EXAM-3 -A4 FALL 2010

MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

22

PRINT NAME		()
Last Name,	First Name MI	(What you v	vish to be	e called)
ID#	EXAM DATE	Friday, Oct.	<u>29, 2010</u>	11:30am
I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.		Date _		Signature
			Score	s
SIGNATURE	DATE	page	points	score
INSTRUCTIONS: Besides this c	over page, there are 12 pages of questions	1	6	
and problems on this exam. MA	KE SURE YOU HAVE ALL THE	2	7	
	ou will receive a grade of zero for that cam. If you cannot read anything, raise	3	10	
your hand and I will come to you	4	6		
	a straight edge are all that you may have IO CALCULATORS! NO SCRATCH	5	9	
	am sheets if necessary. You may remove	6	12	
	name on all sheets. Pages 1-12 are Fill- Γrue/False. Expect no part credit on these	7	16	
pages. For each Fill-in-the Blank	8	3		
answer in the blank provided. No and write the corresponding letter	9	9		
provided. Then circle this letter of	10	6		
pages. However, to insure credit and carefully. Your entire solution	11	9		
answer. SHOW YOUR WORK expressed in your best mathemati	12	7		
given as deemed appropriate. Pro	13			
computations as time allows. G	OD LUCK!!	14		
REQUES'	T FOR REGRADE	15		
Please regard the following prob	blems for the reasons I have indicated:	16		
(e.g., I do not understand what I	I did wrong on page)	17		
		18		
		19	+	
	within a week of the date the exam is ets as necessary to explain your reasons.)	20	+	
I swear and/or affirm that upon	21	+	+	
nothing on this exam except or	21			

changing anything is considered to be cheating.)

Total	100	
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PRINT NAME		() ID No	
	Last Name, First Name	II, What you wish to be called	

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

The dimension of the null space of the linear operator L[y] = y'' + y that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Since the operator L[y] = y'' + y has constant coefficients, we assume a solution of the homogeneous equation L[y] = 0 of the form $y = e^{rx}$. This leads to the two linearly independent solutions $y_1 = \cos(x)$ and $y_2 = \sin(x)$ so that a basis of the nullspace of L is $B = \{\cos(x), \sin(x)\}$. Hence we can deduce that $y_c = c_1 \cos(x) + c_2 \sin(x)$ is the general solution of the homogeneous equation y'' + y = 0.

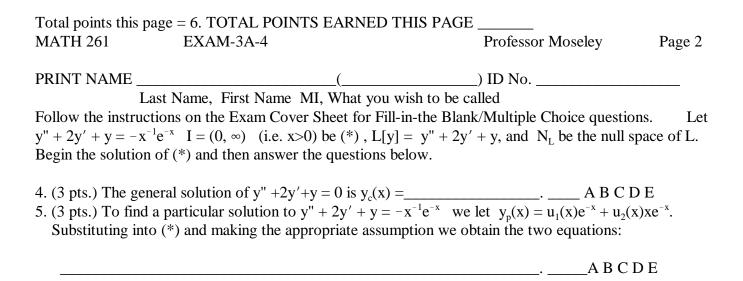
To use the linear theory to obtain the general solution of the nonhomogeneous equation L[y] = g(x), we need a particular solution, y_p , to y'' + y = g(x). We have studied two techniques for this purpose (attendance is required):

- i) Undetermined Coefficients (also called judicious guessing)
- ii) Variation of Parameters (also called variation of constants)

For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation y'' + y = g(x):

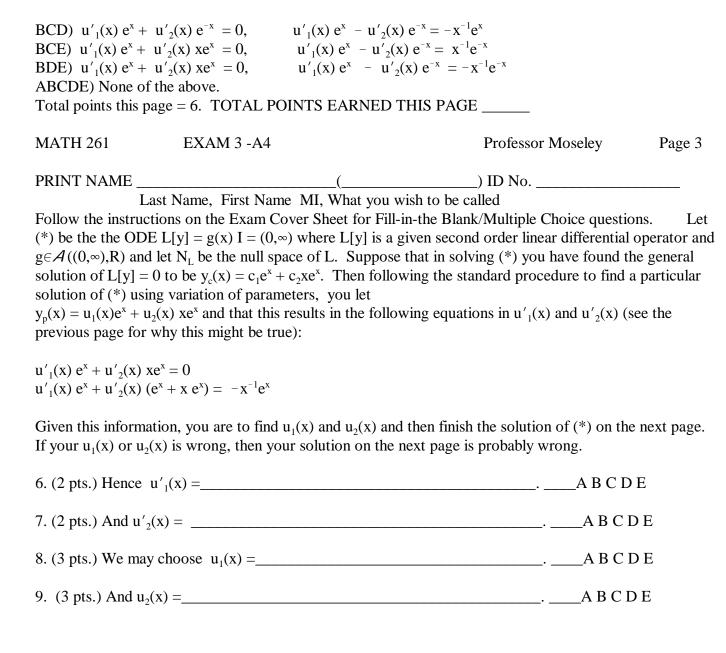
1. (2 pts.)
$$g(x) = \sec(x)$$
 _____ A B C D E
2. (2 pts.) $g(x) = x^{-1}e^{-x}$ ____ A B C D E
3. (2 pts.) $g(x) = x e^{x}$ ____ A B C D E

- A) Neither technique works to find y_p .
- B) Only Undetermined Coefficients works to find y_p.
- C) Only Variation of Parameters works to find y_p.
- D) Either technique works to find y_p .
- E) Not enough information is given.
- AB) Too much information is given.
- AC) All of the above statements are true.
- AD) None of the above statements are true.



Possible answers this page

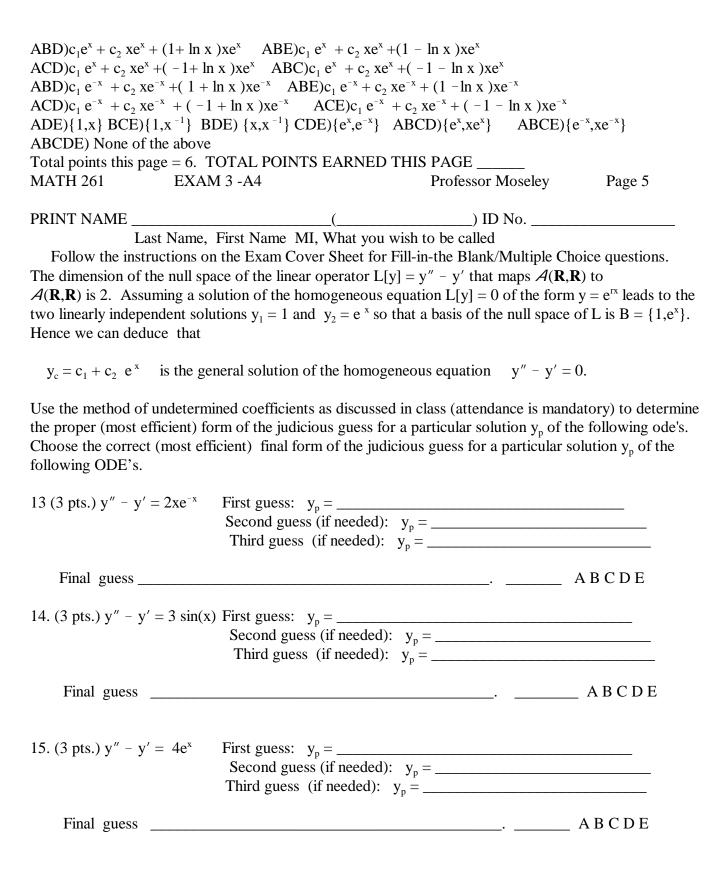
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A) c_1 \cos(x) + c_2 \sin(x) B) c_1 \cos(2x) + c_2 \sin(2x), C) c_1 e^x + c_2 e^{-x} D) c_1 x + c_2 E) c_1 e^x + c_2 x e^x AB) c_1 e^{-x}
+ c_2 x e^{-x} AC) r = \pm i AD) r = \pm 1 AE) r = \pm 2i BC) r = \pm 2i BD) 1,1
                                                                                                           BE) -1.-1
                                                      u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = xe^{x}
CD) u'_{1}(x) e^{x} + u'_{2}(x) xe^{x} = 0,
                                                      u'_{1}(x) e^{x} + u'_{2}(x) (e^{x}+xe^{x}) = -xe^{x}
CE) u'_1(x) e^x + u'_2(x) x e^x = 0,
                                                     u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = x^{-1}e^{x}
DE) u'_1(x) e^x + u'_2(x) x e^x = 0
                                                     u'_{1}(x) e^{x} + u'_{2}(x) (e^{x}+xe^{x}) = -x^{-1}e^{x}
ABC) u'_{1}(x) e^{x} + u'_{2}(x) xe^{x} = 0,
                                                      -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = xe^{-x}
ABD) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                      -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = -xe^{-x}
ABE) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                        -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = x^{-1}e^{-x} 0
ACD) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                      -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = -x^{-1}e^{-x}
ACE) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                       u'_{1}(x) e^{x} - u'_{2}(x) e^{-x} = x^{-1}e^{x}
ADE) u'_{1}(x) e^{x} + u'_{2}(x) e^{-x} = 0,
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Possible answers this page. A) 1 B) -1 C) x D) -x E) x^{-1} AB) $-x^{-1}$ AC) x^{-2} AD) $-x^{-2}$ AE) $\ln x$ BC) $-\ln x$ BD) x ln x BE) $-x \ln x$ CD) $(1/2) x^2 \ln x - (1/4)x^2$ CE) $-(\frac{1}{2}) x^2 \ln x + (\frac{1}{4})x^2$ $DE)e^{x}$ $ABC)-e^{x}$ $ABD)xe^{x}$ $ABE)-xe^{x}$ $BCD)e^{-x}$ BCE) $-e^{-x}$ $BDE)xe^{-x}$ CDE) $-xe^{-x}$ ABCDE) None of the above Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _____ MATH 261 EXAM 3 -A4 Professor Moseley Page 4 PRINT NAME _______ (_______) ID No. ______ Last Name, First Name MI, What you wish to be called Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions Let (*), L[y], and N_L be as on the previous page. Using the data from the previous page, find a particular solution and the general solution to (*). Also find a basis for N₁. 10 (2 pts.) A particular solution to (*) is $y_p(x) =$ _______. A B C D E (Recall that particular solutions are not unique. Use the procedure given in class. Attendance is mandatory.) 11 (2 pts.) The general solution to (*) may be written as $y(x) = \underline{\hspace{1cm}} A B C D E$ 12. (2 pts.) A basis for N_L is ______. A B C D E (Recall that a basis is not unique. Use the procedure given in class. Attendance is mandatory.)

Possible answers this page.

A)
$$x^{-1}$$
 B) $-x^{-1}$ C) x^{-2} D) $-x^{-2}$ E) $\ln x$ AB) $-\ln x$ AC) $x \ln x$ AD) $-x \ln x$ AE) $(1 + \ln x) x e^x$ BC) $(1 - \ln x) x e^x$ BD) $(-1 + \ln x) x e^x$ BE) $(-1 - \ln x) x e^x$ CD) $(1 + \ln x) x e^{-x}$ CE) $(1 - \ln x) x e^{-x}$ DE) $(-1 + \ln x) x e^{-x}$ ABC) $(-1 - \ln x) x e^{-x}$

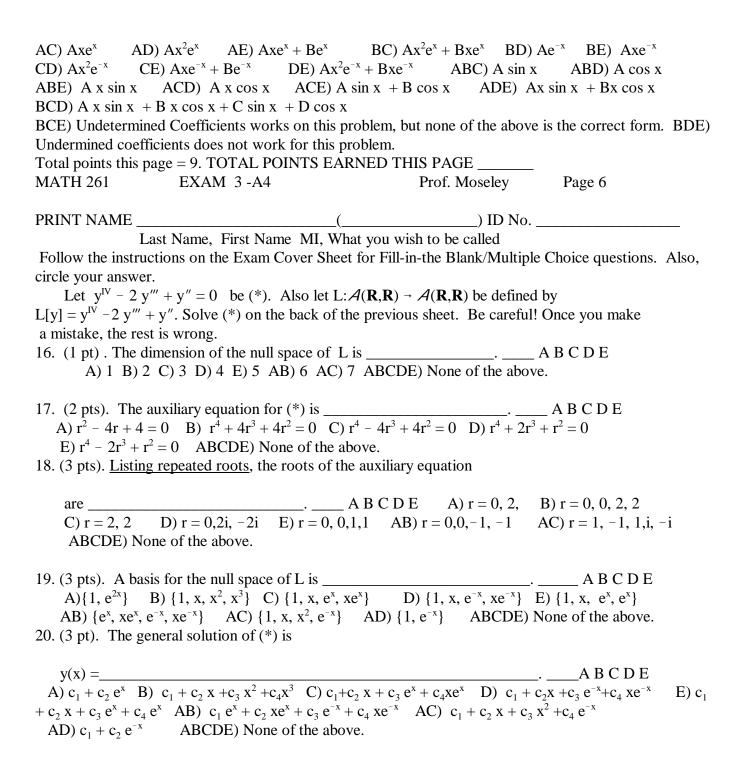


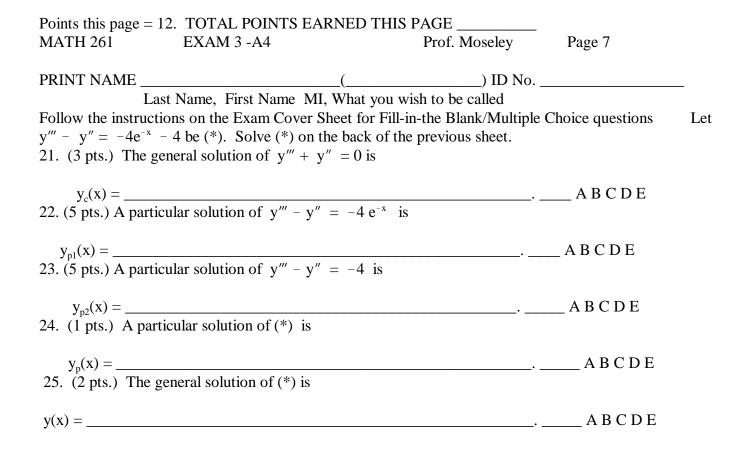
Possible Answers for Final Guesses.

$$B) Ax + B$$

C)
$$Ax^2 + Bx + C$$

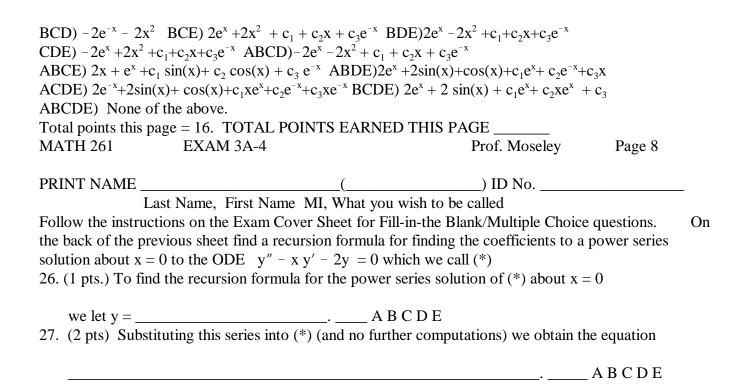
B)
$$Ax + B$$
 C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB) Ae^x



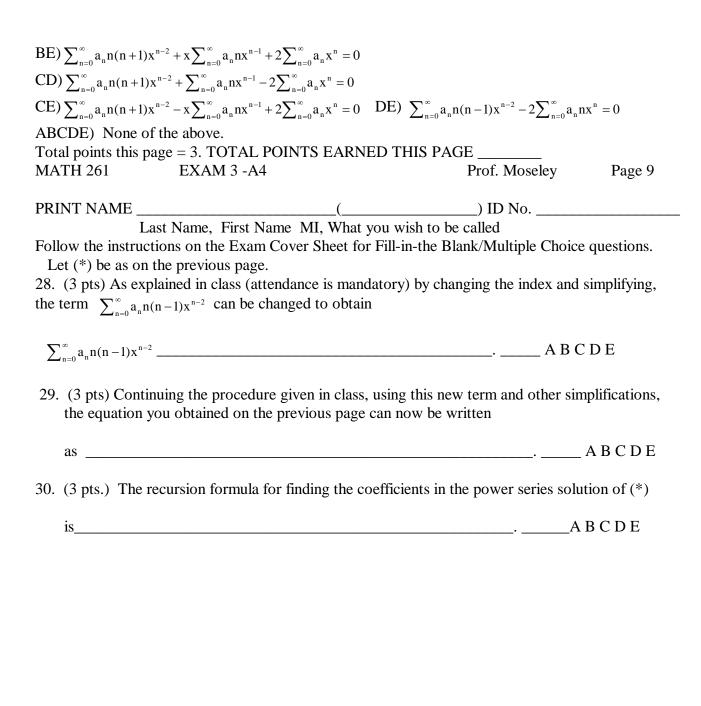


Possible answers this page

A)
$$c_1 + c_2 x + c_3 e^x$$
 B) $c_1 + c_2 x + c_3 e^{-x}$ C) $c_1 + c_2 e^x + c_3 e^{-x}$ D) $c_1 e^x + c_2 \sin(x) + c_3 \cos(x)$ E) $c_1 + c_2 \sin(x) + c_3 \cos(x)$ AB) $c_1 e^{-x} + c_2 \sin(x) + c_3 \cos(x)$ AC)) $2e^x$ AD) $-2e^x$ AE)) $2x e^x$ BC) $-2x e^x$ BD) $2x^2$ BE) $-2x^2$ CD) $2x^2$ CD) $2x^2$ CE) $-2x^2$ DE) $2x^2$ ABC) $2x^2$ ABC) $2x^2$ ACD) $2x^2$ ACD)



$$\begin{split} A) \ \sum\nolimits_{n=1}^{\infty} a_n x^n & B) \ \sum\nolimits_{n=0}^{N} a_n x^n & C) \ \sum\nolimits_{n=0}^{\infty} a_n x^n & D) \ \sum\nolimits_{n=0}^{\infty} a_n (n+1) x^{n+1} & E) \ \sum\nolimits_{n=0}^{\infty} a_{n+2} x^n \\ AB) \ \sum\nolimits_{n=2}^{\infty} a_{n+2} x^n & AC) \ \sum\nolimits_{n=0}^{\infty} a_n n (n-1) x^{n-2} + \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} + 2 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \\ AD) \ \sum\nolimits_{n=0}^{\infty} a_n n (n-1) x^{n-2} + x \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} + 2 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \quad AE) \ \sum\nolimits_{n=0}^{\infty} a_n n (n-1) x^{n-2} + \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} - 2 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \\ BC) \ \sum\nolimits_{n=0}^{\infty} a_n n (n-1) x^{n-2} - x \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} + 2 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \\ BD) \ \sum\nolimits_{n=0}^{\infty} a_n n (n-1) x^{n-2} - \sum\nolimits_{n=0}^{\infty} a_n n x^{n-1} - 2 \sum\nolimits_{n=0}^{\infty} a_n x^n = 0 \\ \end{split}$$



Possible answers this page.

$$A) \ \sum\nolimits_{n = 0}^\infty {{a_{n + 2}}(n + 2)(n + 1){x^n}} \qquad B) \ \sum\nolimits_{n = 0}^\infty {{a_{n + 2}}(n + 2){x^n}} \qquad C) \ \sum\nolimits_{n = 1}^\infty {{a_{n + 2}}(n + 1){x^n}}$$

B)
$$\sum_{n=0}^{\infty} a_{n+2}(n+2)x^n$$

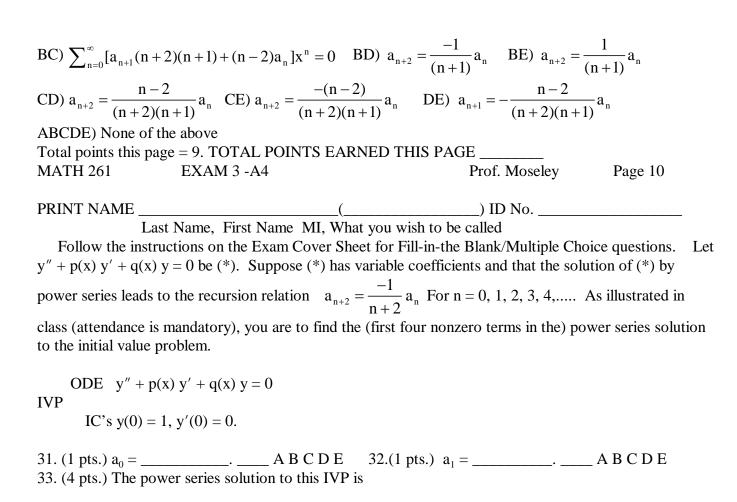
C)
$$\sum_{n=1}^{\infty} a_{n+2}(n+1)x^{n}$$

$$D) \sum_{n=0}^{\infty} a_n (n+2)(n+1) x^n$$

$$D) \sum\nolimits_{n = 0}^\infty {{a_n}(n + 2)(n + 1){x^n}} \qquad E) \sum\nolimits_{n = 0}^\infty {{a_{n + 1}}(n + 2)(n + 1){x^n}}$$

$$AB) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + (n+2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+1)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n}]x^{n} = 0 \ AC) \ \sum\nolimits_{n=0}^{\infty} [(n+2)(n+2)a_{n+2} + (n-2)a_{n+2} + (n-2)a_{n+2}$$

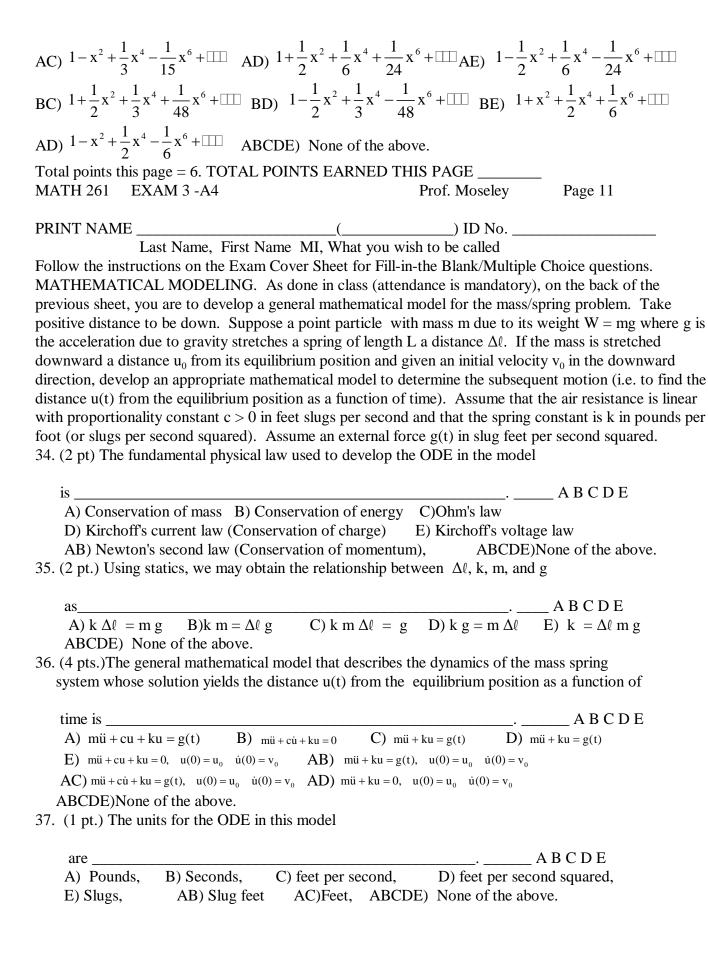
$$AD) \ \sum\nolimits_{n=1}^{\infty} [a_{n+2}(n+2)(n+1) - (n-2)a_{n}]x^{n} = 0 \quad AE) \sum\nolimits_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) - (n+2)a_{n}]x^{n} = 0$$



 $y_1(x) = \underline{\hspace{1cm}}$. A B C D E

Possible answers this page.

C) -1 D) 2 E) -2 AB)
$$1+x^2+\frac{1}{3}x^4+\frac{1}{15}x^6+$$



Total points this p	age = 9. TOTAL POIN	NTS EARNED T	THIS PAGE	
MATH 261	EXAM 3 -A4		Prof. Moseley Page 12	
PRINT NAME		() ID No	_
L	ast Name, First Name	MI, What you v	wish to be called	
Follow the instruc	tions on the Exam Cov	er Sheet for Fill-	-in-the Blank/Multiple Choice questions.	Also,
circle your answer	•			
MATHEMATICA	AL MODELING. Cons	sider the following	ng problem (DO NOT SOLVE!):	
A mass weigh	ing 4 lbs. stretches a sp	oring (which is 10	0 ft. long) 6 inches. If the mass is raised	3 inches
above its equilibria	um position and given a	an initial velocity	of 6 ft./sec. downward, determine the st	ubsequent
motion (i.e. find the	ne distance from the eq	uilibrium positio	n as a function of time). Assume that the	e air
	gible and that there is n			
			en above to the model you developed on	
	obtain the specific mode	el for this proble	m. DO NOT SOLVE! Then answer the	questions
below.				
38. (3 pts.) The sp	oring constant k in pour	nds per foot (or s	slugs per second squared)is	
k =			A B C D E	
39. (2 pts.) The C	DE in the specific mat	hematical model	for the mass spring system from the	
· • /	-		on from the equilibrium position as a	
function of time	is		A B C D E	
<u> </u>			al model for the mass spring system	
from the data	above whose solution y	yields the distanc	ee u(t) down from the equilibrium	
position as a fu	unction of time is $u(0) =$	=	A B C D E	3
41. (1 pt.) The in	itial velocity for the spe	ecific mathematic	cal model for the mass spring system	
from the data a	above whose solution y	ields the distance	e u(t) down from the equilibrium	
position as a fu	unction of time is $\dot{u}(0)$	=	. ABCDE	

CE) 1/6 DE) 1/3 ABC) 1/4 ABD) -1/6 ABE) -1/3 ACD) -1/4 ADE) $\frac{1}{8}\ddot{u} + 2u + 24u = \sin(t)$

 $BCD)\frac{1}{8}\ddot{u} + 2\dot{u} + 24u = 0 \quad BDE) \quad \frac{1}{8}\ddot{u} + 24u = 0 \quad \quad CDE) \quad \frac{1}{8}\ddot{u} + 24u = 0 \quad \quad ABCD)\frac{1}{8}\ddot{u} + 48u = 0 \quad ABCE)\frac{1}{8}\ddot{u} + 48u = 0$

ABDE) $\frac{1}{4}\ddot{u}+12u=0$ ABCDE) None of the above.

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _____