Related Rate Problems Involving Areas

Strategy to tackle the problem

(1) Identify the *given* variables and the variable whose rate is to be minimized. Convert the verbal descriptions into algebraic expressions so that we can see the given information more transparently.
(2) Identify the relation that allow us to write down an equation (referred as the first equation), involving the variables whose rates of change either are given or are to be determined.
(3) Compute the derivative both sides of the first equation, with respect to the time t. We call the resulting equation the second equation. (We may need to apply Chain Rule and/or implicit differentiation skills here).
(4) After completing the differentiation step above, substitute into the second equation all known information to determine the wanted rate.

Example 1 The radius r of a circle is increasing at a rate of 2 inches per minute. Find the rate of change of the area when $r = 6$ inches.

Solution: (1) Let A denote the area. Here we can perform the math modelling as shown in the table below. The involved variables are the area A and the radius r, both are functions of the time t.

Verbal expressions	Corresponding math model
r is increased at a rate of 2 inches per minute	$\frac{dr}{dt} = 2$ inches/minute
find the rate of change of the area	find $\frac{dA}{dt} =?$

(2) From geometry, we know that $A = \pi r^2$, this is the first equation. (Note that we want $\frac{dA}{dt}$, and A and r are involved in this equation). (3) Compute the derivative both

sides of the first equation, with respect to the time t, to get the second equation:

$$\frac{dA}{dt} = \pi \cdot 2r \cdot \frac{dr}{dt}.$$

Here, as $r = r(t)$ is a function of t, we applied Chain Rule to get $\frac{dr^2}{dt} = 2r\frac{dr}{dt}$.

(4) Substitute the given data $r = 2$, and $\frac{dr}{dt} = 2$ into the second equation, to get the answer

$$\frac{dA}{dt} = \pi \cdot 2(2)(2) = 8\pi \text{ square inches/minutre.}$$