EXAM-3 MATH 261: Elementary Differential Equations MATH 261
FALL 2005 EXAMINATION COVER PAGE Professor Moseley

PRINT NAME ____ (________________________)
Last Name, First Name MI (What you wish to be called)

ID # ____ EXAM DATE Friday October 28, 2005

I swear and/or affirm that all of the work presented on this exam is my own and that I have neither given nor received any help during the exam.

_______________________________ _______________
SIGNATURE DATE

Scores

page	points	score
1	8	
2	9	
3	14	
4	16	
5	8	
6	9	
7	14	
8	11	
9	6	
10	5	
11		
12		
13		
14		
15		
16		
17		
18		
19		
20		
21		
22		
Total	100	

INSTRUCTIONS

1. Besides this cover page, there are 10 pages of questions and problems on this exam. **MAKE SURE YOU HAVE ALL THE PAGES**. If a page is missing, you will receive a grade of zero for that page. Read through the entire exam. If you cannot read anything, raise your hand and I will come to you.
2. Place your I.D. on your desk during the exam. Your I.D., this exam, and a straight edge are all that you may have on your desk during the exam. **NO CALCULATORS! NO SCRATCH PAPER!** Use the back of the exam sheets if necessary. You may remove the staple if you wish. Print your name on all sheets.
3. Pages 1-10 are multiple choice. Expect no part credit on these pages. There are no free response pages. However, to insure credit, you should explain your solutions fully and carefully. Your entire solution may be graded, not just your final answer. **SHOW YOUR WORK!** Every thought you have should be expressed in your best mathematics. Partial credit will be given as deemed appropriate. **Proofread your solutions** and **check your computations** as time allows. **GOOD LUCK**!!!!!!!!!!

REQUEST FOR REGRADE

Please regard the following problems for the reasons I have indicated: (e.g., I do not understand what I did wrong on page ____.)

(Regrades should be requested within a week of the date the exam is returned. Attach additional sheets as necessary to explain your reasons.) I swear and/or affirm that upon the return of this exam I have **written nothing on this exam** except on this REGRADE FORM. (Writing or changing anything is considered to be cheating.)

Date _______ Signature_______________________________

PRINT NAME __________________________(_______________) ID No. ____________________

Last Name, First Name MI, What you wish to be called

The **dimension of the null space** of the **linear operator** $L[y] = y'' + y$ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Since the operator $L[y] = y'' + y$ has constant coefficients, we assume a solution of the **homogeneous equation** $L[y] = 0$ of the form $y = e^{rx}$. This leads to the two **linearly independent solutions** $y_1 = \cos(x)$ and $y_2 = \sin(x)$. Hence we can deduce that

$y_h = c_1 \cos(x) + c_2 \sin(x)$ is the **general solution** of $y'' + y = 0$.

To use the linear theory to obtain the **general solution of the nonhomogeneous equation** $L[y] = g(x)$, we need a particular solution, y_p, to $y'' + y = g(x)$. We have studied two techniques for this purpose (attendance is required):

i) Undetermined Coefficients (also called judicious guessing)
ii) Variation of Parameters (also called variation of constants)

(8 pts.) For each of the functions g(x) given below, circle the correct answer that describes which of these techniques can be used to find y_p for the nonhomogeneous equation $y'' + y = g(x)$:

1. $g(x) = \sin(x)$ A B C D E

2. $g(x) = e^{-x}$ A B C D E

3. $g(x) = x^{-1} e^x$ A B C D E

4. $g(x) = \tan(x)$ A B C D E

A. Neither technique works to find y_p.

B. Only Undetermined Coefficients works to find y_p.

C. Only Variation of Parameters works to find y_p.

D. Either technique works to find y_p.

E. None of the above statements are true.

PRINT NAME _________________________(______________) ID No. __________________
Last Name, First Name MI, What you wish to be called

(9 pts.) The **dimension of the null space** of the **linear operator** L[y] = y″ − y′ that maps $\mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ is 2. Assuming a solution of the **homogeneous equation** L[y] = 0 of the form $y = e^{rx}$ leads to the two **linearly independent solutions** $y_1 = 1$ and $y_2 = e^{x}$. Hence we can deduce that

$y_h = c_1 + c_2\, e^{x}$ is the **general solution** of the homogeneous equation $y'' - y' = 0$.

Use the method discussed in class (attendance is mandatory) to determine the **proper (most efficient) form** of the judicious guess for a **particular solution** y_p of the following ode's. Circle the correct (most efficient) form of the judicious guess for a particular solution y_p of the following ode's

5. $y'' - y' = 2e^{x}$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

6. $y'' - y' = 3 \sin x$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

7. $y'' - y' = -4xe^{-x}$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

Possible Answers

A. A B. $Ax + B$ C. $Ax^2 + Bx + C$ D. Ax^2 E. $Ax^2 + Bx$

AB. Ae^{x} AC. Axe^{x} AD. Ax^2e^{x} AE. $Axe^{x} + Be^{x}$ BC. $Ax^2e^{x} + Bxe^{x}$

BD. Ae^{-x} BE. Axe^{-x} CD Ax^2e^{-x} CE. $Axe^{-x} + Be^{-x}$ DE. $Ax^2e^{-x} + Bxe^{-x}$

ABC. A sin x BE. cos x ABD A x sin x ABE A x cos x ACD. A sin x + B cos x

ACE   A x sin x + B x cos x   ABCD   None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ___________________________(_______________) ID No. ____________________
Last Name, First Name MI, What you wish to be called

Your are to solve $y'' + y = 3x + 2e^x$.

8. (3 pts.) The general solution of $y'' + y = 0$ is: A. $c_1x + c_2$ B. $c_1e^x + c_2xe^x$ C. $c_1e^{-x} + c_2xe^{-x}$ D. $c_1\sin(x) + c_2\cos(x)$ E. $c_1 x\sin(x) + c_2 x\cos(x)$ AB. $c_1e^x + c_2e^{-x}$ AC.None of the above.

9. (4 pts.) A particular solution of $y'' + y = 3x$ is: A. $3x + 1$ B. $3e^x$ C. $3x$ D. $3\sin(x) + \cos(x)$ E. $3x\sin(x) + x\cos(x)$ AB. $3e^x + e^{-x}$ AC. None of the above.

10. (4 pts.) A particular solution of $y'' + y = 2e^x$ is: A. $x + 1$ B. e^x C. $3x$ D. $3\sin(x) + \cos(x)$ E. $3x\sin(x) + x\cos(x)$ AB. $3e^x + e^{-x}$ AC.None of the above.

11. (1 pts.) A particular solution of $y'' + y = 3x + 2e^x$ is: A. $3x + e^x$ B. e^x C. $3x$ D. $3\sin(x) + \cos(x)$ E. $3x\sin(x) + x\cos(x)$ AB. $3x + e^{-x}$ AC.None of the above.

12. (2 pts.) The general solution of $y'' + y = 3x + 2e^x$ is: A. $3x + e^x + c_1x + c_2$,
B. $3x + e^x + c_1e^x + c_2xe^x$, C. $3x + e^x + c_1e^{-x} + c_2xe^{-x}$, D. $3x + e^x + c_1\sin(x) + c_2\cos(x)$,
E. $3x + e^x + c_1 x\sin(x) + c_2 x\cos(x)$, AB. $3x + e^x + c_1e^x + c_2e^{-x}$, AC. $e^x + c_1x + c_2$,
AD. $e^x + c_1e^x + c_2xe^x$, AE. $e^x + c_1e^{-x} + c_2xe^{-x}$, BC. $e^x + c_1\sin(x) + c_2\cos(x)$,
BD. $3x + c_1 x\sin(x) + c_2 x\cos(x)$, BE. $3x + c_1e^x + c_2e^{-x}$, CD. $3x + c_1x + c_2$,
CE. $3x + c_1e^x + c_2xe^x$, ABC. $3x + c_1e^{-x} + c_2xe^{-x}$, ABD. $3x + c_1\sin(x) + c_2\cos(x)$,
ABE. $3x + c_1 x\sin(x) + c_2 x\cos(x)$, ACD. $3x + c_1e^x + c_2e^{-x}$, ACE. None of the above.

PRINT NAME ________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

You are to solve $y'' + y = \tan(x)$ $I = (0, \pi/2)$ (i.e. $0 < x < \pi/2$). Let $L[y] = y'' + y$.

13. (2 pts.) The general solution of $L[y] = 0$ is: A. $c_1 \cos(x) + c_2 \sin(x)$, B. $c_1 \cos(2x) + c_2 \sin(2x)$, C. $c_1 e^x + c_2 e^{-x}$, D. $c_1 x + c_2$, E. $r = \pm i$, AB. $r = \pm 1$, AC. $r = \pm 2i$, AD. None of the above.

14. (3 pts.) To find a particular solution y_p to $L[y] = \tan(x)$ using the technique of variation of parameters, we let $y_p(x) = u_1(x) \cos(x) + u_2(x) \sin(x)$. Substituting into the ODE and making the appropriate assumption we obtain:
A. $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0$, $-u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$,
B. $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0$, $-u'_1(x) \sin(x) + u'_2(x) \cos(x) = \tan(x)$,
C. $u'_1(x) \cos(x) + u'_2(x) \sin(x) = \tan(x)$, $-u'_1(x) \sin(x) + u'_2(x) \cos(x) = 0$,
D $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0$, $-u'_1(x) \sin(x) + u'_2(x) \cos(x) = \sin(x)$,
E. $u'_1(x) \cos(x) + u'_2(x) \sin(x) = 0$, $-u'_1(x) \sin(x) + u'_2(x) \cos(x) = \cos(x)$,
AB. None of the above.

15. (3 pts.) Solving we obtain: A. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = \sin(x)$,
B. $u'_1(x) = \sin(x)$, $u'_2(x) = -\sin^2(x)/\cos(x)$, C. $u'_1(x) = 1$, $u'_2(x) = \sin(x)$,
D. $u'_1(x) = -\sin^2(x)/\cos(x)$, $u'_2(x) = 1$, E. $u'_1(x) = 0$, $u'_2(x) = \sin(x)$,
AB. None of the above.

16. (4 pts.) Hence we may choose : A. $u_1(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, $u_2(x) = -\cos(x)$,
B. $u_1(x) = -\cos(x)$, $u_2(x) = -\ln(\tan(x) + \sec(x)) + \sin x$, C. $u_1(x) = x$, $u_2(x) = -\cos(x)$,
D. $u_1(x) = -\ln(\tan(x) + \sec(x))$, $u_2(x) = x$, E. $u_1(x) = 1$, $u_2(x) = -\cos(x)$,
AB. None of the above.

17. (2 pts.) Hence a particular solution to $L[y] = \tan(x)$ is: A. $y_p(x) = -\ln(\tan(x) + \sec(x))$,
B. $-[\cos(x)] \ln(\tan(x) + \sec(x))$, C. $-[\sin(x)]\ln(\tan(x) + \sec(x))$,
D. $-[\tan(x)] \ln(\tan(x) + \sec(x))$, E. $\sin(x) \cos(x)$, AB. $2 \sin(x) \cos(x)$,
AC. $-[\sin(x) \cos(x)] \ln(\tan(x) + \sec(x))$, AD None of the above

18. (2 pts.) Hence the general solution of $L[y] = \tan(x)$ is:
A. $-\ln(\tan(x) + \sec(x)) + c_1 \cos(x) + c_2 \sin(x)$,
B. $-[\cos(x)] \ln(\tan(x) + \sec(x)) + c_1 \cos(x) + c_2 \sin(x)$,
C. $-[\sin(x)]\ln(\tan(x) + \sec(x)) + c_1 \cos(x) + c_2 \sin(x)$,
D. $\sin(x) \cos(x) + c_1 e^x + c_2 e^{-x}$, E. $-\ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$
AB. $-[\cos(x)] \ln(\tan(x) + \sec(x)) + c_1 e^x + c_2 e^{-x}$, AC. None of the above.

PRINT NAME ________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

You are to solve $y^{IV} - 4y''' + 4y'' = 0$ in steps by **circling the correct answers** to the following questions. Let $L: \mathcal{A}(\mathbf{R},\mathbf{R})$ to $\mathcal{A}(\mathbf{R},\mathbf{R})$ be defined by $L[y] = y^{IV} - 4y''' + 4y''$. Be careful as once you make a mistake, the rest is wrong.

19. (1 pt). The order of the ODE is A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

20. (1 pt) . The dimension of the null space for the linear operator $L[y] = y^{IV} - 4y''' + 4y''$ is: A) 1 B) 2 C) 3 D) 4 E) 5 AB) 6 AC) 7 AD) None of the above.

21. (1 pts). The auxiliary equation for this equation is: A) $r^2 - 4r + 4 = 0$, B) $r^4 - 4r^2 + 4 = 0$, C) $r^4 - 4r^3 + 4r^2 = 0$, D) $r^6 + 4r^3 + 4r^2 = 0$, E) $r^6 - 4r^3 + 4r^2 = 0$, AB) None of the above.

22. (2 pts). Listing repeated roots, the roots of the auxiliary equation are: A) r = 0, 2, B) r = 0, 0, 2, 2 C) r = 2, 2 D) r = 0,4, E) r = 0, 2,4 AB) r = 0, 0, −2, −2 AC) r = 0, 0 2i, −2i AD) None of the above.

23. (2 pts). A basis for the null space of the linear operator L is : A) $\{1, x, e^{-2x}, xe^{-2x}\}$ B) $\{1, x, e^{2x}, xe^{2x}\}$ C) $\{1, x, x^2, e^{2x}\}$ D) $\{1, e^{2x}\}$ E) $\{1, x, x^2, x^3\}$ AB) $\{e^{2x}, xe^{2x}, e^{-2x}, xe^{-2x}\}$ AC) $\{1, x, x^2, e^{-2x}\}$ AD) $\{1, e^{-2x}\}$ AE) None of the above.

24. (1 pt). The general solution of $y^{IV} - 4y''' + 4y'' = 0$ is: A) $y(x) = c_1 + c_2 x + c_3 e^{-2x} + c_4 xe^{-2x}$ B) $y(x) = c_1 + c_2 x + c_3 e^{2x} + c_4 xe^{2x}$ C) $y)x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{2x}$ D) $y(x) = c_1 + c_2 e^{2x}$ E) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 x^3$ AB) $y(x) = c_1 e^{2x} + c_2 xe^{2x} + c_3 e^{-2x} + c_4 xe^{-2x}$ AC) $y(x) = c_1 + c_2 x + c_3 x^2 + c_4 e^{-2x}$ AD) $y(x) = c_1 + c_2 e^{-2x}$ AE) None of the above.

PRINT NAME __________________________(_____________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

(9 pts.) The **dimension of the null space** of the **linear operator** L[y] = y‴ + y′ that maps A(**R**,**R**) to A(**R**,**R**) is 3. Assuming a solution of the **homogeneous equation** L[y] = 0 of the form $y = e^{rx}$ leads to the three **linearly independent solutions** $y_1 = 1$ and $y_2 = \cos(x)$ and $y_3 = \sin(x)$. Hence we can deduce that

$y_c = c_1 + c_2 \cos(x) + c_3 \sin(x)$ is the **general solution** of $y''' + y' = 0$.

Use the method discussed in class (attendance is mandatory) to determine the **proper (most efficient) form** of the judicious guess for a **particular solution** y_p of the following ode's. Do <u>not</u> give a second or third guess if these are not needed. Put your final guess in the box provided.

25. $y''' + y' = \sin(x)$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

26. $y''' + y' = 4x^2$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

27. $y''' + y' = -4xe^{-x}$ A B C D E AB AC AD AE BC BD BE

CD CE DE ABC ABD ABE ACD ACE ABCD

Possible Answers

A. A B. $Ax + B$ C. $Ax^2 + Bx + C$ D. Ax^2 E. $Ax^2 + Bx$

AB. Ae^x AC. Axe^x AD. Ax^2e^x AE. $Axe^x + Be^x$ BC. $Ax^2e^x + Bxe^x$

BD. Ae^{-x} BE. Axe^{-x} CD Ax^2e^{-x} CE. $Axe^{-x} + Be^{-x}$ DE. $Ax^2e^{-x} + Bxe^{-x}$

ABC. A sin x BE. cos x ABD A x sin x ABE A x cos x ACD. A sin x + B cos x

ACE A x sin x + B x cos x ABCD None of the above

Total points this page = 9. TOTAL POINTS EARNED THIS PAGE _______

PRINT NAME ________________________(_____________) ID No. _________________
Last Name, First Name MI, What you wish to be called

Your are to solve $y''' + y'' = 4e^{x} + 20\cos(2x)$.

28. (3 pts.) The general solution of $y''' + y'' = 0$ is: A. $c_1 + c_2x + c_3e^{x} + c_4xe^{x}$, B. $c_1 + c_2x + c_3e^{-x}$, C. $c_1 + c_2x + c_3\sin(x) + c_4\cos(x)$ D. $c_1e^{x} + c_2\sin(x) + c_3\cos(x)$, AB. $c_1 + c_2x + c_3e^{x} + c_4e^{-x}$ AC. $c_1e^{-x} + c_2\sin(x) + c_3\cos(x)$, AD. $c_1 + c_2x + c_3e^{x}$, AE. $c_1 + c_2x + c_3x^2$, BC. $c_1 + c_2x + c_3x^2 + c_3e^{-x}$, BD. None of the above.

29. (4 pts.) A particular solution of $y''' + y'' = 4e^{x}$ is: A. ½ e^{-x} B. $2e^{-x}$ C. ½ xe^{-x} D. ½ sin(x) E. ½ $xe^{-x} + e^{-x}$ AB. $2xe^{-x} + e^{-x}$ AC. None of the above.

30. (4 pts.) A particular solution of $y''' + y'' = 20\cos(2x)$ is: A. x + 1 B. e^{x} C. 3x D. 3 sin(2x) + cos(2x) E. 3 x sin(2x) + x cos(2x) AB. $3e^{x} + e^{-x}$ AC. None of the above.

31. (1 pts.) A particular solution of $y''' + y'' = 4e^{x} + 20\cos(2x)$ is:
A. ½ $e^{-x} + 3\sin(2x) + \cos(2x)$ B. ½ $e^{x} + 3\sin(2x) + \cos(2x)$
C. $2e^{-x} + 3\sin(2x) + \cos(2x)$ D. $3e^{-x} + 2\sin(2x)$
E. $x + 1 + 2\sin(2x) + \cos(2x)$ AB. $3x + e^{-x}$ AC. None of the above.

32. (2 pts.) The general solution of $y''' + y'' = 4e^{x} + 20\cos(2x)$ is:
A. ½ $e^{-x} + 3\sin(2x) + c_1 + c_2x + c_3x^2$, B. ½ $e^{-x} + 3\sin(2x) + c_1 + c_2x + c_3e^{-x}$,
C. ½ $e^{x} + 3\sin(2x) + \cos(x) + c_1 + c_2x + c_3x^2$,
D. ½ $e^{-x} + 2\sin(2x) + \cos(2x) + c_1 + c_2x + c_3e^{x}$ E. $3x + e^{x} + c_1x\sin(x) + c_2x\cos(x) + c_3e^{-x}$,
AB. $e^{x} + 3\sin(2x) + \cos(x) + c_1e^{x} + c_2e^{-x} + c_3x + c_4$
AC. $e^{x} + 2\sin(2x) + \cos(2x) + c_1e^{x} + c_2xe^{x} + c_3$
AD. ½ $e^{-x} + 2\sin(2x) + \cos(2x) + c_1xe^{x} + c_2e^{-x} + c_3xe^{-x}$ AE. $e^{x} + c_1\sin(x) + c_2\cos(x)$
BC. $3x + c_1x\sin(x) + c_2x\cos(x)$ BD. $3x + c_1e^{x} + c_2e^{-x}$ BE. $3x + c_1x + c_2$
CD. $3x + c_1e^{x} + c_2xe^{x}$ CE. $3x + c_1e^{-x} + c_2xe^{-x}$ ABC. $3x + c_1\sin(x) + c_2\cos(x)$
ABD. $3x + c_1x\sin(x) + c_2x\cos(x)$ ABE. $3x + c_1e^{x} + c_2e^{-x}$ ACD. None of the above.

PRINT NAME ________________________(_____________) ID No. __________________
Last Name, First Name MI, What you wish to be called

33. (2. pts.) To find the recursion formula for the power series solution about x = 0 of the equation y'' + x y' −2 y = 0 we let: A. $y = \sum_{n=1}^{\infty} a_n x^n$, B. $y = \sum_{n=0}^{N} a_n x^n$, C. $y = \sum_{n=0}^{\infty} a_n x^n$, D. $y = \sum_{n=0}^{\infty} a_n (n+1) x^{n+1}$, E. $y = \sum_{n=0}^{\infty} a_{n+2} x^n$, AB. $y = \sum_{n=2}^{\infty} a_{n+2} x^n$, AC. None of the above.

34. (4 pts) Substituting into the ODE we obtain:

A. $\sum_{n=0}^{\infty} a_n n(n+1)x^n + \sum_{n=0}^{\infty} a_n n x^n - 2\sum_{n=1}^{\infty} a_n x^n = 0$, B. $\sum_{n=0}^{\infty} a_n x^{n-2} + \sum_{n=0}^{\infty} a_n n x^n + \sum_{n=0}^{\infty} a_n x^n = 0$,

C. $\sum_{n=1}^{\infty} a_n n(n-1)x^n + \sum_{n=1}^{\infty} a_n n x^n - 2\sum_{n=1}^{\infty} a_n x^n = 0$, D. $\sum_{n=0}^{\infty} a_n n(n-1)x^n + \sum_{n=0}^{\infty} a_n n x^n = 0$,

E. $\sum_{n=0}^{\infty} a_n n(n+1)x^{n-2} + \sum_{n=0}^{\infty} a_n n x^n - 2\sum_{n=1}^{\infty} a_n x^n = 0$

AB. $\sum_{n=0}^{\infty} a_n n(n-1)x^{n-2} + \sum_{n=0}^{\infty} a_n n x^n - 2\sum_{n=0}^{\infty} a_n x^n = 0$, AC. None of the above.

35. (5 pts.) The recursion formula for finding the coefficients in the power series is:

A. $a_{n+2} = \frac{n-2}{(n+2)(n+1)} a_n$, B. $a_{n+2} = -\frac{n+2}{(n+2)(n+1)} a_n$, C. $a_{n+2} = -\frac{n-2}{(n+2)(n-1)} a_n$,

D. $a_{n+2} = -\frac{n-2}{(n+2)(n+1)} a_n$, E. $a_{n+2} = -\frac{n-1}{(n+2)(n+1)} a_n$,

AB. $a_{n+2} = -\frac{n-2}{(n+2)(n+3)} a_n$, AC. $a_{n+2} = -\frac{n-3}{(n+2)(n+1)} a_n$,

AD. None of these is a possible recursion formula.

PRINT NAME ________________________(_____________) SS No. __________________
Last Name, First Name MI, What you wish to be called

MATHEMATICAL MODELING. As done in class (attendance is mandatory), you are to develop a **general mathematical model** for the mass/spring problem. Take positive distance to be down. Suppose a mass m due to its weight W = mg where g is the acceleration due to gravity stretches a spring of length L a distance $\Delta\ell$. If the mass is stretched downward a distance u_0 from its equilibrium position and given an initial velocity v_0, develop an appropriate mathematical model to determine the subsequent motion (i.e. to find the distance u(t) from the equilibrium position as a function of time). Assume that the air resistance is c in feet slugs per second and that the spring constant is k in pounds per foot (or slugs per second squared). Assume an external force g(t) in slug feet per second squared.

36. (1 pt) The fundamental physical law needed to develop the model is: A. Ohm's law, B. Conservation of mass, C. Conservation of energy, D. Kirchoff's law E. Newton's second law AB. None of the above.

37. (1 pt.) The relationship between $\Delta\ell$, k, m, and g is: A. $k = \Delta\ell\, m\, g$, B. $k\,\Delta\ell = m\, g$, C. $k\, m = \Delta\ell\, g$ D. $k\, g = m\,\Delta\ell$ E. $m\,\Delta\ell = k\, g$, AB. None of the above.

38. (3 pts.)The mathematical model for the mass spring system whose solution yields the distance u(t) from the equilibrium position as a function of time is A. $m\ddot{u} + cu + ku = g(t)$,
B. $m\ddot{u} + c\dot{u} + ku = g(t)$ C. . $m\ddot{u} + ku = g(t)$ D. $m\ddot{u} + c\dot{u} + ku = g(t),\ \ u(0) = u_0\ \ \dot{u}(0) = v_0$
E. $m\ddot{u} + cu + ku = g(t),\ \ u(0) = u_0\ \ \dot{u}(0) = v_0$ AB. $m\ddot{u} + ku = g(t),\ \ u(0) = u_0\ \ \dot{u}(0) = v_0$,
AC. None of the above.

39. (1 pt.) The units for the ODE in the model are A. Feet, B. Seconds, C. feet per second, D. feet per second squared, E. Pounds, AB.Slugs, AC. Slug feet, AD. None of the above.

PRINT NAME _________________________(______________) ID No. ___________________
Last Name, First Name MI, What you wish to be called

(5 pts.) MATHEMATICAL MODELING. Consider the following problem (**DO NOT SOLVE!**):
A mass weighing 4 lbs. stretches a spring (which is 10 ft. long) 2 inches. If the mass is raised 3 inches above its equilibrium position and given an initial velocity of 5 ft./sec. (upward), determine the subsequent motion (i.e. find the distance from the equilibrium position as a function of time). Assume that the air resistance is negligible.

Apply the data given above to the model you developed on the previous page to obtain the **specific model** for this problem. **DO NOT SOLVE!**

40. (2 pts.) The spring constant k in pounds per foot (or slugs per second squared). is: k = 2,
B. k = 24 C. k = 4, D. k = 2/5, E. k = 10, AB. k = 5/2 AC. None of the above.

41. (3 pts.) The specific mathematical model for the mass spring system whose solution yields the distance u(t) down from the equilibrium position as a function of time is _____.
A. $\frac{1}{8}\ddot{u} + 2u + 24u = \sin(t)$, B. $\frac{1}{8}\ddot{u} + 2\dot{u} + 24u = 0$ C. . $\frac{1}{8}\ddot{u} + 24u = 0$, D. . $\frac{1}{8}\ddot{u} + 2\dot{u} + 24u = 0, \;\; u(0) = 3 \;\; \dot{u}(0) = 5$
E. $\frac{1}{8}\ddot{u} + 24u = \sin(t), \;\; u(0) = \frac{1}{4} \;\; \dot{u}(0) = 5$ AB. $\frac{1}{8}\ddot{u} + 24 = 0, \;\; u(0) = -\frac{1}{4} \;\; \dot{u}(0) = -5$,
AC. None of the above.

Total points this page = 5. TOTAL POINTS EARNED THIS PAGE ______