MATH 251 - QUIZ 3

NAME:

I.D.:

Instruction: Circle your answers and show all your work CLEARLY. Solutions with answer only and without supporting procedures will have little credit.

1. Write an equation for the surface generated by revolving the curve $y^2 = 4x$ on the xy-plane around the x-axis.

Solution Consider a point $P(x, y, z)$ on the surface. Then there is a point $Q(x, y_1, 0)$ on the curve $y^2 = 4x$ rotating about the x-axis to get P. Both P and Q must have the saem distance to the x-axis, or more precisely, to the point $C(x, 0, 0)$ on the x-axis. As $|\overline{PC}| = |\overline{QC}|$, we have

$$(x-x)^2 + (y-0)^2 + (z-0)^2 = (x-x)^2 + (y_1 - 0)^2 + (0-0)^2,$$

and so $y_1^2 = y^2 + z^2$. As (x, y_1) is a point on the curve, we also have $y_1^2 = 4x$. Combine these equations to get the answer

$$y^2 + z^2 = 4x.$$

2. Convert the equation $x^2 + y^2 + z^2 = x + y + z$ to both cylindrical and spherical coordinates.

Solution For cylindrical coordinates, apply the conversion formulas to get

$$r^2 \cos^2\theta + r^2 \sin^2\theta + z^2 = r\cos\theta + r\sin\theta + z.$$

For spherical coordinates, apply the conversion formulas to get

$$\rho^2 = \rho\sin\phi\cos\theta + \rho\sin\phi\sin\theta + \rho\cos\phi \text{ or } \rho = \sin\phi\cos\theta + \sin\phi\sin\theta + \cos\phi.$$