Math 156 Fall 2015 Quiz 2

Name:

Evaluate each of the following

1: $\int \tan^3(x)\sec(x)dx$. (Exercise 25 in Section 6.2).

Solution The differentiation formula $(\sec(x))' = \sec(x)\tan(x)$ suggests that $u = \sec(x)$. Thus $du = \sec(x)\tan(x)dx$ and $\tan^2(x) = \sec^2(x) - 1 = u^2 - 1$.

$$\int \tan^3(x)\sec(x)dx = \int (u^2-1)du = \frac{u^3}{3} - u + C = \frac{\sec^3(x)}{3} - \sec(x) + C.$$

2: $\int \frac{x^3}{\sqrt{x^2+9}}dx$. (Exercise 39 in Section 6.2).

Solution Among the quantities $\sqrt{x^2+9}, x$ and 3, $\sqrt{x^2+9}$ is largest and should be the hypothenuse of the right triangle. Therefore, set $x = 3\tan(\theta)$. Then $dx = 3\sec^2(\theta)d(\theta)$ and $\sqrt{x^2+9} = 9\tan^2(\theta)+9 = \sqrt{(3\sec(\theta))^2} = 3\sec(\theta)$. Hence (you can use the result in Problem 1, or use the substitution $u = \sec(\theta)$), the answer is

$$\begin{aligned}
\int \frac{x^3}{\sqrt{x^2+9}}dx &= \int \frac{(3\tan(\theta))^3 \cdot 3\sec^2(\theta)d(\theta)}{3\sec(\theta)} \\
&= 3^3 \int \tan^3(\theta)\sec^2(\theta)d(\theta) = 3^3\left(\frac{\sec^3(\theta)}{3} - \sec(\theta)\right) + C \\
&= 3^2\left(\frac{\sqrt{x^2+9}}{3}\right)^3 - 3^3\left(\frac{\sqrt{x^2+9}}{3}\right) + C = \frac{(\sqrt{x^2+9})^3}{3} - 9\sqrt{x^2+9} + C.
\end{aligned}$$

3: Evaluate the integral $\int \frac{10}{(x-1)(x^2+9)}dx$.

Solution Use the partial fraction we obtained above.

$$\begin{aligned}
\int \frac{10}{(x-1)(x^2+9)}dx &= \int \left(\frac{1}{x-1} - \int \frac{x+1}{x^2+9}\right) dx \\
&= \int \frac{dx}{x-1} \text{ (Use } u = x-1) - \int \frac{x}{x^2+9}dx \text{ (Use } w = x^2+9) \\
&\quad - \int \frac{1}{x^2+9}dx \text{ (Use Formula 17 in the integration table)} \\
&= \ln|x-1| - \frac{1}{2}\ln(x^2+9) - \frac{1}{3}\tan^{-1}(\frac{x}{3}) + C.
\end{aligned}$$