MATH 251 - QUIZ 7

1. Compute the double integral

$$\int_0^{\pi/2}\int_1^e \frac{\sin y}{x}dxdy.$$

Solution

$$\int_0^{\pi/2}\int_0^e \frac{\sin y}{x}dxdy = \int_0^{\pi/2} \sin y dy \int_1^e \frac{1}{x}dx = \Big[-\cos y\Big]_0^{\frac{\pi}{2}}\Big[\ln x\Big]_1^e = (0-(-1))(\ln e - \ln 1) = 1.$$

2. Compute the double integral of $f(x,y) = 1-x$ over the triangle R whose vertices are $(0,0), (1,1)$ and $(-2,1)$.

Solution Note that the line linking $(0,0)$ and $(1,1)$ has equation $y = x$ and the line linking $(0,0)$ and $(-2,1)$ has equation $-2y = x$. We can view the region as vertically simple as well as horizontally simple. One way of doing this is:

$$\begin{aligned}
\text{Ans:} \quad &= \int_{-2}^0\int_{-\frac{x}{2}}^1 (1-x)dydx + \int_0^1\int_x^1 (1-x)dydx = \int_{-2}^0 [(1-x)y]_{-\frac{x}{2}}^1 dx + \int_0^1 [(1-x)y]_x^1 dx \\
&= \int_{-2}^0 \left(1-\frac{x}{2}-\frac{x^2}{2}\right)dx + \int_0^1 (1-2x+x^2)dx = \left[x-\frac{x^2}{4}-\frac{x^3}{6}\right]_{-2}^0 + \left[x-x^2+\frac{x^3}{3}\right]_0^1 \\
&= 0-\left(-2-1+\frac{4}{3}\right)+\left(1-1+\frac{1}{3}\right) = 2.
\end{aligned}$$

Another way of computing the integral is:

$$\begin{aligned}
\text{Ans:} \quad &= \int_0^1\int_{-2y}^y (1-x)dxdy = \int_0^1 \left[x-\frac{x^2}{2}\right]_{-2y}^y dy \\
&= \int_0^1 \left[y-\frac{y^2}{2}-\left((-2y)-\frac{4y^2}{2}\right)\right]dy = \int_0^1 \left[3y+\frac{3y^2}{2}\right]dy \\
&= \left[\frac{3y^2}{2}+\frac{y^3}{2}\right]_0^1 = 2.
\end{aligned}$$