Determine the relationship of two lines

Example: Given two lines

$$L_1: \qquad x - 2 = \frac{y+1}{2} = \frac{z-3}{3}$$

$$L_2: \qquad \frac{x-5}{3} = \frac{y-1}{2} = \frac{z-4}{1}$$

Determine if they are parallel, skew or intersecting.

Solution: The lines L_1 and L_2 are parallel to $\mathbf{v}_1 = (1, 2, 3)$ and $\mathbf{v}_2 = (3, 2, 1)$. Since $\mathbf{v}_1$ cannot be written as a scalar product of $\mathbf{v}_2$, the two lines are not parallel.

Now we try to solve the system of equations, obtained from the symmetric equations of the two lines:

$$\begin{cases} 2x - 4 & = & y + 1 \\ 3x - 6 & = & z - 3 \\ 2x - 10 & = & 3y - 3 \\ x - 5 & = & 3z - 12 \end{cases}$$

Solving the first three equations to get $x = 2, y = -1$ and $z = 3$ (subtract the third from the first side by side to get $6 = -2y + 2$, and so $y = -1$. Substitute $y = -1$ in the first to get $x = 2$, and so from the second, $z = 3$).

If $x = 2, y = -1$ and $z = 3$ also satisfy the fourth equation, then L_1 and L_2 intersect at $P(2, -1, 3)$; otherwise the two lines are skew. In this example, $x = 2, y = -1$ and $z = 3$ do satisfy the fourth equation, and so L_1 and L_2 intersect at $P(2, -1, 3)$.