EXAM-3A-1 MATH 261: Elementary Differential Equations MATH 261
FALL 2009 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____(________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday, Oct. 30, 2009 2:30pm

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

Date _______	Signature____

_______________________________ _______________
SIGNATURE DATE

INSTRUCTIONS: Besides this cover page, there are 12 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets. Pages 1-12 are Fill-in-the Blank/Multiple Choice or True/False. Expect no part credit on these pages. For each Fill-in-the Blank/Multiple Choice question write your answer in the blank provided. Next find your answer from the list given and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics on this paper. Partial credit will be given as deemed appropriate. Proofread your solutions and check your computations as time allows. GOOD LUCK!!

page	Scores points	score
1	6	
2	7	
3	10	
4	6	
5	9	
6	12	
7	16	
8	3	
9	9	
10	6	
11	9	
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REQUEST FOR REGRADE

Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Total	100	

PRINT NAME __________________________(__________________) ID No. ____________________
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

The dimension of the null space of the linear operator $L[y] = y'' + y$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Since the operator $L[y] = y'' + y$ has constant coefficients, we assume a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$. This leads to the two linearly independent solutions $y_1 = \cos(x)$ and $y_2 = \sin(x)$ so that a basis of the nullspace of L is $B = \{\cos(x), \sin(x)\}$. Hence we can deduce that $y_c = c_1 \cos(x) + c_2 \sin(x)$ is the general solution of the homogeneous equation $y'' + y = 0$.

To use the linear theory to obtain the general solution of the nonhomogeneous equation $L[y] = g(x)$, we need a particular solution, y_p, to $y'' + y = g(x)$. We have studied two techniques for this purpose (attendance is required):

i) Undetermined Coefficients (also called judicious guessing)
ii) Variation of Parameters (also called variation of constants)

For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation $y'' + y = g(x)$:

1. (2 pts.) $g(x) = x^{-1} e^x$ ____ . _______ A B C D E

2. (2 pts.) $g(x) = e^{-x}$____. ______ A B C D E

3. (2 pts.) $g(x) = \tan(x)$ _______________________________________. ______ A B C D E

A) Neither technique works to find y_p.
B) Only Undetermined Coefficients works to find y_p.
C) Only Variation of Parameters works to find y_p.
D) Either technique works to find y_p.
E) Not enough information is given.
AB) Too much information is given.
AC) All of the above statements are true.
AD) None of the above statements are true.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(_________________) ID No. __________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Let $y'' - 2y' + y = x^{-1}e^x$ $I = (0, \infty)$ (i.e. $x>0$) be (*) , $L[y] = y'' - 2y' + y$, and N_L be the null space of L. Begin the solution of (*) and then answer the questions below.

4. (3 pts.) The general solution of $y'' - 2y' + y = 0$ is $y_c(x) =$_________________. ____ A B C D E
5. (4 pts.) To find a particular solution to $y'' - 2y' + y = x^{-1}e^x$ we let $y_p(x) = u_1(x)e^x + u_2(x)xe^x$. Substituting into (*) and making the appropriate assumption we obtain the two equations:

____. _____A B C D E

Possible answers this page

A) $c_1 \cos(x) + c_2 \sin(x)$ B) $c_1 \cos(2x) + c_2 \sin(2x)$, C) $c_1 x + c_2$ D) $c_1 e^x + c_2 e^{-x}$ E) $c_1 e^x + c_2 xe^x$ AB) $c_1 e^{-x} + c_2 xe^{-x}$ AC) $r = \pm i$ AD) $r = \pm 1$ AE) $r = \pm 2i$ BC) $r = \pm 2i$ BD) 1,1 BE) $-1,-1$

CD) $u'_1(x) e^x + u'_2(x) xe^x = 0$, $u'_1(x) e^x + u'_2(x) (e^x+xe^x) = xe^x$

CE) $u'_1(x) e^x + u'_2(x) xe^x = 0$, $u'_1(x) e^x + u'_2(x) (e^x+xe^x) = -xe^x$

DE) $u'_1(x) e^x + u'_2(x) xe^x = 0$ $u'_1(x) e^x + u'_2(x) (e^x+xe^x) = x^{-1}e^x$

ABC) $u'_1(x) e^x + u'_2(x) xe^x = 0$, $u'_1(x) e^x + u'_2(x) (e^x+xe^x) = -x^{-1}e^x$

ABD) $u'_1(x) e^{-x} + u'_2(x) xe^{-x} = 0$, $-u'_1(x) e^{-x} + u'_2(x) (e^{-x} - xe^{-x}) = xe^{-x}$

ABE) $u'_1(x) e^{-x} + u'_2(x) xe^{-x} = 0$, $-u'_1(x) e^{-x} + u'_2(x) (e^{-x} - xe^{-x}) = -xe^{-x}$

ACD) $u'_1(x) e^{-x} + u'_2(x) xe^{-x} = 0$, $-u'_1(x) e^{-x} + u'_2(x) (e^{-x} - xe^{-x}) = x^{-1}e^{-x}$

ACE) $u'_1(x) e^{-x} + u'_2(x) xe^{-x} = 0$, $-u'_1(x) e^{-x} + u'_2(x) (e^{-x} - xe^{-x}) = -x^{-1}e^{-x}$

ADE) $u'_1(x) e^x + u'_2(x) e^{-x} = 0$, $u'_1(x) e^x - u'_2(x) e^{-x} = x^{-1}e^x$

BCD) $u'_1(x) e^x + u'_2(x) e^{-x} = 0$, $u'_1(x) e^x - u'_2(x) e^{-x} = -x^{-1}e^x$

BCE) $u'_1(x)\, e^x + u'_2(x)\, xe^x = 0$, $u'_1(x)\, e^x - u'_2(x)\, e^{-x} = x^{-1}e^{-x}$
BDE) $u'_1(x)\, e^x + u'_2(x)\, xe^x = 0$, $u'_1(x)\, e^x - u'_2(x)\, e^{-x} = -x^{-1}e^{-x}$
ABCDE) None of the above.
Total points this page = 7. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME ________________________(________________) ID No. __________________
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Let (*) be the the ODE L[y] = g(x) I = (0,∞) where L[y] is a given second order linear differential operator and $g \in \mathcal{A}((0,\infty),R)$ and let N_L be the null space of L. Suppose that in solving (*) you have found the general solution of L[y] = 0 to be $y_c(x) = c_1e^{-x} + c_2\, xe^{-x}$. Then following the standard procedure to find a particular solution of (*) using variation of parameters, you let
$y_p(x) = u_1(x)e^{-x} + u_2(x)\, xe^{-x}$ and that this results in the following equations in $u'_1(x)$ and $u'_2(x)$ (see the previous page for why this might be true):

$$u'_1(x)\, e^{-x} + u'_2(x)\, xe^{-x} = 0$$
$$-u'_1(x)\, e^{-x} + u'_2(x)\, (e^{-x} - x\, e^{-x}) = x^{-1}e^{-x}$$

Given this information, you are to find $u_1(x)$ and $u_2(x)$ and then finish the solution of (*) on the next page. If your $u_1(x)$ or $u_2(x)$ is wrong, then your solution on the next page is probably wrong.

6. (2 pts.) Hence $u'_1(x)$ =____. ____A B C D E

7. (2 pts.) And $u'_2(x)$ = ____. ____A B C D E

8. (3 pts.) We may choose $u_1(x)$ =________________________________. ____A B C D E

9. (3 pts.) And $u_2(x)$ =____. ____A B C D E

Possible answers this page.

A) 1   B) -1 C) x   D) $-x$ E) x^{-1} AB) $-x^{-1}$ AC) x^{-2} AD) $-x^{-2}$ AE) ln x   BC) $-\ln x$
BD) x ln x   BE) $-x \ln x$ CD) $(1/2)\, x^2 \ln x - (1/4)x^2$ CE) $-(½)\, x^2 \ln x + (1/4)x^2$
DE)e^x ABC)$-e^x$ ABD)xe^x ABE)$-xe^x$ BCD)e^{-x} BCE) $-e^{-x}$ BDE)xe^{-x} CDE) $-xe^{-x}$
ABCDE) None of the above
Total points this page = 10. TOTAL POINTS EARNED THIS PAGE ______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Let (*) , L[y], and N_L be as on the previous page. Using the data from the previous page, find a particular solution and the general solution to (*).

10 (2 pts.) A particular solution to (*) is

$y_p(x)$ = ____. ___A B C D E

(Recall that particular solutions are not unique. Use the procedure given in class. (Attendance is mandatory.)

11 (2 pts.) The general solution to (*) may be written as

y(x) = ____. ___A B C D E

12. (2 pts.) A basis for N_L is ____. ___A B C D E

(Recall that a basis is not unique. Use the procedure given in class. (Attendance is mandatory.)

Possible answers this page.
A) x^{-1} B) $-x^{-1}$ C) x^{-2} D) $-x^{-2}$ E) ln x   AB) $-\ln x$ AC) x ln x   AD) $-x \ln x$
AE)$(1+\ln x)xe^x$ BC)$(1-\ln x)xe^x$ BD)$(-1+\ln x)xe^x$ BE)$(-1-\ln x)xe^x$
CD)$(1+\ln x)xe^{-x}$ CE)$(1-\ln x)xe^{-x}$ DE) $(-1+\ln x)xe^{-x}$ ABC)$(-1-\ln x)xe^{-x}$
ABD)$c_1e^x + c_2\, xe^x + (1+\ln x)xe^x$ ABE)$c_1 e^x + c_2\, xe^x + (1-\ln x)xe^x$

ACD)$c_1 e^x + c_2 xe^x + (-1 + \ln x)xe^x$ ABC)$c_1 e^x + c_2 xe^x + (-1 - \ln x)xe^x$
ABD)$c_1 e^{-x} + c_2 xe^{-x} + (1 + \ln x)xe^{-x}$ ABE)$c_1 e^{-x} + c_2 xe^{-x} + (1 - \ln x)xe^{-x}$
ACD)$c_1 e^{-x} + c_2 xe^{-x} + (-1 + \ln x)xe^{-x}$ ACE)$c_1 e^{-x} + c_2 xe^{-x} + (-1 - \ln x)xe^{-x}$
ADE)$\{1,x\}$ BCE)$\{1,x^{-1}\}$ BDE) $\{x,x^{-1}\}$ CDE)$\{e^x,e^{-x}\}$ ABCD)$\{e^x,xe^x\}$ ABCE)$\{e^{-x},xe^{-x}\}$ ABCDE) None of the above
Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

PRINT NAME _________________________(________________) ID No. __________________
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.
The dimension of the null space of the linear operator $L[y] = y'' - y'$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^x$ so that a basis of the null space of L is $B = \{1,e^x\}$. Hence we can deduce that

$y_c = c_1 + c_2 e^x$ is the general solution of the homogeneous equation $y'' - y' = 0$.

Use the method of undetermined coefficients as discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Choose the correct (most efficient) final form of the judicious guess for a particular solution y_p of the following ODE's.

13 (3 pts.) $y'' - y' = 2\sin x$ First guess: y_p = ______________________________
Second guess (if needed): y_p = ________________________
Third guess (if needed): y_p = ________________________

Final guess ____. _______ A B C D E

14. (3 pts.) $y'' - y' = 3e^x$ First guess: y_p = ______________________________
Second guess (if needed): y_p = ________________________
Third guess (if needed): y_p = ________________________

Final guess ____. _______ A B C D E

15. (3 pts.) $y'' - y' = 4xe^{-x}$ First guess: y_p = ______________________________
Second guess (if needed): y_p = ________________________
Third guess (if needed): y_p = ________________________

Final guess ____. _______ A B C D E

Possible Answers for Final Guesses.
A) A B) Ax + B C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB) Ae^x

AC) Axe^x AD) Ax^2e^x AE) $Axe^x + Be^x$ BC) $Ax^2e^x + Bxe^x$ BD) Ae^{-x} BE) Axe^{-x}
CD) Ax^2e^{-x} CE) $Axe^{-x} + Be^{-x}$ DE) $Ax^2e^{-x} + Bxe^{-x}$ ABC) A sin x ABD) A cos x
ABE) A x sin x ACD) A x cos x ACE) A sin x + B cos x ADE) Ax sin x + Bx cos x
BCD) A x sin x + B x cos x + C sin x + D cos x
BCE) Undetermined Coefficients works on this problem, but none of the above is the correct form. BDE) Undermined coefficients does not work for this problem.
Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME __________________________(_________________) ID No. __________________
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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

Let $y^{IV} - 4y''' + 4y'' = 0$ be (*). Also let $L: A(\mathbf{R},\mathbf{R}) \to A(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$. Solve (*) on the back of the previous sheet. Be careful! Once you make a mistake, the rest is wrong.

16. (1 pt) . The dimension of the null space of L is ________________. ____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

17. (2 pts). The auxiliary equation for (*) is ________________________. ____ A B C D E
A) $r^2 - 4r + 4 = 0$ B) $r^4 - 4r^2 + 4 = 0$ C) $r^4 - 4r^3 + 4r^2 = 0$ D) $r^4 + 4r^3 + 4r^2 = 0$
E) $r^6 - 4r^3 + 4r^2 = 0$ AB) None of the above.

18. (3 pts). <u>Listing repeated roots</u>, the roots of the auxiliary equation

are ____________________________. ____ A B C D E A) r = 0, 2, B) r = 0, 0, 2, 2
C) r = 2, 2 D) r = 0,2i, −2i E) r = 0,2,−2 AB) r = 0, 0,−2, −2 AC) r = 0, 0 2i, −2i
AD) None of the above.

19. (3 pts). A basis for the null space of L is ___________________________. _____ A B C D E
A) $\{1, e^{2x}\}$ B) $\{1, x, x^2, x^3\}$ C) $\{1, x, e^{2x}, xe^{2x}\}$ D) $\{1, x, e^{-2x}, xe^{-2x}\}$ E) $\{1, x, e^{2x}, e^{2x}\}$
AB) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ AC) $\{1, x, x^2, e^{-2x}\}$ AD) $\{1, e^{-2x}\}$ AE) None of the above.

20. (3 pt). The general solution of (*) is

y(x) =____. ____A B C D E
A) $c_1 + c_2 e^{2x}$ B) $c_1 + c_2 x + c_3 x^2 + c_4 x^3$ C) $c_1 + c_2 x + c_3 e^{2x} + c_4 xe^{2x}$ D) $c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$ E) $c_1 + c_2 x + c_3 e^{2x} + c_4 e^{2x}$ AB) $c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$ AC) $c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $c_1 + c_2 e^{-2x}$ AE) None of the above.

Points this page = 12. TOTAL POINTS EARNED THIS PAGE __________

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PRINT NAME _________________________(________________) ID No. __________________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions

Let $y''' + y'' = 4e^x + 4$ be (*). Solve (*) on the back of the previous sheet.

21. (3 pts.) The general solution of $y''' + y'' = 0$ is

$y_c(x)$ = ____. ____ A B C D E

22. (5 pts.) A particular solution of $y''' + y'' = 4e^x$ is

$y_{p1}(x)$ = ____. ____ A B C D E

23. (5 pts.) A particular solution of $y''' + y'' = 4$ is

$y_{p2}(x)$ = ____. _____ A B C D E

24. (1 pts.) A particular solution of (*) is

$y_p(x)$ = ____. _____ A B C D E

25. (2 pts.) The general solution of (*) is

$y(x)$ = ____. _____ A B C D E

Possible answers this page

A) $c_1 + c_2x + c_3e^x$ B) $c_1 + c_2x + c_3e^{-x}$ C) $c_1 + c_2e^x + c_3e^{-x}$ D) $c_1e^x + c_2\sin(x) + c_3\cos(x)$

E) $c_1 + c_2\sin(x) + c_3\cos(x)$ AB) $c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$ AC)) $2e^x$ AD) $-2e^x$ AE)) $2x\,e^x$

BC) $-2x\,e^x$ BD) $2x^2$ BE) $-2x^2$ CD) $2\cos(x)$ CE) $-2\cos(x)$ DE) $2\sin(x)$ ABC) $-2\sin(x)$
ABD) $2e^x+2x^2$ ABE) $2e^x-2x^2$ ACD) $-2e^x+2x^2$ ACE) $-2e^x-2x^2$ ADE) $2e^x+2\sin(x)$
BCD) $2x-2\sin(x)$ BCE) $2e^x+2x^2+c_1+c_2x+c_3e^{-x}$ BDE) $2e^x-2x^2+c_1+c_2x+c_3e^{-x}$
CDE) $-2e^x+2x^2+c_1+c_2x+c_3e^{-x}$ ABCD) $-2e^x-2x^2+c_1+c_2x+c_3e^{-x}$
ABCE) $2x+e^x+c_1\sin(x)+c_2\cos(x)+c_3e^{-x}$ ABDE) $2e^x+2\sin(x)+\cos(x)+c_1e^x+c_2e^{-x}+c_3x$
ACDE) $2e^{-x}+2\sin(x)+\cos(x)+c_1xe^x+c_2e^{-x}+c_3xe^{-x}$ BCDE) $2e^x+2\sin(x)+c_1e^x+c_2xe^x+c_3$
ABCDE) None of the above.

Total points this page = 16. TOTAL POINTS EARNED THIS PAGE _______

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. On the back of the previous sheet find a recursion formula for finding the coefficients to a power series solution about x = 0 to the ODE $y'' + x\,y' + 2y = 0$ which we call (*)

26. (1 pts.) To find the recursion formula for the power series solution of (*) about x = 0

we let y = _________________________. ____ A B C D E

27. (2 pts) Substituting this series into (*) (and no further computations) we obtain the equation

____. _____ A B C D E

A) $\sum_{n=1}^{\infty} a_n x^n$ B) $\sum_{n=0}^{N} a_n x^n$ C) $\sum_{n=0}^{\infty} a_n x^n$ D) $\sum_{n=0}^{\infty} a_n (n+1)x^{n+1}$ E) $\sum_{n=0}^{\infty} a_{n+2} x^n$

AB) $\sum_{n=2}^{\infty} a_{n+2} x^n$ AC) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=0}^{\infty} a_n n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$

AD) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} + x\sum_{n=0}^{\infty} a_n n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$ AE) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=0}^{\infty} a_n n x^{n-1} - 2\sum_{n=0}^{\infty} a_n x^n = 0$

BC) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} - x\sum_{n=0}^{\infty} a_n nx^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$

BD) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} - \sum_{n=0}^{\infty} a_n nx^{n-1} - 2\sum_{n=0}^{\infty} a_n x^n = 0$

BE) $\sum_{n=0}^{\infty} a_n n(n+1)x^{n-2} + x\sum_{n=0}^{\infty} a_n nx^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$

CD) $\sum_{n=0}^{\infty} a_n n(n+1)x^{n-2} + \sum_{n=0}^{\infty} a_n nx^{n-1} - 2\sum_{n=0}^{\infty} a_n x^n = 0$

CE) $\sum_{n=0}^{\infty} a_n n(n+1)x^{n-2} - x\sum_{n=0}^{\infty} a_n nx^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$ DE) $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} - 2\sum_{n=0}^{\infty} a_n nx^n = 0$

ABCDE) None of the above.

Total points this page = 3. TOTAL POINTS EARNED THIS PAGE ________

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Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.

Let (*) be as on the previous page.

28. (3 pts) As explained in class (attendance is mandatory) by changing the index and simplifying, the term $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2}$ can be changed to obtain

$\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2}$ ____. _____ A B C D E

29. (3 pts) Continuing the procedure given in class, using this new term and other simplifications, the equation you obtained on the previous page can now be written

as ____. _____ A B C D E

30. (3 pts.) The recursion formula for finding the coefficients in the power series solution of (*)

is____. ______A B C D E

Possible answers this page.

A) $\sum_{n=0}^{\infty} a_{n+2}(n+2)(n+1)x^n$ B) $\sum_{n=0}^{\infty} a_{n+2}(n+2)x^n$ C) $\sum_{n=1}^{\infty} a_{n+2}(n+1)x^n$

D) $\sum_{n=0}^{\infty} a_n(n+2)(n+1)x^n$ E) $\sum_{n=0}^{\infty} a_{n+1}(n+2)(n+1)x^n$

AB) $\sum_{n=0}^{\infty}[(n+2)(n+1)a_{n+2}+(n+2)a_n]x^n=0$ AC) $\sum_{n=0}^{\infty}[(n+1)(n+2)a_{n+2}+(n-2)a_n]x^n=0$

AD) $\sum_{n=1}^{\infty}[a_{n+2}(n+2)(n+1)-(n-2)a_n]x^n=0$ AE) $\sum_{n=0}^{\infty}[a_{n+2}(n+2)(n+1)-(n+2)a_n]x^n=0$

BC) $\sum_{n=0}^{\infty}[a_{n+1}(n+2)(n+1)+(n-2)a_n]x^n=0$ BD) $a_{n+2}=\frac{-1}{(n+1)}a_n$ BE) $a_{n+2}=\frac{1}{(n+1)}a_n$

CD) $a_{n+2}=\frac{n-2}{(n+2)(n+1)}a_n$ CE) $a_{n+2}=\frac{-(n-2)}{(n+2)(n+1)}a_n$ DE) $a_{n+1}=-\frac{n-2}{(n+2)(n+1)}a_n$

ABCDE) None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Let $y'' + p(x)\,y' + q(x)\,y = 0$ be (*). Suppose (*) has variable coefficients and that the solution of (*) by power series leads to the recursion relation $a_{n+2}=\frac{1}{n+1}a_n$ For n =0, 1, 2, 3, 4,..... As illustrated in class (attendance is mandatory), you are to find the (first four nonzero terms in the) power series solution to the initial value problem.

ODE $y'' + p(x)\,y' + q(x)\,y = 0$

IVP

IC's $y(0) = 1$, $y'(0) = 0$.

31. (1 pts.) a_0 = ____________. _____ A B C D E 32.(1 pts.) a_1 = ___________. _____ A B C D E

33. (4 pts.) The power series solution to this IVP is

$y_1(x)$ = ____. _____ A B C D E

Possible answers this page.

A) 0   B) 1   C) −1   D) 2   E) −2   AB) $1+x^2+\frac{1}{3}x^4+\frac{1}{15}x^6+\cdots$

AC) $1-x^2+\frac{1}{3}x^4-\frac{1}{15}x^6+\cdots$ AD) $1+\frac{1}{2}x^2+\frac{1}{6}x^4+\frac{1}{24}x^6+\cdots$ AE) $1-\frac{1}{2}x^2+\frac{1}{6}x^4-\frac{1}{24}x^6+\cdots$

BC) $1+\frac{1}{2}x^2+\frac{1}{3}x^4+\frac{1}{48}x^6+\cdots$ BD) $1-\frac{1}{2}x^2+\frac{1}{3}x^4-\frac{1}{48}x^6+\cdots$ BE) $1+x^2+\frac{1}{2}x^4+\frac{1}{6}x^6+\cdots$

AD) $1-x^2+\frac{1}{2}x^4-\frac{1}{6}x^6+\cdots$ ABCDE) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ________

PRINT NAME _________________________(______________) ID No. __________________

Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions.
MATHEMATICAL MODELING. As done in class (attendance is mandatory), on the back of the previous sheet, you are to develop a general mathematical model for the mass/spring problem. Take positive distance to be down. Suppose a point particle with mass m due to its weight W = mg where g is the acceleration due to gravity stretches a spring of length L a distance $\Delta\ell$. If the mass is stretched downward a distance u_0 from its equilibrium position and given an initial velocity v_0 in the downward direction, develop an appropriate mathematical model to determine the subsequent motion (i.e. to find the distance u(t) from the equilibrium position as a function of time). Assume that the air resistance is linear with proportionality constant c > 0 in feet slugs per second and that the spring constant is k in pounds per foot (or slugs per second squared). Assume an external force g(t) in slug feet per second squared.

34. (2 pt) The fundamental physical law used to develop the ODE in the model

is ____. _____ A B C D E

A) Ohm's law,   B) Conservation of mass   C) Conservation of energy
D) Newton's second law (Conservation of momentum)   E) Kirchoff's voltage law
AB) Kirchoff's current law (Conservation of charge)   ABCDE) None of the above.

35. (2 pt.) Using statics, we may obtain the relationship between $\Delta\ell$, k, m, and g

as____. ____ A B C D E

A) $k = \Delta\ell\, m\, g$ B) $k\,\Delta\ell = m\, g$ C) $k\, m = \Delta\ell\, g$ D) $k\, g = m\,\Delta\ell$ E) $k\, m\,\Delta\ell = g$
ABCDE) None of the above.

36. (4 pts.)The general mathematical model that describes the dynamics of the mass spring system whose solution yields the distance u(t) from the equilibrium position as a function of

time is ____. ______ A B C D E

A) $m\ddot{u} + cu + ku = g(t)$ B) $m\ddot{u} + c\dot{u} + ku = 0$ C) $m\ddot{u} + ku = g(t)$ D) $m\ddot{u} + ku = g(t)$
E) $m\ddot{u} + c\dot{u} + ku = g(t),\quad u(0) = u_0 \quad \dot{u}(0) = v_0$ AB) $m\ddot{u} + cu + ku = 0,\quad u(0) = u_0 \quad \dot{u}(0) = v_0$
AC) $m\ddot{u} + ku = g(t),\quad u(0) = u_0 \quad \dot{u}(0) = v_0$ AD) $m\ddot{u} + ku = 0,\quad u(0) = u_0 \quad \dot{u}(0) = v_0$
ABCDE)None of the above.

37. (1 pt.) The units for the ODE in this model

are ____. ______ A B C D E

A) Feet,   B) Seconds   C) feet per second,   D) feet per second squared,   E) Pounds,
AB) Slugs,   AC) Slug feet   AD) None of the above.

PRINT NAME _________________________(_______________) ID No. _________________
Last Name, First Name MI, What you wish to be called

Follow the instructions on the Exam Cover Sheet for Fill-in-the Blank/Multiple Choice questions. Also, circle your answer.

MATHEMATICAL MODELING. Consider the following problem (DO NOT SOLVE!):

A mass weighing 4 lbs. stretches a spring (which is 10 ft. long) 2 inches. If the mass is lowered 4 inches below its equilibrium position and given an initial velocity of 5 ft./sec. upward, determine the subsequent motion (i.e. find the distance from the equilibrium position as a function of time). Assume that the air resistance is negligible and that there is no external force.

On the back of the previous sheet, apply the data given above to the model you developed on the previous page to obtain the specific model for this problem. DO NOT SOLVE! Then answer the questions below.

38. (3 pts.) The spring constant k in pounds per foot (or slugs per second squared)is

k =____. _____ A B C D E

39. (2 pts.) The ODE in the specific mathematical model for the mass spring system from the data above whose solution yields the distance u(t) down from the equilibrium position as a

function of time is ____. _____ A B C D E

40. (1 pt.) The initial condition for the specific mathematical model for the mass spring system from the data above whose solution yields the distance u(t) down from the equilibrium

position as a function of time is u(0) = ______________________________. _____ A B C D E

41. (1 pt.) The initial velocity for the specific mathematical model for the mass spring system from the data above whose solution yields the distance u(t) down from the equilibrium

position as a function of time is $\dot{u}(0)$ = ____________________________. _____ A B C D E

Possible answrs this page.

A)0 B)2 C)4 D)5 E) 6 AB)16 AC) 24 AD)32 AE)48 BC)96 BD) −4 BE) −5 CD) −6

CE) 1/6 DE)1/3 ABC)1/4 ABD) −1/6 ABE) −1/3 ACD) −1/4 ADE) $\frac{1}{8}\ddot{u}+2u+24u=\sin(t)$

BCD) $\frac{1}{8}\ddot{u}+2\dot{u}+24u=0$ BDE) $\frac{1}{8}\ddot{u}+24u=0$ CDE) $\frac{1}{8}\ddot{u}+24u=0$ ABCD) $\frac{1}{8}\ddot{u}+48u=0$ ABCE) $\frac{1}{8}\ddot{u}+48u=0$

ABDE) $\frac{1}{8}\ddot{u}+12u=0$ ABCDE) None of the above.

Total points this page = 7. TOTAL POINTS EARNED THIS PAGE ______