Review for Chapter 13

1. Given a parametric planar curve $x = f(t)$, $y = g(t)$, where $a \le t \le b$, how to eliminate the parameter? (Use substitutions, or use trigonometry indentities, *etc*).

How to parametrize a curve $f(x, y) = 0$? (For polar form $r = f(\theta)$, one can set $x(t) = f(t)\cos t$ and $y(t) = f(t)\sin t$; for the special form $y = f(x)$, one can set $x(t) = t$ and $y(t) = f(t)$.)

2. Differentiation of parametric curves and the slope.

$$\frac{dy}{dx} = \frac{dy/dt}{dx/dt}, \quad \frac{d^2y}{dx^2} = \frac{dy'/dt}{dx/dt}.$$

For polar coordinates,

$$\frac{dy}{dx} = \frac{r'\sin\theta + r\cos\theta}{r'\cos\theta - r\sin\theta}, \quad \cot\phi = \frac{1}{r}\frac{dr}{d\theta},$$

where ϕ denotes the angle between the tangent line at P and the radius OP from the origin.

3. Review of integration in Chapter 6 and how to apply the formulas to paramatric curves.

Area under curve:	$A = \int y dx$
Vol. of rev. around x-axis:	$V_x = \int \pi y^2 dx$
Vol. of rev. around y-axis:	$V_y = \int \pi x^2 dy$
Arc length:	$s = \int ds = \int \sqrt{(dx)^2 + (dy)^2}$
Area of surf. of rev. around x-axis:	$A_x = \int 2\pi \lvert y \rvert ds$
Area of surf. of rev. around y-axis:	$A_y = \int 2\pi \lvert x \rvert ds$

4. Operations of vectors.

The addition and the multiplication of vectors behave pretty much the same way as the operations of numbers with some exceptions (see (7) on page 633, (9) on page 634 and Theorem 3 on page 643). The following are some reminders:
(i) The dot product of two vectors outputs a number.
(ii) The scalar product and the vector product output vectors.
(iii) The vector product of two vectors is not commutative.

5. Some important facts:
(i) If $\vec{a} \neq \vec{0}$, then $\vec{u} = \vec{a}/|\vec{a}|$ is the unit vector that has the same direction of $\vec{a}$;
(ii) Perpendicular test: $\vec{a}\vec{b} = 0$ iff $\vec{a}$ and $\vec{b}$ are perpendicular;
(iii) Parallel test: $\vec{a} \times \vec{b} = \vec{0}$ iff $\vec{a}$ and $\vec{b}$ are parallel;
(iv) $\vec{a} \times \vec{b}$ is perpendicular to both $\vec{a}$ and $\vec{b}$;
(v) $\vec{a}\vec{b} = |\vec{a}||\vec{b}|\cos\theta$;
(vi) $|\vec{a} \times \vec{b}| = |\vec{a}||\vec{b}|\sin\theta$;
where θ in (v) and (vi) is the angle between $\vec{a}$ and $\vec{b}$.