MATH 261: STUDY GUIDE AND ASSIGNMENT SHEET

Text: Elementary Differential Equations and Boundary Value Problems (Ninth Edition) by W.E. Boyce and R.C. DiPrima. Assignments are from this text except that LAN means "Linear Algebra Notes" by James L. Moseley and (A) indicates additional problems which are to be solved in the same assignment. These should be considered as minimal. Students who have difficulty as well as students who wish to acquire better than an average proficiency should work additional problems. The objective of this course is to provide a reasonably complete coverage of the techniques for solving first and second order ordinary differential equations (ODE's) with selected applications as well as an introduction to 1)Difference Equations 2)Numerical Techniques 3)Higher Order Linear ODE.'s, 4)Power Series Solutions, 5)Laplace Transforms, 6)Theory of Linear Equations, 7)Eigenvalue Problems for Matrices with Complex Entries, 8)Systems of ODE.'s , 9) Boundary Value Problems (BVP.'s), 10)Eigenvalue Problems for ODE.'s, 11)Partial Differential Equations (PDE.'s), and/or 12)Fourier Series. Additional coverage of these topics is contained in other courses. A change from the previous syllabus is that a better job of teaching the Linear Theory is attempted. Thus after first order ODE's are covered, a review of Vector Space Theory is included. Although a deep understanding is not expected, coverage of what the Dimension of the Null Space means is expected. There are 57 assignments. At least 47 should be covered. Those with an asterisk(*) can be omitted without seriously impairing the objective of the course. Four hour (50 min.) exams and a comprehensive final should be given.

Assgn.# § #	Material to be studied	Problems to be worked
	FIRST ORDER TECHNIQUES FOR ODES	
1. §1.1,§1.2, §1.3, 1.4 §2.1	Order, Linear vs. Nonlinear, Verifying a Solution, First Order Linear: Solution by Integrating Factor	p.24 #1 ,2,3,4,5,6 ,7,8,9 p.39 #1c,2c,3c,5c,6c
2. §1.2,§ 2.1 §2.4,§2.8	Linear: General Solution, Initial Value Problem (IV P), Exist. & Uniq. Theorem (no proof), Interval of Validity	p.39 #9c,11c,12c,13,15 p.15 #1 a,2a,3,4,5
3. §2.2	Separable Equations	p.47 #1,3,5,7,9,13,15
4. §2.4	Nonlin: Gen'l Sol'n, Imp licit Sol'n, Exist.,Uniq. (no prf)	p.75 #1,3,5,7,9,11
5. §1.1, § 2.1 §2.4	Direction Fields, Isoclines, Integral Curves (You may or may not be able to draw the direction field without a computer.)	p.39#1a,b,2a,b, p75#17,19 p.7#1,3,15,16,26,29
6. §2.6	Exact Equations	p.99 # 1,3,5,7,9,11,13
7. §2.2, § 2.4	Substitutions: 1) Bernoulli's Eq. 2) Homogeneous Eq.	p.75 # 27,28,29,30 p.47parts a,b,c,d,e for #30,31,33,35,37
8. §2.9MP	REVIEW	p.132 # 1,3,7,11 ,17,20,2 1,24.
	MATH MODELING I :App l. of First Order ODE 's	
9. §1.2, 2.3	Radioactive Decay, Continuous Compounding.	p.15 #12,13,14, p.59 #,7,8,9
*10. §1.2 , 2.3	Mixing (Tank) problems, Newton's Law of Cooling	p.59 #1,2,3,4,16,17,18
*11. §2.5	Population Dynamics	p.15# 7 ,8, p.88 # 1,2,3,4,5 ,6
12. §1.2 , §2.3	Elementary Mechanics, Newton's Second Law of Motion	p15 #9,10,11, p.59#20,21,22
	FIRST ORDER DIFFERENCE EQUATIONS	
*13. §2.9	Linear First Order Difference Equations	p. 130 # 1,2,3,5,7
	A NUMERICAL TECHNIQUE	
14 §2.7	Euler's Numerical Method for First Order ODE's	p.109 # 1,2

	SECOND ORDER I: Solution by First Order Techniques	
15. Mis.Pro Page132	Second Order: Solutions by First Order Techniques, Initial Value Problems (IV Ps), Linear and Nonlinear Eqs.	p.132 #36,38,40,42,44,46,48,49,50
	THEORY OF LINEAR EQ. I: Brief Review of Linear Algebra	
16 §4.2 ,§3.4	**R** and **C** as Alg. Fields, De Moivre's Thm., Euler's Form.	p.231#1,2,3,4,7,9, p.163#1,3,5
17. §7.2, LAN Chap. 3	Review o f Matrix Algebra, Abstract Linear Algebra, Def'n of a Vector S pace, $\mathbf{R}^n$, Function Spaces, Subspaces	p.371 #1,2,3,4,6a,7a,8,10,12,20 LAN Chap.3 p.4 #1-8, p.8 #1-4, p.12 #1-5, (A)Is W $=\{(x,y,0):x,y \in \mathbf{R}\}$a subspace of $\mathbf{R}^3$
18. §7.3 LAN Chap. 5	Linear Indep. in $\mathbf{R}^n$, Linear Algebraic Eqs., Writing Sol'ns Lin. Indep. of Functions	p.383 # 1,3,4,5,6 ,7,8 p.224 #7,8,9 LAN Chap.5, p.6 #1-12,p.11 #1-11,p.14 #1-8, p.(A)Are these sets linearly indep.?1.$\{e^{3x},e^{3(x-1)}\}$ 2.$\{3t-5, 9t-15\}$ 3. $\{x^3,\lvert x\rvert^3\}$
19. Review of Linear Algebra LAN Chap. 5	Review o f Concepts: Spanning Set, Basis, Dimension. Overview of the Theory of Linear Eqs. of the First Kind: Linear Operators, Linear Homogenous (Vector) Eqs. , Null Space, Basis of the Null Space, Dimension of the Null Space. General Solution of the Homogeneous Equation. Particular Solution of the Nonhomogeneous Equation. General Solution of the Nonhomogeneous Equation,	LAN Chap. 5 p.3#1-5,(A) Let $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}$ and 1) Compute $T([1,2]^T)$. 2) Show that T is a linear oper. 3) Solve $T(\vec{x}) \vec{0}$. 4)Find a basis for the null space of T.

	SECOND ORDE R II: Theory & Solution of Linear Eqs.	
20. §3.2	Linear Differential Operators. Linear Homogeneous Eqs. Linear Indep. of Solutions, Superposition, Wronskian, Existence and Uniqueness of Solution to I.V.P.	p.155 #1,3,5,7,8,9,13,14,15 (A) Let $L[y] = y''+y' + y$. Compute $L[e^x]$, $L[\sin(x)]$, and $L[3 +4t +5t^2]$
21. §3.1,§10.1	Linear Homogeneous Eqs. with Constant Coefficients: Distinct Real Roots, I.V.P.'s and Boundary Value Problems (B.V .P.'s)	p. 144 #1,3,5,7,9,10,11,12 A) (1) $y''- y = 0$, $y(0) = 0$, $y(1) = 0$ 2) $y''- y = 0$, $y(0) = 0$, $y(1) = 1$
22. §3.2	Linear Homo. Eqs: Fundamental Set, General Solution, Linear Independence of Solutions	p. 155 #22,23,24,28
23. §3.3	L. H. E. w. Const. Coef.: Complex Rts., Euler's Formula	p.163 # 7,9,11,13,17,19,21
24. §3.4	L. H. E. w. Con.. Coef.: Repeat Rts., Reduction of Order	p.171 #1,3,5,7,9,11,23,25,27
25. §3.5	Linear Nonhom. Eqs.: Method of Undet. Coefficients	p.183 #1,2,3,8,11,13,16
26. §3.5	Linear Nonhomogeneous Eqs.: Method of Undetermined Coefficients, Use of the Superposition Principle	p.183 # 4,5,7,9,10,14,15,17,18
27. §3.6	Linear Nonhomogeneous Eq s.: Variation of Parameters	p.189 #1,3,5,7,9,11,13
	MATH MODELING II :Appl. of Second Order OD E's	
28. §3.7	Mech. and Elect. Vibrations (Spring Prob & Elect. Cir.)	p.202 #1,2,5,6,8,9,12,13
*29. §3.8	Forced Vibrations	p.215 # 1,3,5,6,7 ,8,9
	HIGHER ORDER LINEAR ODE'S	
30. §4.1 §4.2	Higher Ord. Lin. ODE's: Theory & Tech.for Solving Linear Hom. Eq s. with Constant Coefficients	p.224 #1,2,7,9,11,13 p.231 #11,14, 15,17, 18,29,31

31. §4.3 ,4.4	Tech.for Solving Linear Nonhom. Eqs.: Meth.of Undeter. Coef., Variation of Para meters (No Problems)	p.237 #1,2,3,4,5,6,9,11,13,15
	POWER SERIES SOLUTIONS	
32. §5.1	Review of Power Series, Taylor Series, Change of Index in Sigma Notation	p.249 #1,3,5,7,9,11,13, 15,19, 20,21
33. §5.2	Solution o f Second Order Linear Equations (with Variable Coefficients) Using Power Series, Recursion Relation	p.259 # 1,2,5,7,9 (A) 1) $x^2y'' - 4xy' + 4y = 0$ 2) $xy'' + y' - y = 0$
	LAPLACE TRANSFORMS I: Def'n, Comp., and Prop.	
34. §6.1	Improper Integrals. Definition of Laplace Transform . Piecewise Continuous. Func., Exponential Order	p.311 #1,3,5a,6,15,21,23
35. §6.2	Linearity of the Laplace Transform, Use of Knowledge of Elementary Transform Pairs and Linearity to Obtain Transforms, Use of Table. Other Properties of the Laplace Transform: Transform of the Derivative, the nth Derivative, the Integral, $\mathcal{L}\{t\, f(t)\}(s) = -d\, \mathcal{L}\{f(t)\}(s)/ds$, $\mathcal{L}\{e^{at} f(t)\}(s) = \mathcal{L}\{f(t)\}(s-a)$	(A) Use a table and linearity to find the Laplace Transform of 1) $f(t) = 3+4t$, 2) $f(t) = 3e^{2t} + 5e^{-6t}$ 3) $f(t) = 7\sin(2t) + 8\cos(3t)$ 4) $f(t) = 4 + 3t + 2t^2 + 5t^3$ 5) $f(t) = 3 + t^2 + 2e^{2t} + 5\cos 3t$ 6) $f(t) = 2t + 3te^{2t} + 2t \sin 3t$ 7) $f(t) = 3e^t \sin 2t + 4e^{-2t} \sin 2t$
	THEORY OF LINEAR EQS . II: One-to-One Mappings	
36.	One-to-One functions, One-to-One Linear Operators. If we restrict ourselves to continuous functions of exponential order, then Lerch's Theorem assures us that the Laplace transform,$\mathcal{L}$, is a one-to-one (linear) operator. Hence the inverse transform,$\mathcal{L}^{-1}$, exists and can be shown to be linear. To understand what this means, we review the concepts of a one-to-one function f:**R**→**R** and a one-to-one linear operator from $\mathbf{R}^n$ to $\mathbf{R}^m$(m>n) .	(A) 1) Show that the function $f(x) = 3x + 2$ is one-to-one. 2) Let $T(\vec{x}) = A\vec{x}$ where $A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 0 \end{bmatrix}$ and $\vec{x} = [x\ y]^T$. Show that T is one-to-one.
	LAPLACE TRANS FORM S II: Inverse Trans. & IV P's	
37. §6.2	Exist. and Linearity of Inverse Transform. Computation of Inverse Transforms: Partial Fractions, Use of Table .	p.320 #1,2,3,4,5,6,7,8,9,10
38. §6.2	Solution of I.V.P.	p.320 #11,12,13,14,15,16,24 (A) $y''+y = 1$, $y(0) = 0$, $y'(0) = 0$
*39. §6.3	Step Functions	p.328 #1,2,3,4,7,9,13,15,17
*40. §6.4	Discontinuous Forcing Functions	p.336 #1,3,5,7,9,11

	SYSTEMS O F O.D.E.'s I : Sol'n by Scalar Tech.	
41. §7.1	Solution using Elimination	p.359 #1,2, 8,9,10,11,12
	THEORY OF LINEAR EQS III: Calc., Vec t., & EVP 's	
42. §7.1,§7.2 §7.3 §7.4	Calculus of Matrix and "Vector" functions, Eigenvalue Problems for Matrices Containing Real and Complex Entries.	p.371 # 21, p.383 #13,15,16,18,21,22 (A) Solve $A\vec{x} = \lambda\vec{x}$ $A = \begin{bmatrix} i & i \\ 0 & -1 \end{bmatrix}$

SYS. OF O.D .E.'s II: Theory & Sol'n by Matrix Techs.		
43. §7.4 §7.5	Theory of First Order Systems: Solution of First Order System, Wronskian, Fundamental Solution Set, General Solution, Homogeneous Linear System s with Constant Coefficients	p. 371 # 22,23, p.389 #4 ,6 (A) Convert to first order system and solve. Also find the Wronsk.. of the Fund. Sol'n Set 1) $y'' - y = 0$, 2) $y'' + 3y' + 2y = 0$
44. §7.5	Hom. Linear Sys. of ODE's with C onst, Coeff., Solution Using Matrix Techniques, Distinct Roots	p.398 #1a,3a,5a,7a,10,12 (Dir. fields and traject. need not be drawn.)
*45. §7.6	Hom. Linear Systems with Complex Eigenvalues	p.409 # 1a,2a,3a,5a,7 (No dir.f. or traj.)
*46. §7.8	Hom. Linear Systems with Repeated Eigenvalues	p.428 # 1c,2c,3c,5c,7c ,8c (No d .f. or traj.)
B.V.P .'s AND EIGE NVA LUE P ROB LEM O.D.E .'s		
47. §10 .1	Two Point Boundary Value Problems, Self adjoint Eigenvalue Problems for O.D .E.'s	p.583 #1,2,3,4,11,12 (A) Solve 1) $y''+y = 0$, $y(0)=0$, $y(1)=0$ 2) $y''+y = 0$, $y(0)= 0$, $y(\pi)=0$ 3) $y''+y = 0$, $y(0)= 0$, $y(\pi)=1$
FOURIER SERIES		
48. § 10 .2	Periodicity, Periodic Extent., Properties. of Trig Funcs	p.592 #1,2,3,4,5,6,7,8,9,10
49. § 10 .2	Computation of Fourier Series Coefficients.	p.592 #13,15,16,17,19,20
50. §10 .3	Fourier Theorem: Convergence of the Series	p.600 #1,2,3,4,5,9,10
51. §10 .4	Even and Odd Functions	p.608 #1,2,3,4,5,6,7,8,9,11,13
52. §10 .4	Sine and Cosine Series	p.608 #15,16,17,18,19,21,22,23,24,26
PARTIAL DIFF. EQS. Heat Conduction Model		
53 §10.1,10.5 App. A, B	Math Modeling: Derivation of the Heat Conduction Model, PDE 's, BC's, and IC's	p.583 # 14,15,16 (A) What phys. law is used to derive the wave eq.?
54. §10 .5	Sep. of Variables, General Sol'n of PDE and BC's.	p.618 # 1,2,3,4,5,6,7,8
55. §10 .5	Solution o f the Heat Conduction Model with Homogeneous BC's Using Fourier Series	p.618 #9,10,11, (A)Solve $u_t = u_{xx}$ $0 < x < \pi$, $t > 0$ $u(0,t) = 0$, $u(\pi,t) = 0$, $t > 0$ $u(x,0) = x$ $0 < x < \pi$
*56. §10 .6	Nonhomogeneous Boundary Conditions	p.627 # 1,3,5,7,9
*57. §10 .6	Bar with Insulated Ends	p.627 #12,13,14
*58.		