Convert from one coordinate system to another

Useful Facts

Cylindrical	Rectangle	Spherical	Rectangle
$\begin{cases} r^2 &= x^2 + y^2 \\ \theta &= \tan^{-1} \frac{y}{x} \\ z &= z \end{cases}$	$\begin{cases} x &= r\cos\theta \\ y &= r\sin\theta \\ z &= z \end{cases}$	$\begin{cases} \rho^2 &= x^2 + y^2 + z^2 \\ \phi &= \tan^{-1} \frac{\sqrt{x^2+y^2}}{z} \\ \theta &= \tan^{-1} \frac{y}{x} \end{cases}$	$\begin{cases} x &= \rho\sin\phi\cos\theta \\ y &= \rho\sin\phi\sin\theta \\ z &= \rho\cos\phi \end{cases}$

Example (1) : Find the rectangular coordinates of the point $P(2, \frac{3\pi}{4}, 3)$, given in cylindrical coordinates.

Solution: Here $r = 2$, $\theta = \frac{3\pi}{4}$ and $z = 3$. Note that $\cos\frac{3\pi}{4} = -\frac{\sqrt{2}}{2}$, and $\sin\frac{3\pi}{4} = \frac{\sqrt{2}}{2}$.

$$\begin{cases} x = 2\cos\frac{3\pi}{4} = -\sqrt{2} \\ y = 2\cos\frac{3\pi}{4} = \sqrt{2} \\ z = 3. \end{cases} \tag{1}$$

Example (2) : Find the rectangular coordinates of the point $P(3, \frac{\pi}{2}, \pi)$, given in spherical coordinates.

Solution: Here $\rho = 3$, $\phi = \frac{\pi}{2}$ and $\theta = \pi$. Note that $\cos\frac{\pi}{2} = 0$, $\cos\pi = -1$, $\sin\frac{\pi}{2} = 1$ and $\sin\pi = 0$.

$$\begin{cases} x = 3\sin\frac{\pi}{2}\cos\pi = -3 \\ y = 3\sin\frac{\pi}{2}\sin\pi = 0 \\ z = 3\cos\frac{\pi}{2} = 0. \end{cases} \tag{2}$$

Example (3) : Find the cylindrical and spherical coordinates of the point $P(2, 1, -2)$, given in rectangular coordinates.

Solution: For cylindrical coordinates,

$$\begin{cases} r = \sqrt{2^2 + 1^2} = \sqrt{5} \\ \theta = \tan^{-1}\frac{1}{2} \\ z = -2. \end{cases} \tag{3}$$

For spherical coordinates, note that $\sqrt{x^2 + y^2} = \sqrt{5}$.

$$\begin{cases} \rho = \sqrt{2^2 + 1^2 + (-2)^2} = \sqrt{9} = 3 \\ \phi = \frac{\pi}{2} + \tan^{-1}\frac{\sqrt{5}}{2} \\ \theta = \tan^{-1}\frac{1}{2}. \end{cases} \tag{4}$$