EXAM-3A-1	
FALL 2009	

MATH 261: Elementary Differential Equations EXAMINATION COVER PAGE

MATH 261 Professor Moseley

PRINT NAME			()
Last Name,	First Name	MI	(What you v	vish to be	e called)
ID #		EXAM DATE	Friday, Oct.	30, 2009	2:30pm
I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.			Date _		Signature
			page	Scores points	s score
SIGNATURE	DA	ATE		6	
INSTRUCTIONS: Besides this cand problems on this exam. MA	-		7		
PAGES . If a page is missing, you page. Read through the entire expansion.	ou will receive a grade of	zero for that	3	10	
your hand and I will come to you			4	6	
exam. Your I.D., this exam, and			5	9	
on your desk during the exam. PAPER! Use the back of the ex			6	12	
the staple if you wish. Print your			7	16	
in-the Blank/Multiple Choice or 'pages. For each Fill-in-the Blank			-		
answer in the blank provided. N	ext find your answer from	the list given	8	3	
and write the corresponding letter			9	9	
provided. Then circle this letter pages. However, to insure credit		-	10	6	
and carefully. Your entire solution	on may be graded, not jus	t your final	11	9	
answer. SHOW YOUR WORK expressed in your best mathemat	• •		12	7	
given as deemed appropriate. Pr	13				
computations as time allows.	GOOD LUCK!!				
DEOLIEC	T EOD DECDADE		14		
	T FOR REGRADE		15		
Please regard the following pro (e.g., I do not understand what			16		
			17		
			18		
(Regrades should be requested	within a weak of the data the even is	19			
		s as necessary to explain your reasons.)	20		
I swear and/or affirm that upon	21				
nothing on this exam except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)					

MATH 261 EXAM 3A-1 Fall 2009 Professor Moseley Page 1

PRINT NAME ______(_____) ID No. _______

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

The dimension of the null space of the linear operator L[y] = y'' + y that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Since the operator L[y] = y'' + y has constant coefficients, we assume a solution of the homogeneous equation L[y] = 0 of the form $y = e^{rx}$. This leads to the two linearly independent solutions $y_1 = \cos(x)$ and $y_2 = \sin(x)$ so that a basis of the nullspace of L is $B = \{\cos(x), \sin(x)\}$. Hence we can deduce that $y_c = c_1 \cos(x) + c_2 \sin(x)$ is the general solution of the homogeneous equation y'' + y = 0.

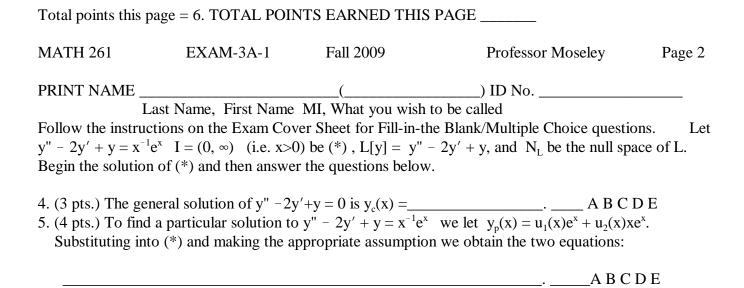
To use the linear theory to obtain the general solution of the nonhomogeneous equation L[y] = g(x), we need a particular solution, y_p , to y'' + y = g(x). We have studied two techniques for this purpose (attendance is required):

- i) Undetermined Coefficients (also called judicious guessing)
- ii) Variation of Parameters (also called variation of constants)

For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation y'' + y = g(x):

1. (2 pts.)
$$g(x) = x^{-1} e^{x}$$
 _____ A B C D E
2. (2 pts.) $g(x) = e^{-x}$ _____ A B C D E
3. (2 pts.) $g(x) = \tan(x)$ ____ A B C D E

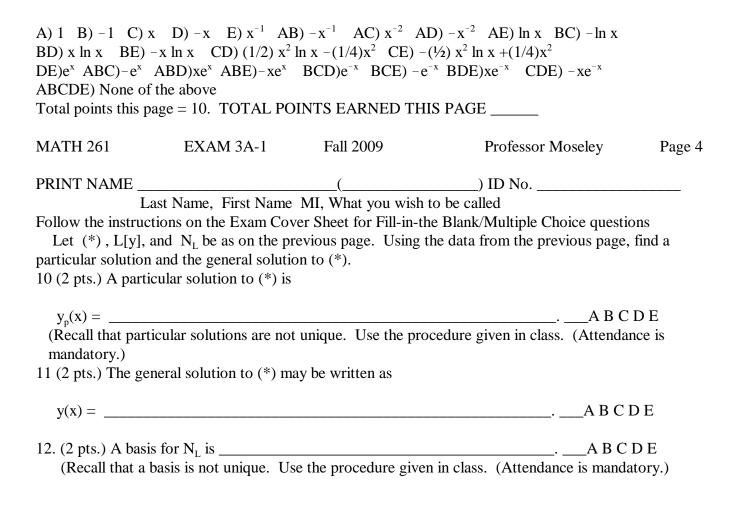
- A) Neither technique works to find y_p.
- B) Only Undetermined Coefficients works to find y_p .
- C) Only Variation of Parameters works to find y_p .
- D) Either technique works to find y_p .
- E) Not enough information is given.
- AB) Too much information is given.
- AC) All of the above statements are true.
- AD) None of the above statements are true.



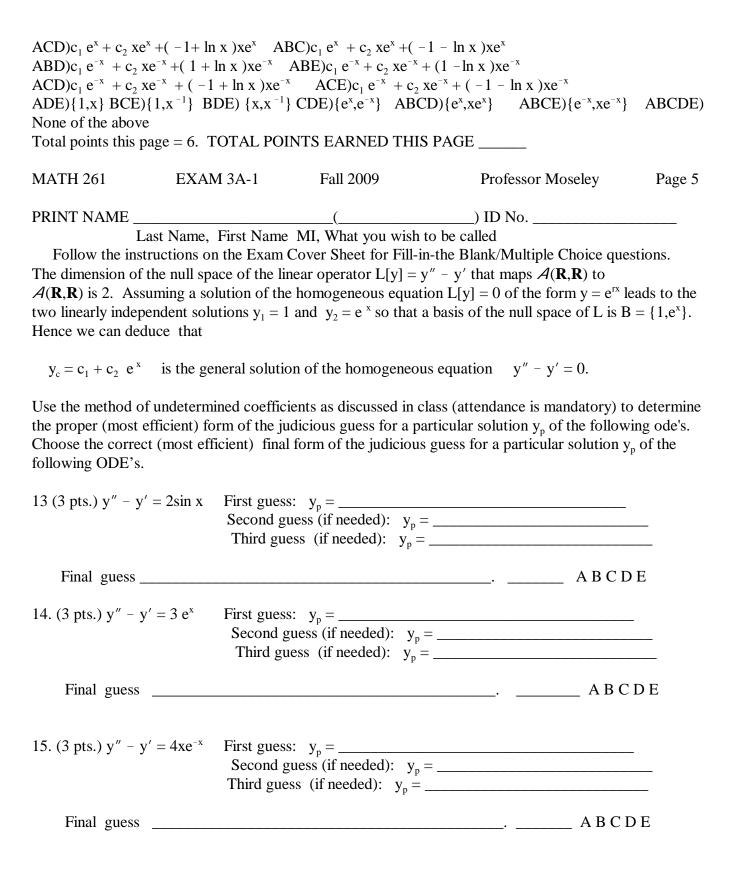
Possible answers this page

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A)c_1\cos(x) + c_2\sin(x) B)c_1\cos(2x) + c_2\sin(2x), C)c_1x + c_2 D)c_1e^x + c_2e^{-x} E)c_1e^x + c_2xe^x AB)c_1e^{-x} + c_2xe^{-x}
c_2 x e^{-x} AC) r = \pm i AD) r = \pm 1 AE) r = \pm 2i BC) r = \pm 2i BD) 1,1
                                                                                                      BE) -1,-1
CD) u'_{1}(x) e^{x} + u'_{2}(x) xe^{x} = 0,
                                                      u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = xe^{x}
                                                    u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = -xe^{x}
CE) u'_{1}(x) e^{x} + u'_{2}(x) xe^{x} = 0,
DE) u'_1(x) e^x + u'_2(x) x e^x = 0
                                                    u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = x^{-1}e^{x}
                                                      u'_{1}(x) e^{x} + u'_{2}(x) (e^{x} + xe^{x}) = -x^{-1}e^{x}
ABC) u'_{1}(x) e^{x} + u'_{2}(x) xe^{x} = 0,
                                                       -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = xe^{-x}
ABD) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
ABE) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                       -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = -xe^{-x}
ACD) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                       -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = x^{-1}e^{-x}
                                                         -u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - xe^{-x}) = -x^{-1}e^{-x}
ACE) u'_{1}(x) e^{-x} + u'_{2}(x) x e^{-x} = 0,
                                                       u'_{1}(x) e^{x} - u'_{2}(x) e^{-x} = x^{-1}e^{x}
ADE) u'_{1}(x) e^{x} + u'_{2}(x) e^{-x} = 0,
BCD) u'_{1}(x) e^{x} + u'_{2}(x) e^{-x} = 0,
                                                       u'_{1}(x) e^{x} - u'_{2}(x) e^{-x} = -x^{-1}e^{x}
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ABCDE) None of the above. Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _____ MATH 261 EXAM 3A-1 Fall 2009 Professor Moseley Page 3 PRINT NAME ______ (______) ID No. _____ Last Name, First Name MI, What you wish to be called Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. (*) be the ODE L[y] = g(x) $I = (0, \infty)$ where L[y] is a given second order linear differential operator and $g \in \mathcal{A}((0,\infty),R)$ and let N_L be the null space of L. Suppose that in solving (*) you have found the general solution of L[y] = 0 to be $y_c(x) = c_1 e^{-x} + c_2 x e^{-x}$. Then following the standard procedure to find a particular solution of (*) using variation of parameters, you let $y_n(x) = u_1(x)e^{-x} + u_2(x) xe^{-x}$ and that this results in the following equations in $u'_1(x)$ and $u'_2(x)$ (see the previous page for why this might be true): $u'_{1}(x) e^{-x} + u'_{2}(x) xe^{-x} = 0$ $-u'_{1}(x) e^{-x} + u'_{2}(x) (e^{-x} - x e^{-x}) = x^{-1}e^{-x}$ Given this information, you are to find $u_1(x)$ and $u_2(x)$ and then finish the solution of (*) on the next page. If your $u_1(x)$ or $u_2(x)$ is wrong, then your solution on the next page is probably wrong. 6. (2 pts.) Hence $u'_{1}(x) =$ ______. ___ A B C D E 7. (2 pts.) And $u'_{2}(x) =$. A B C D E 8. (3 pts.) We may choose $u_1(x) =$ _____. ___. A B C D E 9. (3 pts.) And $u_2(x) =$ _____. A B C D E



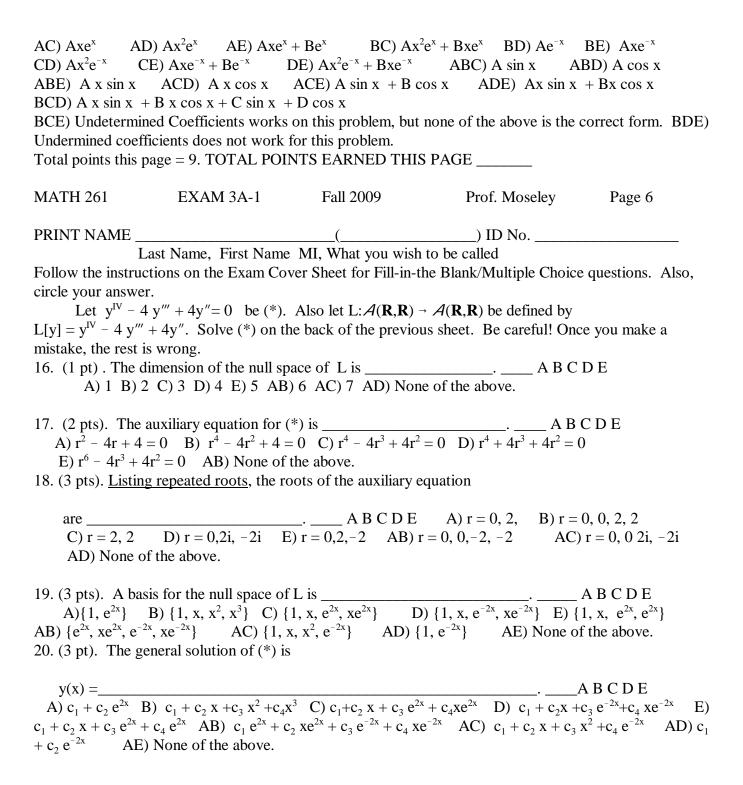
Possible answers this page.



Possible Answers for Final Guesses.

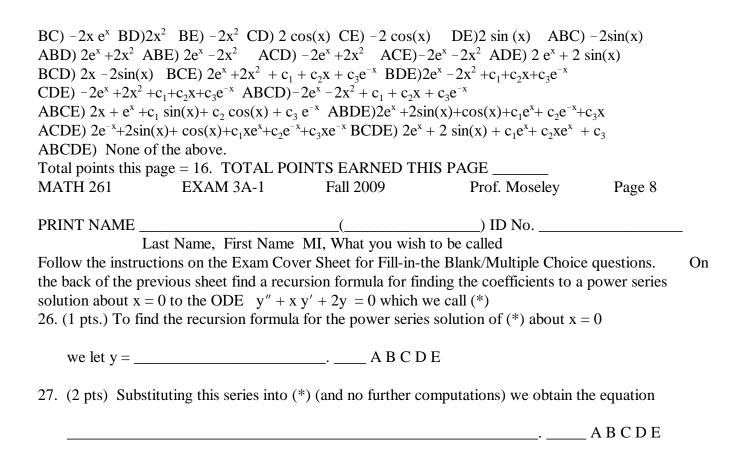
- A) A

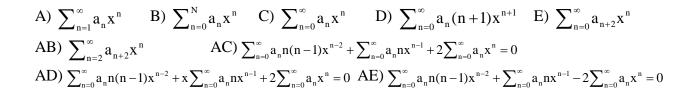
- B) Ax + B C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB) Ae^x



Points this page =	= 12. TOTAL POINTS	S EARNED THIS PA	GE	
MATH 261	EXAM 3A-1	Fall 2009	Prof. Moseley	Page 7
Follow the in Let $y''' + y'' = 4$		Cover Sheet for Fill- *) on the back of the	to be called in-the Blank/Multiple Chaprevious sheet.	
$y_{c}(x) = $	ticular solution of v'''	$+ v'' = 4 e^x$ is	A	BCDE
			A I	
$y_{p2}(x) = \underline{\hspace{1cm}}$	rticular solution of y" -		A	ВСDЕ
$y_p(x) =$ 25. (2 pts.) The	e general solution of (*)	ı is	·	ABCDE
			·	ABCDE

Possible answers this page A)
$$c_1 + c_2 x + c_3 e^x$$
 B) $c_1 + c_2 x + c_3 e^{-x}$ C) $c_1 + c_2 e^x + c_3 e^{-x}$ D) $c_1 e^x + c_2 \sin(x) + c_3 \cos(x)$ E) $c_1 + c_2 \sin(x) + c_3 \cos(x)$ AB) $c_1 e^{-x} + c_2 \sin(x) + c_3 \cos(x)$ AC)) $2e^x$ AD) $-2e^x$ AE)) $2x$ e^x





Possible answers this page.

A)
$$\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n$$
 B) $\sum_{n=0}^{\infty} a_{n+2}(n+2)x^n$ C) $\sum_{n=1}^{\infty} a_{n+2}(n+1)x^n$ D) $\sum_{n=0}^{\infty} a_n(n+2)(n+1)x^n$ E) $\sum_{n=0}^{\infty} a_{n+1}(n+2)(n+1)x^n$

B)
$$\sum_{n=0}^{\infty} a_{n+2}(n+2)x^n$$

C)
$$\sum_{n=1}^{\infty} a_{n+2}(n+1)x^{n}$$

ABCDE

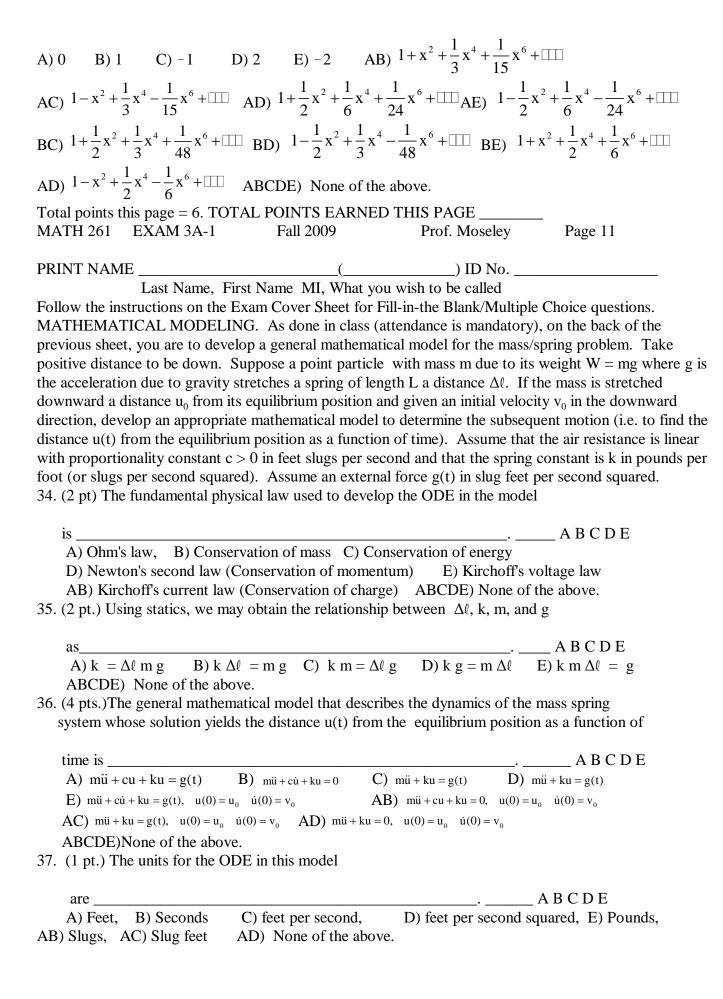
D)
$$\sum_{n=0}^{\infty} a_n(n+2)(n+1)x^n$$

E)
$$\sum_{n=0}^{\infty} a_{n+1}(n+2)(n+1)x^n$$

AB)
$$\sum_{n=0}^{\infty} [(n+2)(n+1)a_{n+2} + (n+2)a_n] x^n = 0$$
 AC) $\sum_{n=0}^{\infty} [(n+1)(n+2)a_{n+2} + (n-2)a_n] x^n = 0$ AD) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) - (n-2)a_n] x^n = 0$ AE) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) - (n+2)a_n] x^n = 0$ BD) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) - (n+2)a_n] x^n = 0$ BD) $\sum_{n=0}^{\infty} [a_{n+1}(n+2)(n+1) + (n-2)a_n] x^n = 0$ BD) $\sum_{n=0}^{\infty} [a_{n+1}(n+2)(n+1) - (n+2)a_n] x^$

 $y_1(x) =$ _______. A B C D E

33. (4 pts.) The power series solution to this IVP is



Total points this p	bage = 9. TOTAL POI	NTS EARNED THI	S PAGE	
MATH 261	EXAM 3A-1	Fall 2009	Prof. Moseley Page 12	
PRINT NAME _		() ID No	
	ast Name, First Name			
			the Blank/Multiple Choice questions. Als	so,
circle your answer	r.			
MATHEMATICA	AL MODELING. Con	sider the following	problem (DO NOT SOLVE!):	
A mass weigh	ing 4 lbs. stretches a sp	oring (which is 10 ft	. long) 2 inches. If the mass is lowered 4	
inches below its e	quilibrium position and	l given an initial velo	ocity of 5 ft./sec. upward, determine the	
subsequent motio	n (i.e. find the distance	from the equilibrium	n position as a function of time). Assume	that
	is negligible and that th			
			above to the model you developed on the	
	obtain the specific mod	el for this problem.	DO NOT SOLVE! Then answer the ques	tions
below.				
38. (3 pts.) The sp	oring constant k in pou	nds per foot (or slug	gs per second squared)is	
k =			A B C D E	
data above who	ose solution yields the o	distance u(t) down	r the mass spring system from the from the equilibrium position as a A B C D E	
			model for the mass spring system	
	-		(t) down from the equilibrium	
position as a fu	unction of time is u(0)	=	A B C D E	
			model for the mass spring system	
from the data	above whose solution y	vields the distance u	(t) down from the equilibrium	
position as a fu	unction of time is $\dot{u}(0)$	=	A B C D E	
Possible answrs th	1 0			
A)0 B)2 C)4 D)	5 E) 6 AB)16 AC) 2	4 AD)32 AE)48 B	C)96 BD) -4 BE) -5 CD) -6	
			D) $-1/4$ ADE) $\frac{1}{8}\ddot{u} + 2u + 24u = \sin(t)$	
$BCD)\frac{1}{8}\ddot{\mathbf{u}} + 2\dot{\mathbf{u}} + 24\mathbf{u}$	$= 0 \text{ BDE}) \frac{1}{8} \ddot{u} + 24 u = 0$	CDE) $\frac{1}{8}\ddot{u} + 24u = 0$	ABCD) $\frac{1}{8}\ddot{u} + 48u = 0$ ABCE) $\frac{1}{8}\ddot{u} + 48u = 0$	
ABDE) $\frac{1}{8}\ddot{u} + 12u = 0$	ABCDE) None of t	the above.		

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE _____