}{s}(1+e^{-8s})$, AE. $\frac{5}{s}(1-e^{-8s})$, AE. $\frac{5}{s}(1-e^{8s})$,

BC. None of the above.

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(12 pts.) Compute the Laplace transform of the following functions. Then circle the letter corresponding to the Laplace transform of the function.

13. $f(t) = 2 + 3t$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE

14. $f(t) = 2\,e^{2t} + 3\,e^{-3t}$ A B C D E AB AC AD AE BC BD
BE CD CE DE ABC ABD ABE

15 $f(t) = 2\sin(2t) + 3\cos(3t)$ A B C D E AB AC AD AE BC
BD BE CD CE DE ABC ABD ABE

A. $\frac{2}{s}+\frac{3}{s^2}$, B. $\frac{2}{s^2}+\frac{3}{s^3}$, C. $\frac{2}{s^2}+\frac{3/2}{s^3}$, D. $\frac{2}{s^2}+\frac{1}{s^3}$, E. $\frac{1}{s}+\frac{1}{s^2}$, AB. $2+\frac{3}{s}$

AC. $\frac{2}{s-2}+\frac{3}{s+3}$, AD. $\frac{2}{s+2}+\frac{3}{s-3}$, AE. $\frac{2}{s-3}+\frac{3}{s+2}$, BC. $\frac{2}{(s-3)^2}+\frac{3}{(s+3)^3}$,

BD. $\frac{2}{s-4}+\frac{4}{s+2}$, BE. $\frac{2}{s^2+1}+\frac{3s}{s^2+4}$, CD. $\frac{2s}{s^2+4}+\frac{3}{s+9}$, CE. $\frac{2}{s^2-1}+\frac{3}{s^2-4}$,

DE. $\frac{2}{s^2+4}+\frac{3}{s^2+9}$, ABC. $\frac{2}{s^2+4}+\frac{2s}{s^2+9}$, ABD. $\frac{2s}{s^2+4}+\frac{3}{s^2+9}$ ABE. None of the above.

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(12 pts.) Compute the inverse Laplace transform of the following functions:

16. $F(s) = \frac{2}{s} + \frac{3}{s+2}$ A B C D E AB AC AD AE BC BD BE
CD CE CF DE

17. $F(s) = \frac{2s+4}{s^2+9}$ A B C D E AB AC AD AE BC BD BE
CD CE CF DE

18 $F(s) = \frac{2s+3}{s^2 - 2s + 2}$ A B C D E AB AC AD AE BC BD
BE
CD CE CF DE

A. $2 + 2e^{-2t}$, B. $3 + 2e^{-2t}$, C. $2 + 2e^{-3t}$, D. $2 + 3e^{-2t}$, E. $2 + e^{-2t}$, AB. $2\cos 3t + (4/3)\sin 3t$,
AC. $2\cos 3t + 4\sin 3t$, AD. $2\cos 2t + (4/3)\sin 2t$, AE. $3\cos 3t + (4/3)\sin 3t$,

BC. $2\cos 3t + 3\sin 3t$, BD. $2e^t\cos 3t + 5e^t\sin 3t$, BE. $2e^t\cos t + 5e^t\sin t$,
CD. $2e^t\cos 3t + 2e^t\sin 3t$, CE. $5e^t\cos 3t + 5e^t\sin 3t$, DE. None of the above
Total points this page = 12. TOTAL POINTS EARNED THIS PAGE ________

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Consider the IVP: ODE $y'' + 4y = 0$ IC's $y(0) = 2$, $y'(0) = 0$

19. (3 pts.) Letting $Y = \mathscr{L}\{y(t)\}(s)$, compute the Laplace transform of the ODE using the IC's. Circle the correct answer. Be careful, if you miss this question, you will also miss the next question.
A. $s^2Y - 2s - 1 + 4Y = 0$, B. $s^2Y - 2s + 4Y = 0$, C. $s^2Y - 2 + 4Y = 0$,
D. $s^2Y - s - 1 + 4Y = 0$, E. $s^2Y + s + 1 - 4Y = 0$, AB. $s^2Y + s + 1 + 4Y = 0$,

AC. $s^2Y - 2s - 2 + 4Y = 0$, AD. $s^2Y + 2s + 2 + 4Y = 0$, AE. $s^2Y - 2s + 4Y = 0$,
BC. $s^2Y - s - 1 + 4Y = 0$, BD. None of the above

20. (3 pts.) Now use algebra to solve in the frequency domain for the Laplace transform of the solution to the IVP.
A. $Y = \dfrac{2s+1}{s^2+4}$, B. $Y = \dfrac{2s}{s^2+4}$, C. $Y = \dfrac{-2}{s^2+4}$, D. $Y = \dfrac{s+1}{s^2+4}$,
E. $Y = \dfrac{-s-1}{s^2+4}$, AB. $Y = \dfrac{2s+2}{s^2+4}$, AC. $Y = \dfrac{2s+1}{s^2+4}$, AD. $Y = \dfrac{2s+1}{s^2+4}$,
AE. $Y = \dfrac{2s+1}{s^2+4}$, BC. $Y = \dfrac{2s+1}{s^2+4}$, BD. None of the above

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(5 pts.) True or false. Eigenvalue Problems for Complex Matrices.
Assume A is an n×n square matrix of possibly complex numbers. Under this hypothesis, determine which of the following is true and which is false.

True or False   21. A real square matrix will have only real eigenvalues.

True or False   22. A real square matrix will have only distinct eigenvalues.

True or False 23. A real symmetric matrix will have only real eigenvalues.

True or False   24. A Hermitian matrix will have only real eigenvalues.

True or False   25. A Hermitian matrix will have only distinct eigenvalues.

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Consider the matrix $A = \begin{bmatrix} i & 3 \\ 0 & 1 \end{bmatrix} \in \mathbf{C}^{2x2}$.

26. (1 pt.) The degree of the polynomial where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is _____. A. 1, B. 2, C.3, D. 4, E. 5, AB.6, AC. 7, AD. None of the above.

27. (2 pt.) The polynomial $p(\lambda)$ where the solution of $p(\lambda) = 0$ yields the eigenvalues of A is _____.. A. $p(\lambda) = (\lambda+1)(\lambda - i)$, B. $p(\lambda) = (\lambda-1)(\lambda + i)$, C. $p(\lambda) = (\lambda-1)(\lambda - i)$, D. $p(\lambda) = (\lambda+1)(\lambda + i)$, E. $p(\lambda) = (\lambda-3)(\lambda - i)$, AB. $p(\lambda) = (\lambda-1)(\lambda - 3)$, AC. $p(\lambda) = (\lambda+1)(\lambda + 3)$, AD. None of the above.

28. (1 pt.) Counting repeated roots, the matrix A given above has how many eigenvalues?

A. 0, B 1, C. 2, D. 3, E 4, AB. 5, AC. 6, AD. 7, AE. 8.

29. (2 pts.) The eigenvalues of A are ______..

A. $\lambda_1 = 2$, $\lambda_2 = i$, B. $\lambda_1 = 1$, $\lambda_2 = 2i$, C. $\lambda_1 = 1$, $\lambda_2 = i$, D. $\lambda_1 = 3$, $\lambda_2 = i$, E. $\lambda_1 = 1$, $\lambda_2 = 3i$,
AB. $\lambda_1 = 2$, $\lambda_2 = 2i$, AC. $\lambda_1 = 2$, $\lambda_2 = 3i$, AD. $\lambda_1 = -1$, $\lambda_2 = -i$, AE. $\lambda_1 = -2$, $\lambda_2 = i$, BC. $\lambda_1 = -1$, $\lambda_2 = -2i$, BD. $\lambda_1 = 2$, $\lambda_2 = -i$, BE. $\lambda_1 = -2$, $\lambda_2 = -2i$, CD. None of the above

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30. (5 pts.) $\lambda = 2$ is an eigenvalue of the matrix $A = \begin{bmatrix} 3 & -1 \\ 4 & -2 \end{bmatrix}$. Using the conventions discussed in class (attendance is mandatory), find a basis B for the eigenspace associated with this eigenvalue. Such vectors are usually called the eigenvectors associated with the eigenvalue $\lambda = 2$. Circle the correct answer. Recall that basis sets are not unique so that conventions discussed in class are mandatory.

A. $B = \{[1,1]^T, [4,4]^T\}$, B. $B = \{[1,1]^T\}$, C. $B = \{[1,2]^T\}$, D. $B = \{[1,2]^T, [4,8]^T\}$
E. $B = \{[2,1]^T\}$ AB. $B = \{[1,3]^T\}$ AC. $B = \{[1,4]^T\}$, AD. $B = \{[4,1]^T\}$
AE. $B = \{[3,1]^T\}$, BC. $B = \varnothing$ BD. None of the above.

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31. (3 pts.) Let S = $\{\vec{v}_1,\vec{v}_2,...,\vec{v}_n\}$ $\subseteq$V where V is a vector space. Choose the completion of the following definition of what it means for S to be linearly independent.

Definition. The set S = $\{\vec{v}_1,\vec{v}_2,\ldots,\vec{v}_n\}\subseteq$V where V is a vector space is linearly independent if

A. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n=\vec{0}$ has only the trivial solution $c_1 = c_2 = \cdots = c_n = 0$.

B. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n=\vec{0}$ has an infinite number of solutions.

C. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n=\vec{0}$ has a solution other than the trivial solution.

D. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n=\vec{0}$ has at least two solutions.

E. The vector equation $c_1\vec{v}_1+c_2\vec{v}_2+...+c_n\vec{v}_n=\vec{0}$ has no solution.

AB. The associated matrix is nonsingular. AC. The associated matrix is singular

32. (3 pts.)Now apply this definition to the space of time varying "vectors" $\vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^3)$. That is, explain what is necessary for the set S ={ $[x_1(t), y_1(t), z_1(t)]^T$, $[x_2(t), y_2(t), z_2(t)]^T$, ... , $[x_n(t), y_n(t), z_n(t)]^T$ } $\subseteq \vec{\mathcal{A}}(\mathbf{R},\mathbf{R}^3)$ to be linearly independent. (The transpose notation is used to save space.)

A. The vector equation $c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$ has only the trivial solution $c_1 = c_2 = \cdots = c_n = 0$.

B. The vector equation
$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$
has an infinite number of solutions

C. The vector equation
$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$
has a solution other than the trivial solution.

D. The vector equation
$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$
has at least two solutions.

E. The vector equation
$c_1 [x_1(t), y_1(t), z_1(t)]^T + c_2 [x_2(t), y_2(t), z_2(t)]^T + \cdots + c_n [x_n(t), y_n(t), z_n(t)]^T = [0, 0, 0]^T$ $\forall t \in \mathbf{R}$
has no solution.

AB. The associated matrix is nonsingular. AC. The associated matrix is singular

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33. (4 pts.) You are to determine Directly Using the Definition (DUD) if the following set of time varying "vectors" are linearly independent. Let S= $\{\vec{x}_1,\vec{x}_2\} \subseteq \mathcal{A}\ (\mathbf{R},\mathbf{R}^2)$ where

$\vec{x}_1 = \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix}$ and $\vec{x}_2 = \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix}$. Thus you are to determine if the set S is linearly independent.

Circle the statement that is true.

A. S is linearly independent as $c_1 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + c_2 \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t \in \mathbf{R}$

implies $c_1 = 0$ and $c_2 = 0$.

B. S is linearly independent as $-2 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t \in \mathbf{R}$.

C. S is linearly dependent as $c_1 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + c_2 \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t \in \mathbf{R}$ implies

$c_1 = 0$ and $c_2 = 0$.

D. S is linearly dependent as $-2 \begin{bmatrix} 3e^t \\ 4e^t \end{bmatrix} + \begin{bmatrix} 6e^t \\ 8e^{-t} \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \end{bmatrix}$ $\forall t \in \mathbf{R}$.

E. S is neither linearly independent or linearly dependent as the definition does not apply.

AB. None of the above statements are true.

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34. (4 pts.) Consider the scalar equation $u'' + 4u' - 2z = 0$ where $u = u(t)$ (i.e. the dependent variable u is a function of the independent variable t so that $u' = du/dt$ and $u'' = du/dt$). Convert this to a system of two first order equations by letting $u = x$ and $u' = y$ (i.e. obtain two first order scalar equations in x and y; you may think of $u = x$ as the position and $u' = y$ as the velocity of a point particle). Now write this system of two scalar equations in the vector form $\vec{x}' = A\vec{x}$ where $\vec{x} = \begin{bmatrix} x \\ y \end{bmatrix}$ and A is a 2x2 matrix. This system is_____.

A. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, B. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 4 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, C. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

D. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, E. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AB. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$

AC. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 4 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AD. $\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 2 & -4 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$, AE. None of the above.

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35. (5 pts.) Using the procedure illustrated in class (attendance is mandatory), eliminate x_2 in the following system to obtain a single second order ODE in x_1.

$x_1' = 3x_1 - 2x_2$
$x_2' = 2x_1 - 2x_2$

This ODE is: A. $x_1'' + x_1' + 2x_1 = 0$, B. $x_1'' + x_1' - 2x_1 = 0$, C. $x_1'' - x_1' + 2x_1 = 0$,
D. $x_1'' - x_1' - 2x_1 = 0$, E. $x_1'' + 2x_1' + x_1 = 0$, AB. $x_1'' + 2x_1' - x_1 = 0$,
AC. $x_1'' - 2x_1' + x_1 = 0$,
AD. $x_1'' - 2x_1' - x_1 = 0$, AE. $x_1'' + 2x_1' + 2x_1 = 0$, BC. $x_1'' + 2x_1' - 2x_1 = 0$,
BD. $x_1'' - 2x_1' + 2x_1 = 0$, BE. $x_1'' - x_1' - 2x_1 = 0$, CD. None of the above.

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Write in the correct letter(s) for the possible answers given below.

DEFINITION. Let f:X→Y. Then f is one-to-one if ∀ x_1, x_2∈X, we have: 36.(2 pts.) ___________

.

THEOREM. Let T:V→W be a linear operator where V and W are vector spaces over the same field **K**. If the null space N_T is $\{\vec{0}\}$, then T is a one-to-one mapping.

Proof. We begin our proof of the theorem by first proving the following lemma:

Lemma. If N_T = $\{\vec{0}\}$ and $T(\vec{v}) = \vec{0}$, then $\vec{v} = \vec{0}$.

Proof of lemma: Let us assume 37.(1 pts.) _______ (give the hypothesis of the lemma) . By the definition

of the null space, N_T = { $\vec{v}$ ∈V: 38.(2 pts.) _________} so that 39.(1 pts.) ________ implies that $\vec{v}$ ∈N_T.

Since 40.(1 pts.) ________, we have that 41 (1pt.) _____________ as was to be proved.

QED for lemma.

Having finished the proof of the lemma, we now continue with the proof of the theorem. To show that T

is one-to-one, we assume 42.(1 pts.) ____________ and show that 43.(1 pts.) ____________.

We use the STATEMENT/REASON FORMAT.

STATEMENT	REASON
$T(\vec{v}_1) = T(\vec{v}_2)$	44. (1 pt.) ________
45. (1pt.) ________	Vector algebra in W
$\vec{v}_1 - \vec{v}_2 = \vec{0}$	46(1 pt.) ________
$\vec{v}_1 = \vec{v}_2$	47. (1 pt.) ________

Hence T is one-to-one as was to be proved.

QED for the theorem.

Possible answers.

A. $f(x_1) = f(x_2)$ implies $x_1 = x_2$, B. $f(x_1) = f(x_2)$, C.$x_1 = x_2$, D. $x_1 = x_2$ implies $f(x_1) = f(x_2)$, E. N_T = $\{\vec{0}\}$ and T($\vec{v}$) = $\vec{0}$, AB. $T(\alpha\vec{v}_1) = \alpha T(\vec{v}_1)$, AC. $T(\alpha\vec{v}_1 + \beta\vec{v}_2) = \alpha T(\vec{v}_1) + \beta T(\vec{v}_2)$,

AD. T($\vec{v}$) = $\vec{0}$, AE. N_T = $\{\vec{0}\}$, BC. Hypothesis (or Given), BD. $T(\vec{v}_1) = T(\vec{v}_2)$,

BE. The lemma proved above, CD. Vector algebra in V, CE. Vector algebra in W

DE. $\vec{v}$ ∈N_T ABC. $\vec{v}_1 = \vec{v}_2$, ABD. $T(\vec{v}_1 - \vec{v}_2)$, ABE $T(\vec{v}_1 - \vec{v}_2) = \vec{0}$, BCD. $\vec{v}_1 - \vec{v}_2 = \vec{0}$

BCE. $\vec{v} = \vec{0}$, BDE. T is a one-to-one mapping CDE. T is a linaer operator, ABCD. Definition of T, ABCE .Theorems from Calculus, ACDE. Definition of T, BCDE, None of the above.

Total points this page = 14. TOTAL POINTS EARNED THIS PAGE ________

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48. (5 pts.) You are to solve the following Boundary Value Problem (BVP).

BVP
- ODE $y'' + y = 0.$
- BC's $y(0) = 0, \quad y(\pi) = 0$

The solution set S for this BVP is S = _______.

A. $\emptyset$ (i.e. the BVP has no solution) B. $\{\sin x\}$ C. $\{\cos x\}$

D. $\{c \sin x : c \in \mathbf{R}\}$ E. $\{c \cos x : c \in \mathbf{R}\}$ AB. $\{\tan x\}$ AC. $\{c \tan x : c \in \mathbf{R}\}$,

AD. None of the above sets is the solution set for this boundary value problem.

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TABLE OF LAPLACE TRANSFORMS THAT NEED NOT BE MEMORIZED

$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$	Domain F(s)
t^n n = positive integer	$\frac{n!}{s^{n+1}}$	$s > 0$
sinh (at)	$\frac{a}{s^2 - a^2}$	s > *a *
cosh (at)	$\frac{s}{s^2 - a^2}$	s > *a *
$e^{at}\sin(bt)$	$\frac{b}{(s-a)^2 + b^2}$	$s > a$
$e^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2 + b^2}$	$s > a$
$t^n e^{at}$ n = positive integer	$\frac{n!}{(s-a)^{n+1}}$	$s > a$
$u(t)$	$\frac{1}{s}$	$s > 0$
$u(t-c)$	$\frac{e^{-cs}}{s}$	$s > 0$
$e^{ct}f(t)$	$F(s-c)$	
$f(ct)$ $c > 0$	$\frac{1}{c} F\left(\frac{s}{c}\right)$	
$\delta(t)$	1	
$\delta(t-c)$	e^{-cs}	