EXAM-3	-B2
FALL 201	2

MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

PRINT NAME		()
Last Name,	First Name MI	(What	you v	wish to be	e called)
ID#	EXAM DAT	E <u>Friday,</u>	Oct.	29, 2012	11:30am
	ne work presented on this exam is my or received any help during the exam.	wn	Date _		Signatur
				Scores	
SIGNATURE	DATE	<u> </u>	age	points	score
INSTRUCTIONS: Besides this c	over page, there are 12 pages of question	ons	1	6	
	KE SURE YOU HAVE ALL THE		2	7	
	u will receive a grade of zero for that		3	10	
	am. If you cannot read anything, raisePlace your I.D. on your desk during t	ho	4	6	
	a straight edge are all that you may hav				
on your desk during the exam. N	O CALCULATORS! NO SCRATC	H _	5	9	
	am sheets if necessary. You may remove		6	12	
	name on all sheets. Pages 1-12 are Fill True/False. Expect no part credit on the		7	16	
<u> •</u>	Multiple Choice question write your	_	8	3	
	ext find your answer from the list given	_			
	r or letters for your answer in the blank or letters. There are no free response	_	9	9	
	, you should explain your solutions fully	y	10	6	
	on may be graded, not just your final		11	9	
	I Every thought you have should be cs on this paper. Partial credit will be		12	7	
=	poofread your solutions and check your	_		 	
	OOD LUCK!!	_	13		
		- ,	14		
REQUES'	Γ FOR REGRADE		15		
Please regard the following problem (e.g., I do not understand what I di	blems for the reasons I have indicated: I did wrong on page)		16		
			17		
		\dashv \lceil	18		
		\dashv \sqcap	19		
1	within a week of the date the exam is	,	20		
returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have written			21		
	n this REGRADE FORM. (Writing or				
changing anything is considered	to be cheating.)		22		

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. We do not solve the differential equation L[y] = g(x) where L[y] = y'' + y is a linear operator that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ by isolating the unknown function. We use the linear theory. The dimension of the null space of L[y] is 2. Since the operator L[y] = y'' + y has constant coefficients, we assume a solution of the homogeneous equation L[y] = 0 of the form $y = e^{rx}$. This leads to the two linearly independent solutions $y_1 = \cos(x)$ and $y_2 = \sin(x)$ so that a basis of the nullspace of L is $B = \{\cos(x), \sin(x)\}$. Hence we can deduce that $y_c = c_1 \cos(x) + c_2 \sin(x)$ is the general solution of the homogeneous equation y'' + y = 0.

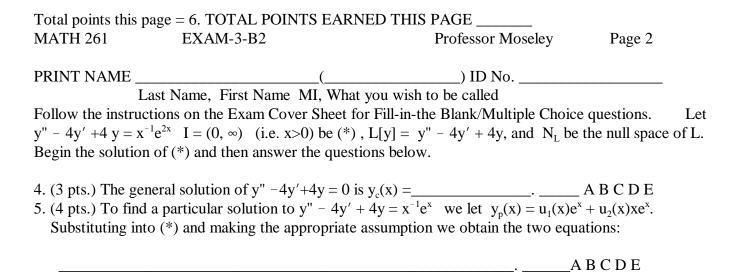
To use the linear theory to obtain the general solution of the nonhomogeneous equation L[y] = g(x), we need a particular solution, y_p , to y'' + y = g(x). We have studied two techniques for this purpose (attendance is required):

- i) Undetermined Coefficients (also called judicious guessing)
- ii) Variation of Parameters (also called variation of constants)

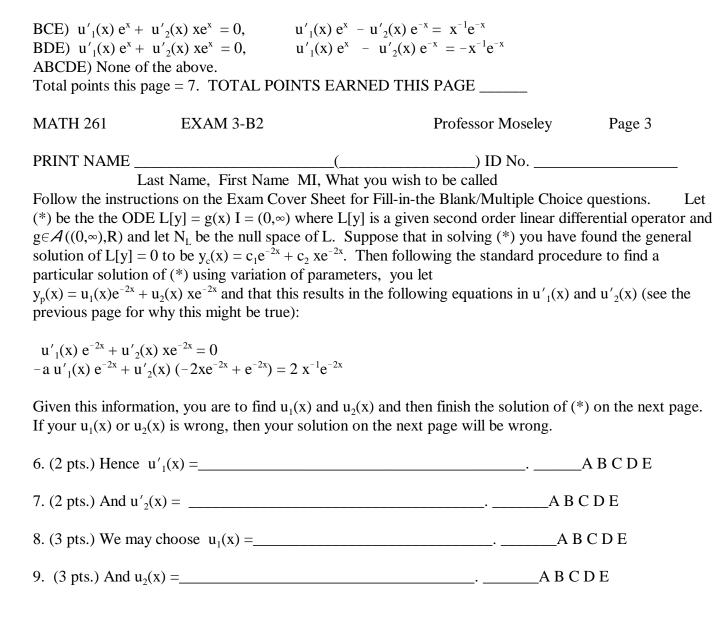
For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation y'' + y = g(x):

1. (2 pts.)
$$g(x) = 2x^{-1} e^{x}$$
 _____ A B C D E
2. (2 pts.) $g(x) = 3 e^{x}$ _____ A B C D E
3. (2 pts.) $g(x) = 4 \sec(x)$ _____ A B C D E

- A) Neither technique works to find y_p.
- B) Only Undetermined Coefficients works to find y_n.
- C) Only Variation of Parameters works to find y_p.
- D) Either technique works to find y_p .
- E) Not enough information is given.
- AB) Too much information is given.
- AC) All of the above statements are true.
- AD) None of the above statements are true.



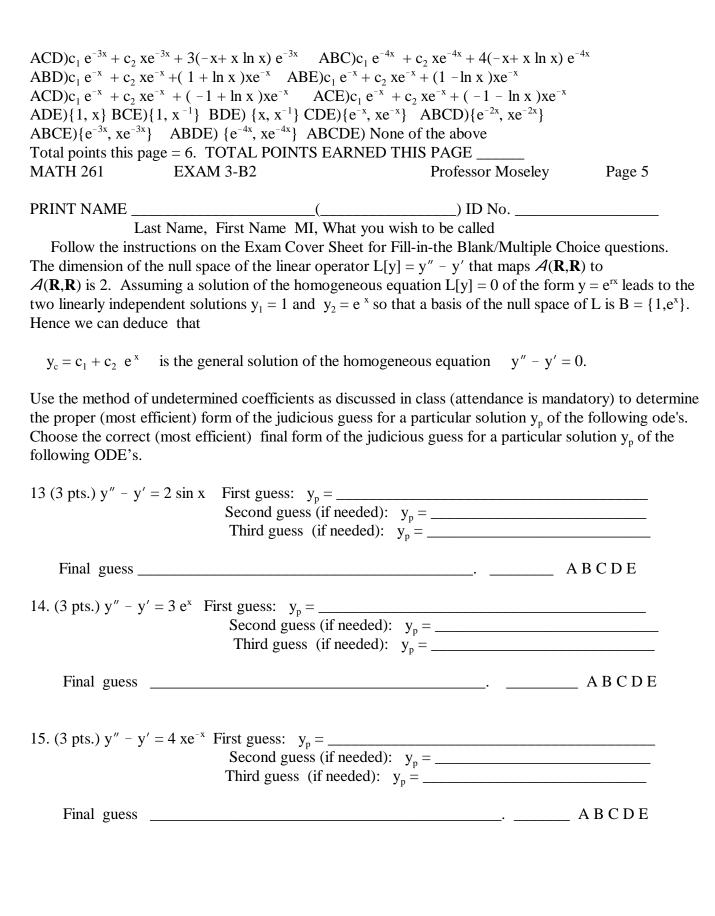
Possible answers this page



Possible answers this	page.		
A) 1 B) 2 C) 3 D) 4	E) -1 AB) -2 AC) -3 A	D) -4 AE) x BC) 2x BD) 3x BE) 4x CD) -x	
		$2x^{-1}$ BCD) $3x^{-1}$ BCE) $4x^{-1}$ BDE) $\ln x$	
, ,	3 ln x ABCE) 4 ln x ABC		
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MATH 261	EXAM 3-B2	Professor Moseley Page 4	
PRINT NAME	(_) ID No	
	Name, First Name MI, Wh		
Follow the instruction	s on the Exam Cover Sheet	for Fill-in-the Blank/Multiple Choice questions page. Using the data from the previous page, find a	
	I the general solution to (*)		
10 (2 pts.) A particular solution to (*) is			
$y_{n}(x) = $		A B C D E	
(Recall that particul mandatory.)	ar solutions are not unique.	Use the procedure given in class. (Attendance is	
• •	al solution to (*) may be wr	itten as	
y(x) =		A B C D E	
12. (2 pts.) A basis for	r N _L is B =	A B C D E	
		ocedure given in class. (Attendance is mandatory.	

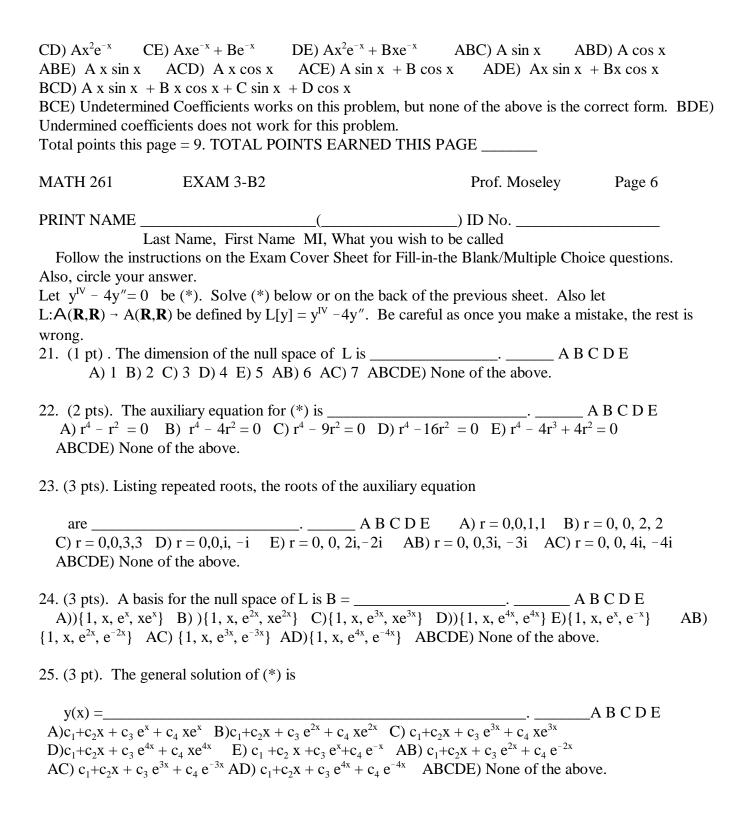
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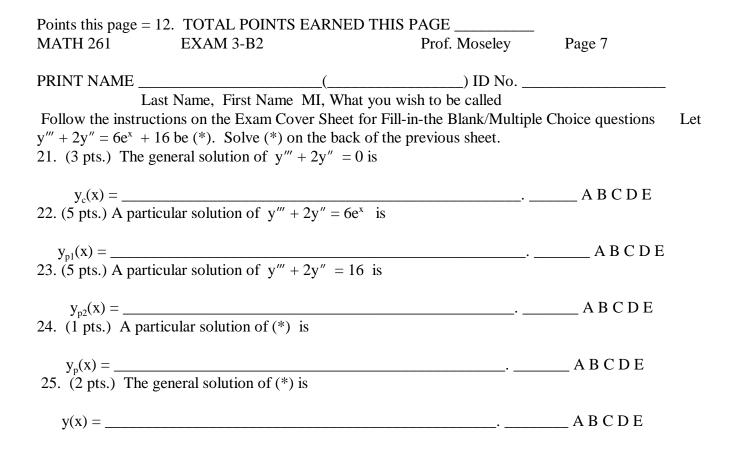
A) x^{-1} B) $-x^{-1}$ C) x^{-2} D) $-x^{-2}$ E) $\ln x$ AB) $-\ln x$ AC) $x \ln x$ AD) $-x \ln x$ AE) $(-x + x \ln x) e^{-x}$ BC) $2 (-x + x \ln x) e^{-2x}$ BD) $3 (-x + x \ln x) e^{-3x}$ BE) $4 (-x + x \ln x) e^{-4x}$ CD) $(1 + \ln x) x e^{-x}$ CE) $2(1 - \ln x) x e^{-x}$ DE) $3(-1 + \ln x) x e^{-x}$ ABC) $4(-1 - \ln x) x e^{-x}$ ABD) $c_1 e^{-x} + c_2 x e^{-x} + (-x + \ln x) e^{-x}$ ABE) $c_1 e^{-2x} + c_2 x e^{-2x} + 2(-x + x \ln x) e^{-2x}$



Possible Answers for Final Guesses.

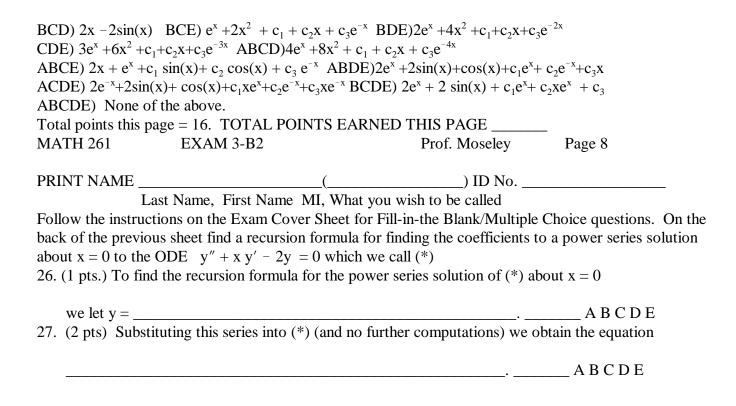
A) A B)
$$Ax + B$$
 C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB) Ae^x AC) Axe^x AD) Ax^2e^x AE) $Axe^x + Be^x$ BC) $Ax^2e^x + Bxe^x$ BD) Ae^{-x} BE) Axe^{-x}





Possible answers this page

A)
$$c_1 + c_2 x + c_3 e^{-x}$$
 B) $c_1 + c_2 x + c_3 e^{-2x}$ C) $c_1 + c_2 x + c_3 e^{-3x}$ D) C) $c_1 + c_2 x + c_3 e^{-4x}$ E) $c_1 + c_2 \sin(x) + c_3 \cos(x)$ AB) $c_1 e^{-x} + c_2 \sin(x) + c_3 \cos(x)$ AC)) e^x AD) $2e^x$ AE)) $3e^x$ BC) $4e^x$ BD) x^2 BE) $2x^2$ CD) $3x^2$ CE) $4x^2$ DE) $6x^2$ ABC) $8x^2$ ABD) $e^x + 2x^2$ ABE) $2e^x + 4x^2$ ACD) $3e^x + 6x^2$ ACE) $4e^x + 8x^2$ ADE) $2e^x + 2\sin(x)$



Possible answers this page

A)
$$\sum_{n=1}^{\infty} a_n x^n$$
 B) $\sum_{n=0}^{N} a_n x^n$ C) $\sum_{n=0}^{\infty} a_n x^n$ D) $\sum_{n=0}^{\infty} a_n (n+1) x^{n+1}$ E) $\sum_{n=0}^{\infty} a_{n+2} x^n$ AB) $\sum_{n=2}^{\infty} a_{n+2} x^n$ AC) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + x \sum_{n=0}^{\infty} a_n n x^{n-1} + \sum_{n=0}^{\infty} a_n x^n = 0$

AD)
$$\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + x \sum_{n=0}^{\infty} a_n n x^{n-1} + 2 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$AE) \ \sum\nolimits_{n = 0}^\infty {{a_n}n(n - 1){x^{n - 2}}} + x\sum\nolimits_{n = 0}^\infty {{a_n}n{x^{n - 1}}} + 3\sum\nolimits_{n = 0}^\infty {{a_n}{x^n}} = 0$$

BC)
$$\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + x \sum_{n=0}^{\infty} a_n n x^{n-1} + 4 \sum_{n=0}^{\infty} a_n x^n = 0$$

$$BD) \sum\nolimits_{n = 0}^\infty {{a_n}n(n - 1){x^{n - 2}}} + \sum\nolimits_{n = 0}^\infty {{a_n}n{x^{n - 1}}} - \sum\nolimits_{n = 0}^\infty {{a_n}{x^n}} \, = 0$$

$$BE) \ \sum\nolimits_{n = 0}^\infty {{a_n}n(n + 1){x^{n - 2}}} + x\sum\nolimits_{n = 0}^\infty {{a_n}n{x^{n - 1}}} - 2\sum\nolimits_{n = 0}^\infty {{a_n}{x^n}} = 0$$

$$CD) \ \sum\nolimits_{n = 0}^\infty {{a_n}n(n + 1){x^{n - 2}}} + \sum\nolimits_{n = 0}^\infty {{a_n}n{x^{n - 1}}} - 3\sum\nolimits_{n = 0}^\infty {{a_n}{x^n}} = 0$$

$$CE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n+1) x^{n-2} + x \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} - 4 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2 \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE) \ \ \sum\nolimits_{n=0}^{\infty} a_n n x^n = 0 \\ DE$$

ABCDE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE ____

MATH 261 EXAM 3-B2 Prof. Moseley

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PRINT NAME ______(_____) ID No. ______

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Let (*) be as on the previous page.

28. (3 pts) As explained in class (attendance is mandatory) by changing the index and simplifying, the term $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2}$ can be changed to obtain

$$\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} = \underline{\hspace{1cm}} A B C D E$$

29. (3 pts) Continuing the procedure given in class, using this new term and other simplifications, the equation you obtained on the previous page can now be written

as ______. ___ A B C D E

30. (3 pts.) The recursion formula for finding the coefficients in the power series solution of (*)

is______. ____A B C D E

Possible answers this page.

$$A) \ \sum\nolimits_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n \qquad B) \ \sum\nolimits_{n=0}^{\infty} a_{n+2}(n+2)x^n \qquad C) \ \sum\nolimits_{n=1}^{\infty} a_{n+2}(n+1)x^n$$

B)
$$\sum_{n=0}^{\infty} a_{n+2}(n+2)x^n$$

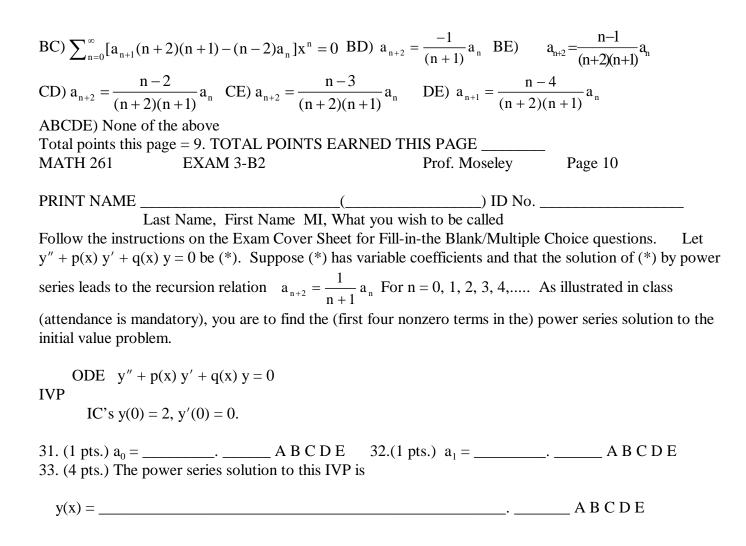
C)
$$\sum_{n=1}^{\infty} a_{n+2}(n+1)x^{n}$$

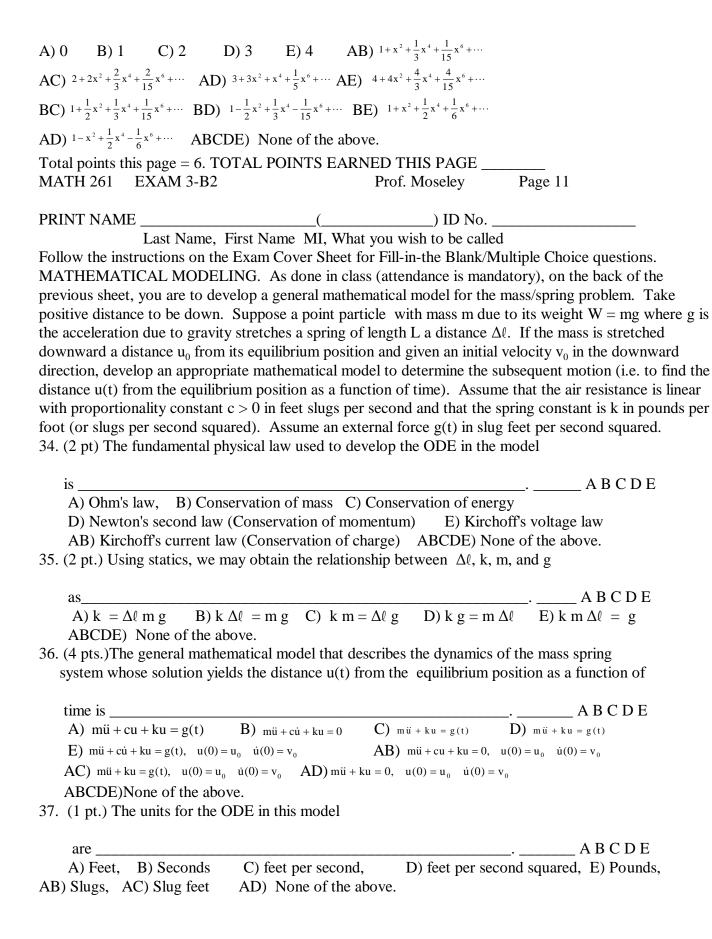
D)
$$\sum_{n=0}^{\infty} a_n(n+2)(n+1)x^n$$

$$D) \sum_{n=0}^{\infty} a_n (n+2)(n+1) x^n \qquad E) \sum_{n=0}^{\infty} a_{n+1} (n+2)(n+1) x^n$$

$$AB) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + (n-1)a_n]x^n = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+1)(n+2)a_{n+2} + (n-2)a_n]x^n = 0 \ AC$$

$$\text{AD) } \sum\nolimits_{n=1}^{\infty} [a_{n+2}(n+2)(n+1) + (n-3)a_n]x^n = 0 \qquad \text{AE) } \sum\nolimits_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) + (n-4)a_n]x^n = 0$$





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				12	
PRINT NAME _		() ID No		
L	Last Name, First Name	MI, What you	wish to be called		
Follow the instruc	ctions on the Exam Cov	ver Sheet for Fill	-in-the Blank/Multiple C	Choice questions.	Also,
circle your answe	r.		_	_	
MATHEMATICA	AL MODELING. Con	sider the followi	ng problem (DO NOT S	SOLVE!):	
A mass weigh	ning 4 lbs. stretches a sp	oring (which is 1	2 ft. long) 3 inches. If t	the mass is lowere	ed 4
inches below its e	quilibrium position and	given an initial	velocity of 4 ft./sec. upw	vard, determine th	he
subsequent motio	n (i.e. find the distance	from the equilib	rium position as a functi	ion of time). Assu	ume that
the air resistance	is negligible and that th	ere is no externa	l force.		
On the back of	of the previous sheet, ap	oply the data give	en above to the model ye	ou developed on t	the
previous page to	obtain the specific mod	el for this proble	m. DO NOT SOLVE!	Then answer the	questions
below.					
38. (3 pts.) The sp	pring constant k in pou	nds per foot (or	slugs per second square	d) is	
1 _r _				ADCDE	
$K = \underline{\qquad}$	ODE in the analific mat	thamatical made	l for the mass spring syst	ABCDE	
			on from the equilibrium		
data above wild	ose solution yields the t	distance u(t) dov	on from the equilibrium	i position as a	
function of time	e is		·	ABCDE	
40. (1 pt.) The ini	itial position for the spe	ecific mathematic	al model for the mass sp	pring system	
from the data	above whose solution	yields the distance	ce u(t) down from the	equilibrium	
position as a fo	unction of time is $u(0)$	=	-	A B C D E	
41. (1 pt.) The in	iitial velocity for the spo	ecific mathematic	cal model for the mass s	pring system	
from the data	above whose solution y	rields the distanc	e u(t) down from the e	equilibrium	
position as a fi	unction of time is $\dot{u}(0)$	· =		ABCDE	

A)1 B)2 C)4 D)5 E) 6 AB)4/3 AC) 8/5 AD)32 AE)48 BC)-3 BD) -4 BE) -5 CD) -6

CE) 1/6 DE)1/3 ABC)1/4 ABD) 1/2 ABE) 5/(12) ACD) 3/2 ADE) $\frac{1}{16}\ddot{u} + u = 0$ BCD) $\frac{1}{8}\ddot{u} + \frac{4}{3}u = 0$

 $BDE) \ \frac{3}{16}\ddot{u} + \frac{3}{2}u = 0 \qquad CDE) \ \frac{1}{4}\ddot{u} + \frac{8}{5}u = 0 \qquad ABCD) \frac{1}{8}\ddot{u} + 3u = 0 \quad ABCE) \frac{1}{8}\ddot{u} + 48u = 0 \quad ABDE) \frac{1}{8}\ddot{u} + 12u = 0$

ABCDE) None of the above.

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _____