EXAM-3 MATH 261: Elementary Differential Equations MATH 261
FALL 2006 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME____ (________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday Ocober 27, 2006

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

________________________________ ________________
SIGNATURE DATE

Scores

page	points	score
1	8	
2	9	
3	14	
4	16	
5	8	
6	9	
7	14	
8	11	
9	6	
10	5	
11		
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18		
19		
20		
21		
22		
Total	100	

INSTRUCTIONS

1. Besides this cover page, there are 10 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-10 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics. Partial credit will be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. **GOOD LUCK**!!!!!!!!!!!

REQUEST FOR REGRADE

Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature_______________________________

PRINT NAME ____________________________(___________________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

The dimension of the null space of the linear operator $L[y] = y'' + y$ that maps $A(\mathbf{R},\mathbf{R})$ to $A(\mathbf{R},\mathbf{R})$ is 2. Since the operator $L[y] = y'' + y$ has constant coefficients, we assume a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$. This leads to the two linearly independent solutions $y_1 = \cos(x)$ and $y_2 = \sin(x)$. Hence we can deduce that

$y_h = c_1 \cos(x) + c_2 \sin(x)$ is the general solution of $y'' + y = 0$.

To use the linear theory to obtain the general solution of the nonhomogeneous equation $L[y] = g(x)$, we need a particular solution, y_p, to $y'' + y = g(x)$. We have studied two techniques for this purpose (attendance is required):

i) Undetermined Coefficients (also called judicious guessing)
ii) Variation of Parameters (also called variation of constants)

For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation $y'' + y = g(x)$:

1. (2 pts.) $g(x) = x^{-1} e^x$ ____ ______ A B C D E

2. (2 pts.) $g(x) = \tan(x)$ ____ ______ A B C D E

3. (2 pts.) $g(x) = \sin(x)$ ____ ______ A B C D E

4. (2 pts.) $g(x) = e^{-x}$ ____ ______ A B C D E

A) Neither technique works to find y_p.

B) Only Undetermined Coefficients works to find y_p.

C) Only Variation of Parameters works to find y_p.

D) Either technique works to find y_p.

ABCDE) None of the above statements are true.

Total points this page = 8. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. ________________
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The dimension of the null space of the linear operator $L[y] = y'' - y'$ that maps $A(\mathbf{R},\mathbf{R})$ to $A(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the two linearly independent solutions $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that

$y_h = c_1 + c_2\ e^{x}$ is the general solution of the homogeneous equation $y'' - y' = 0$.

Use the method discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Circle the correct (most efficient) final form of the judicious guess for a particular solution y_p of the following ode's

5. (3 pts.) $y'' - y' = 2e^{x}$ First guess: y_p = ________________________
Second guess (if needed): y_p = ____________________
Third guess (if needed): y_p = ____________________

Final guess ______________________________. _______ A B C D E

6. (3 pts.) $y'' - y' = 3 \sin x$ First guess: y_p = ________________________
Second guess (if needed): y_p = ____________________
Third guess (if needed): y_p = ____________________

Final guess ______________________________. _______ A B C D E

7. (3 pts.) $y'' - y' = -4xe^{-x}$ First guess: y_p = ________________________
Second guess (if needed): y_p = ____________________
Third guess (if needed): y_p = ____________________

Final guess ______________________________. _______ A B C D E

Possible Answers for Final Guesses.
A) A B) $Ax + B$ C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB) Ae^{x}
AC) Axe^{x} AD) Ax^2e^{x} AE) $Axe^{x} + Be^{x}$ BC) $Ax^2e^{x} + Bxe^{x}$ BD). Ae^{-x} BE) Axe^{-x}
CD) Ax^2e^{-x} CE) $Axe^{-x} + Be^{-x}$ DE) $Ax^2e^{-x} + Bxe^{-x}$ ABC) $A \sin x$ BE. $\cos x$
ABD) $A x \sin x$ ABE) $A x \cos x$ ACD) $A\sin x + B\cos x$ ACE) $Ax \sin x + Bx \cos x$
ABCD) None of the above
Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _________________________(________________) ID No. __________________

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Solve $y'' + y = 3x + 2e^x$ on the back of the previous sheet.

8. (3 pts.) The general solution of $y'' + y = 0$ is ______________________. ____ A B C D E

9. (4 pts.) A particular solution of $y'' + y = 3x$ is ____________________. _____ A B C D E

10. (4 pts.) A particular solution of $y'' + y = 2e^x$ is __________________. _____ A B C D E

11. (1 pts.) A particular solution of $y'' + y = 3x + 2e^x$ is ________________. ____ A B C D E

12. (2 pts.) The general solution of $y'' + y = 3x + 2e^x$ is:________________. ____ A B C D E

A) $c_1x + c_2$ B) $c_1e^x + c_2xe^x$ C) $c_1e^{-x} + c_2xe^{-x}$ D) $c_1 \sin(x) + c_2 \cos(x)$ E) $c_1 x \sin(x) + c_2 x \cos(x)$ AB) $c_1e^x + c_2e^{-x}$ AC) $3x + 1$ AD) $3e^x$ AE) $3x$ BC) $3 \sin(x) + \cos(x)$ BD) $3 x\sin(x) + x\cos(x)$ BE) $3e^x + e^{-x}$ CD) $x + 1$ CE) e^x DE) $3e^x + e^{-x}$ ABC) $3x + e^x$ ABD) $2e^x$ ABE) $3x + 2e^x$ ACD) $3x + e^{-x}$ ACE) $3x + e^x + c_1x + c_2$ ADE) $3x + e^x + c_1e^x + c_2xe^x$ BCD) $3x + e^x + c_1e^{-x} + c_2xe^{-x}$ BCE) $3x + e^x + c_1 \sin(x) + c_2 \cos(x)$ BDE) $3x + e^x + c_1 x\sin(x) + c_2 x\cos(x)$ CDE) $3x + e^x + c_1e^x + c_2e^{-x}$ ABCD) $e^x + c_1x + c_2$, ABCE) $e^x + c_1e^x + c_2xe^x$ ABDE) $e^x + c_1e^{-x} + c_2xe^{-x}$
ACDE) $e^x + c_1 \sin(x) + c_2 \cos(x)$ BCDE) None of the above.

Total points this page = 14. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME _______________________(_______________) ID No. ________________
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Solve the ODE $y'' + y = \tan(x)$ $I = (0, \pi/2)$ (i.e. $0 < x < \pi/2$) on the back of the previous sheet. Also let $L[y] = y'' + y$.

13. (2 pts.) The general solution of $L[y] = 0$ is $y_c(x) =$ ________________. ____ A B C D E
A) $c_1 \cos(x) + c_2 \sin(x)$ B) $c_1 \cos(2x) + c_2 \sin(2x)$, C) $c_1 e^x + c_2 e^{-x}$ D) $c_1 x + c_2$
E) $r = \pm i$ AB) $r = \pm 1$ AC) $r = \pm 2i$ AD) None of the above.

14. (3 pts.) To find a particular solution $y_p(x)$ to $L[y] = \tan(x)$ using the technique of variation of parameters, we let $y_p(x) = u_1(x)\cos(x) + u_2(x)\sin(x)$. Substituting into the ODE and making the

appropriate assumption we obtain:______________________________. _____A B C D E
A) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
B) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \tan(x)$
C) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = \tan(x)$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = 0$
D) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \sin(x)$
E) $u'_1(x)\cos(x) + u'_2(x)\sin(x) = 0$, $-u'_1(x)\sin(x) + u'_2(x)\cos(x) = \cos(x)$
AB) None of the above.

15. (3 pts.) Solving we obtain ____. ____A B C D E A) $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = \sin(x)$ B) $u'_1(x) = \sin(x)$, $u'_2(x) = -\sin^2(x)/\cos(x)$ C) $u'_1(x) = 1$, $u'_2(x) = \sin(x)$ D) $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = 1$
E) $u'_1(x) = 0$, $u'_2(x) = \sin(x)$ AB) None of the above.

16. (4 pts.) Hence we may choose____________________________________. ____A B C D E
A) $u_1(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, $u_2(x) = -\cos(x)$
B) $u_1(x) = -\cos(x)$, $u_2(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, C) $u_1(x) = x$, $u_2(x) = -\cos(x)$, D) $u_1(x) = -\ln(\tan(x) + \sec(x))$, $u_2(x) = x$, E) $u_1(x) = 1$, $u_2(x) = -\cos(x)$, AB)None of the above.

17. (2 pts.) Hence a particular solution to $L[y] = \tan(x)$ is

$y_p(x) =$ ____. ___A B C D E
A) $-\ln(\tan(x) + \sec(x))$ B) $-[\cos(x)]\ln(\tan(x) + \sec(x))$ C) $-[\sin(x)]\ln(\tan(x) + \sec(x))$
D) $-[\tan(x)]\ln(\tan(x) + \sec(x))$ E) $\sin(x)\cos(x)$ AB) $2\sin(x)\cos(x)$
AC) $-[\sin(x)\cos(x)]\ln(\tan(x) + \sec(x))$ AD)None of the above

18. (2 pts.) Hence the general solution of $L[y] = \tan(x)$ is

$y(x) =$ ____. _____A B C D E
A) $-\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$ B) $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$
C) $-[\sin(x)]\ln(\tan(x) + \sec(x)) + c_1\cos(x) + c_2\sin(x)$ D) $\sin(x)\cos(x) + c_1 e^x + c_2 e^{-x}$
E) $-\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$ AB) $-[\cos(x)]\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$
AC) None of the above.

Total points this page = 16. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(__________________) ID No. ___________________

Last Name, First Name MI, What you wish to be called

Solve the ODE $y^{IV} - 4y''' + 4y'' = 0$ on the back of the previous sheet. Let L:$\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$. Be careful as once you make a mistake, the rest is wrong.

19. (1 pt). The order of the ODE given above is __________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

20. (1 pt) . The dimension of the null space of L is _________________. _____ A B C D E
A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

21. (1 pts). The auxiliary equation for this ODE is ________________________. _____ A B C D E
A) $r^2 - 4r + 4 = 0$ B) $r^4 - 4r^2 + 4 = 0$ C) $r^4 - 4r^3 + 4r^2 = 0$ D) $r^6 + 4r^3 + 4r^2 = 0$
E) $r^6 - 4r^3 + 4r^2 = 0$ AB) None of the above.

22. (2 pts). Listing repeated roots, the roots of the auxiliary equation

are _____________________________. _____ A B C D E A) r = 0, 2, B) r = 0, 0, 2, 2
C) r = 2, 2 D) r = 0,4 E) r = 0,2,4 AB) r = 0, 0,−2, −2 AC) r = 0, 0 2i, −2i
AD) None of the above.

23. (2 pts). A basis for the null space of L is ____________________________. _____ A B C D E
A) $\{1, x, e^{-2x}, xe^{-2x}\}$ B) $\{1, x, e^{2x}, xe^{2x}\}$ C) $\{1, x, x^2, e^{2x}\}$ D) $\{1, e^{2x}\}$ E) $\{1, x, x^2, x^3\}$
AB) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ AC) $\{1, x, x^2, e^{-2x}\}$ AD) $\{1, e^{-2x}\}$ AE) None of the above.

24. (1 pt). The general solution of this ODE is _____________________________. _____A B C D E
A) $y(x) = c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$ B) $y(x) = c_1 + c_2 x + c_3 e^{2x} + c_4 xe^{2x}$
C) $y)x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x}$ D) $y(x) = c_1 + c_2 e^{2x}$
E) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$ AB) $y(x) = c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$
AC) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $y(x) = c_1 + c_2 e^{-2x}$ AE) None of the above.

Points this page = 8. TOTAL POINTS EARNED THIS PAGE __________

PRINT NAME ______________________(_______________) ID No. _______________
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(9 pts.) The dimension of the null space of the linear operator $L[y] = y''' + y'$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 3. Assuming a solution of the homogeneous equation $L[y] = 0$ of the form $y = e^{rx}$ leads to the three linearly independent solutions $y_1 = 1$ and $y_2 = \cos(x)$ and $y_3 = \sin(x)$. Hence we can deduce that

$y_c = c_1 + c_2 \cos(x) + c_3 \sin(x)$ is the general solution of $y''' + y' = 0$.

Use the method discussed in class (attendance is mandatory) to determine the proper (most efficient) form of the judicious guess for a particular solution y_p of the following ode's. Do not give a second or third guess if these are not needed. Put your final guess in the box provided.

25. (3 pts.) $y''' + y' = \sin(x)$ First guess: y_p = ______________________
Second guess (if needed): y_p = ______________________
Third guess (if needed): y_p = ______________________

Final guess ______________________________. _____ A B C D E

26. (3 pts.) $y''' + y' = 4x^2$ First guess: y_p = ______________________
Second guess (if needed): y_p = ______________________
Third guess (if needed): y_p = ______________________

Final guess ______________________________. _______ A B C D E

27.(3 pts.) $y''' + y' = -4xe^{-x}$ First guess: y_p = ______________________
Second guess (if needed): y_p = ______________________
Third guess (if needed): y_p = ______________________

Final guess ______________________________. _____ A B C D E

Possible Answers
A) A B) $Ax + B$ C) $Ax^2 + Bx + C$ D) Ax^2 E) $Ax^2 + Bx$ AB. Ae^x
AC) Axe^x AD) Ax^2e^x AE) $Axe^x + Be^x$ BC) $Ax^2e^x + Bxe^x$ BD) Ae^{-x}
BE) Axe^{-x} CD) Ax^2e^{-x} CE.) $Axe^{-x} + Be^{-x}$ DE) $Ax^2e^{-x} + Bxe^{-x}$ ABC) A sin x
BE) cos x ABD) A x sin x ABE) A x cos x ACD) A sin x + B cos x
ACE) A x sin x + B x cos x ABCD) None of the above
Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. _________________
Last Name, First Name MI, What you wish to be called

Solve $y''' + y'' = 4e^x + 20\cos(x)$ on the back of the previous sheet..

28. (3 pts.) The general solution of $y''' + y'' = 0$ is

$y_c(x) =$ ____. ____ A B C D E

A) $c_1 + c_2x + c_3e^x$ B) $c_1 + c_2x + c_3e^{-x}$ C) $c_1 + c_2\sin(x) + c_3\cos(x)$
D) $c_1e^x + c_2\sin(x) + c_3\cos(x)$ AB) $c_1 + c_2e^x + c_3e^{-x}$ AC) $c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$
AD) $c_1 + c_2x + c_3e^{2x}$ AE) $c_1 + c_2x + c_3x^2$ BC) None of the above.

29. (4 pts.) A particular solution of $y''' + y'' = 4e^{-x}$ is

$y_{p1}(x) =$ ____. ____ A B C D E

A) $\frac{1}{2}e^x$ B) $2e^x$ C) $\frac{1}{2}xe^x$ D) $\frac{1}{2}\sin(x)$ E) $\frac{1}{2}xe^x + e^x$ AB) $2xe^x + e^x$
AC) None of the above.

30. (4 pts.) A particular solution of $y''' + y'' = 20\cos(2x)$ is

$y_{p2}(x) =$ ____. _____ A B C D E

A) $20\cos(x)$ B) $20\sin(x)$ C) $3x$ D) $-10\sin(x) - 10\cos(x)$ E) $-10\sin(2x) - 10x\cos(x)$
AB) $10\sin(x) + 10\cos(x)$ AC) None of the above.

31. (1 pts.) A particular solution of $y''' + y'' = 4e^x + 20\cos(2x)$ is

$y_p(x) =$ ____. _____ A B C D E

A) $\frac{1}{2}e^x - 10\sin(x) - 10\cos(x)$ B) $2e^x - 10\sin(x) - 10\cos(x)$ C) $2e^x + 10\sin(x) + 10\cos(x)$
D) $2e^x + 20\sin(x)$ E) $3x - 10\sin(x) - 10\cos(x)$ AB) $2e^x + 20\cos(x)$ AC) None of the above.

32. (2 pts.) The general solution of $y''' + y'' = 4e^x + 20\cos(2x)$ is

$y(x) =$ ____. _____ A B C D E

A) $\frac{1}{2}e^x + 20\sin(x) + c_1 + c_2x + c_3x^2$ B) $2e^x - 10\sin(x) - 10\cos(x) + c_1 + c_2x + c_3e^x$
C) $2e^x + 10\sin(2x) + 10\cos(x) + c_1 + c_2x + c_3e^x$ D) $\frac{1}{2}e^x - 10\sin(x) - 10\cos(x) + c_1 + c_2x + c_3e^x$
E) $3x + e^x + c_1x\sin(x) + c_2x\cos(x) + c_3e^{-x}$ AB) $e^x + 3\sin(2x) + \cos(x) + c_1e^x + c_2e^{-x} + c_3x + c_4$ AC) $e^x + 2\sin(2x) + \cos(2x) + c_1e^x + c_2xe^x + c_3$
AD) $\frac{1}{2}e^{-x} + 2\sin(2x) + \cos(2x) + c_1xe^x + c_2e^{-x} + c_3xe^{-x}$ AE) $e^x + c_1\sin(x) + c_2\cos(x)$
BC) $3x + c_1x\sin(x) + c_2x\cos(x)$ BD) $3x + c_1e^x + c_2e^{-x}$ BE). $3x + c_1x + c_2$
CD) $3x + c_1e^x + c_2xe^x$ CE) $3x + c_1e^{-x} + c_2xe^{-x}$ AB). $3x + c_1\sin(x) + c_2\cos(x)$
ABD) $3x + c_1x\sin(x) + c_2x\cos(x)$ ABE) $3x + c_1e^x + c_2e^{-x}$ ACD) None of the above.

Total points this page = 14. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(________________) ID No. __________________

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On the back of the previous sheet find a recursion formula for finding the coefficients to a power series solution about x = 0 to the ODE $y'' + x y' - 2 y = 0$

33. (1 pts.) To find the recursion formula for the power series solution about x = 0 of this ODE

we let: ______________________________. ____ A B C D E

A) $y = \sum_{n=1}^{\infty} a_n x^n$

B) $y = \sum_{n=0}^{N} a_n x^n$ C) $y = \sum_{n=0}^{\infty} a_n x^n$ D) $y = \sum_{n=0}^{\infty} a_n (n+1) x^{n+1}$ E) $y = \sum_{n=0}^{\infty} a_{n+2} x^n$

AB) $y = \sum_{n=2}^{\infty} a_{n+2} x^n$ AC) None of the above.

34. (2 pts) Substituting into this ODE we obtain

____. _____ A B C D E

A) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + \sum_{n=0}^{\infty} a_n n x^{n-1} - 2\sum_{n=0}^{\infty} a_n x^n = 0$

B) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + x\sum_{n=0}^{\infty} a_n n x^{n-1} - 2\sum_{n=0}^{\infty} a_n x^n = 0$

C) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + x\sum_{n=0}^{\infty} a_n n x^{n-1} + 2\sum_{n=0}^{\infty} a_n x^n = 0$ D) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} - 2\sum_{n=0}^{\infty} a_n n x^n = 0$

E) $\sum_{n=0}^{\infty} a_n n(n-1) x^{n-2} + \sum_{n=0}^{\infty} a_n n x^n + 2\sum_{n=0}^{\infty} a_n x^n = 0$ AC) None of the above.

35. (3 pts) By changing the index and simplifying, this equation can be changed to obtain

____. _____ A B C D E

A) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+3) + (n-2)a_n] x^n = 0$ B) $\sum_{n=0}^{\infty} [a_{n+2} + (n-2)a_n] x^n = 0$

C) $\sum_{n=1}^{\infty} [a_{n+2}(n+2)(n+1) + (n+2)a_n] x^n = 0$ D) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) - n a_n] x^n = 0$

E) $\sum_{n=0}^{\infty} [a_{n+2}(n+2)(n+1) + (n-2)a_n] x^n = 0$ AC) None of the above.

36. (5 pts.) The recursion formula for finding the coefficients in the power series

is____. ______A B C D E

A) $a_{n+2} = -\frac{n-2}{(n+2)(n+3)} a_n$ B) $a_{n+2} = -\frac{n+2}{(n+2)(n+1)} a_n$ C) $a_{n+2} = -\frac{n-2}{(n+2)(n-1)} a_n$

D) $a_{n+2} = -\frac{n-2}{(n+2)(n+1)} a_n$ E) $a_{n+2} = -\frac{n-1}{(n+2)(n+1)} a_n$ AB) $a_{n+2} = -\frac{n}{(n+2)(n+1)} a_n$

AC) $a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_n$ AD) None of these is a possible recursion formula.

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MATHEMATICAL MODELING. As done in class (attendance is mandatory), you are to develop a general mathematical model for the mass/spring problem. Take positive distance to be down. Suppose a mass m due to its weight W = mg where g is the acceleration due to gravity stretches a spring of length L a distance $\Delta\ell$. If the mass is stretched downward a distance u_0 from its equilibrium position and given an initial velocity v_0, develop an appropriate mathematical model to determine the subsequent motion (i.e. to find the distance u(t) from the equilibrium position as a function of time). Assume that the air resistance is c in feet slugs per second and that the spring constant is k in pounds per foot (or slugs per second squared). Assume an external force g(t) in slug feet per second squared.

37. (1 pt) The fundamental physical law needed to develop the model

is ____. _____ A B C D E

A) Ohm's law, B) Conservation of mass C) Conservation of energy D) Kirchoff's law
E) Newton's second law AB) None of the above.

38. (1 pt.) The relationship between $\Delta\ell$, k, m, and g

is____. ____ A B C D E

A) $k = \Delta\ell\, m\, g$ B) $k\,\Delta\ell = m\, g$ C) $k\, m = \Delta\ell\, g$ D) $k\, g = m\,\Delta\ell$ E) $m\,\Delta\ell = k\, g$
AB) None of the above.

39. (3 pts.)The mathematical model for the mass spring system whose solution yields the distance u(t) from the equilibrium position as a function of time

is ____. ______ A B C D E

A) $m\ddot{u} + cu + ku = g(t)$ B) $m\ddot{u} + c\dot{u} + ku = g(t)$ C) $m\ddot{u} + ku = g(t)$
D) $m\ddot{u} + c\dot{u} + ku = g(t), \quad u(0) = u_0 \quad \dot{u}(0) = v_0$ E) $m\ddot{u} + cu + ku = g(t), \quad u(0) = u_0 \quad \dot{u}(0) = v_0$
AB) $m\ddot{u} + ku = g(t), \quad u(0) = u_0 \quad \dot{u}(0) = v_0$ AC) None of the above.

40. (1 pt.) The units for the ODE in the model

are ____. ______ A B C D E

A) Feet, B) Seconds C) feet per second, D) feet per second squared, E) Pounds,
AB) Slugs, AC) Slug feet AD) None of the above.

Total points this page = 6. TOTAL POINTS EARNED THIS PAGE ______

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For each question write your answer in the blank provided. Next find your answer from the list of possible answers listed below and write the corresponding letter or letters for your answer in the blank provided. Then circle this letter or letters. Finally, circle your answer.

MATHEMATICAL MODELING. Consider the following problem (**DO NOT SOLVE!**):

A mass weighing 4 lbs. stretches a spring (which is 10 ft. long) 2 inches. If the mass is raised 3 inches above its equilibrium position and given an initial velocity of 5 ft./sec. (upward), determine the subsequent motion (i.e. find the distance from the equilibrium position as a function of time). Assume that the air resistance is negligible.

Apply the data given above to the model you developed on the previous page to obtain the **specific model** for this problem. **DO NOT SOLVE!**

41. (2 pts.) The spring constant k in pounds per foot (or slugs per second squared)

is ____. _____ A B C D E

A) $k = 2$ B) $k = 24$ C) $k = 4$ D) $k = 2/5$ E) $k = 10$ AB) $k = 5/2$ AC) None of the above.

42. (3 pts.) The specific mathematical model for the mass spring system whose solution yields the distance u(t) down from the equilibrium position as a function of time

is ____. _____ A B C D E

A) $\frac{1}{8}\ddot{u} + 2u + 24u = \sin(t)$ B) $\frac{1}{8}\ddot{u} + 2\dot{u} + 24u = 0$ C) $\frac{1}{8}\ddot{u} + 24u = 0$

D) $\frac{1}{8}\ddot{u} + 2\dot{u} + 24u = 0, \quad u(0) = 3 \quad \dot{u}(0) = 5$ E) $\frac{1}{8}\ddot{u} + 24u = \sin(t), \quad u(0) = \frac{1}{4} \quad \dot{u}(0) = 5$

AB) $\frac{1}{8}\ddot{u} + 24 = 0, \quad u(0) = -\frac{1}{4} \quad \dot{u}(0) = -5$ AC) None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______