MATH 251 - QUIZ 1 SOLUTIONS

1. Let $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}$ and $\mathbf{b} = \mathbf{i} + 3\mathbf{j} - 2\mathbf{k}$.
(1A) Compute $3\mathbf{a} + 4\mathbf{b}$.

$$3\mathbf{a} + 4\mathbf{b} = 3(1, -2, 3) + 4(1, 3, -2) = (3, -6, 9) + (4, 12, -8) = (7, 6, 1).$$

(1B) Compute $(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b})$.

$$(\mathbf{a} + \mathbf{b}) \cdot (\mathbf{a} - \mathbf{b}) = \mathbf{a}^2 - \mathbf{b}^2 = (1 + (-2)^2 + 3^2) - (1 + 3^2 + (-2)^2) = 0.$$

(1C) Find x such that $\mathbf{c} = \mathbf{i} + \mathbf{j} + x\mathbf{k}$ is perpendicular to the vector $\mathbf{a} + \mathbf{b}$.
Note that $\mathbf{c} \cdot (\mathbf{a} + \mathbf{b}) = (1, 1, x) \cdot (2, 1, 1) = 2 + 1 + x$. For $\mathbf{c}$ to be perpendicular to $\mathbf{a} + \mathbf{b}$, the dot product must be zero and so x must satisfy $2 + 1 + x = 0$, yielding $x = -3$.

(1D) Find the components $Comp_{\mathbf{a}}\mathbf{b}$ and $Comp_{\mathbf{b}}\mathbf{a}$.
Apply the fomula in your notes or in your textbook (page 722, Formula (14)) to compute them.

$$Comp_{\mathbf{a}}\mathbf{b} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{b}|} = \frac{1 - 6 - 6}{\sqrt{1 + 9 + 4}} = \frac{-11}{\sqrt{14}}.$$

and

$$Comp_{\mathbf{b}}\mathbf{a} = \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} = \frac{1 - 6 - 6}{\sqrt{1 + 4 + 9}} = \frac{-11}{\sqrt{14}}.$$

(1E) Find the direction cosines (also called directional numbers) of $\mathbf{a}$.
Use the result in (1D) that $|\mathbf{a}| = \sqrt{14}$. Then compute the direction cosines as follows:

$$\cos\alpha = \frac{1}{\sqrt{14}}, \cos\beta = \frac{-2}{\sqrt{14}}, \cos\gamma = \frac{3}{\sqrt{14}}.$$