

Review for Chapter 14

1. Operation of vectors.

The addition and the multiplication of vectors behave pretty much the same way as the operations of numbers with some exceptions (see (7) of page 633, (9) of page 634 and Theorem 3 of page 643). The following are some reminders:

- (i) The dot product of two vectors outputs a number.
- (ii) The scalar product and the vector product output vectors.
- (iii) The vector product of two vectors is not commutative.

2. The distance formula and its relation to dot products.

If \vec{r} has its head in (x_1, y_1, z_1) and tail at (x_2, y_2, z_2) , then $\vec{r} = \langle x_1 - x_2, y_1 - y_2, z_1 - z_2 \rangle$, and

$$|\vec{r}|^2 = \vec{r} \bullet \vec{r} = (x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2.$$

3. Important facts: (the angle θ below is the angle between the two vectors \vec{a} and \vec{b})

- (i) $\vec{a} \bullet \vec{b} = |\vec{a}||\vec{b}| \cos \theta$, and so \vec{a} and \vec{b} are perpendicular if and only if $\vec{a} \bullet \vec{b} = 0$.
- (ii) $|\vec{a} \times \vec{b}|^2 = |\vec{a}|^2 |\vec{b}|^2 \sin^2 \theta$, and so \vec{a} and \vec{b} are parallel to each other if and only if $\vec{a} \times \vec{b} = \vec{0}$, the zero vector.
- (iii) $\vec{a} \times \vec{b}$ is perpendicular to both \vec{a} and \vec{b} .

4. (Section 14.1) Direction angles and numbers.

Let $\vec{r} = \langle x, y, z \rangle$. Let the angles between \vec{r} and the x , y and z axes be α , β and γ , respectively. Then these angles are the direction angles of \vec{r} and the direction numbers are

$$\cos \alpha = \frac{\vec{r} \bullet \vec{i}}{|\vec{r}|}, \quad \cos \beta = \frac{\vec{r} \bullet \vec{j}}{|\vec{r}|}, \quad \cos \gamma = \frac{\vec{r} \bullet \vec{k}}{|\vec{r}|}.$$

5. (Vector components, Section 14.1) The component of \vec{a} along \vec{b} is

$$\text{Comp}_{\vec{b}} \vec{a} = \frac{\vec{a} \bullet \vec{b}}{|\vec{b}|}.$$

6. (Equations of a line, Section 14.3) Let L be a line in space that is parallel to $\vec{n} = \langle a, b, c \rangle$ and passes through (x_0, y_0, z_0) . Then the parametric equation of L is

$$r(t) = \langle at + x_0, bt + y_0, ct + z_0 \rangle,$$

and the symmetric equations of L is

$$\frac{x - x_0}{a} = \frac{y - y_0}{b} = \frac{z - z_0}{c}.$$

7. (Facts about planes, Section 14.3) The plane passes (x_0, y_0, z_0) with normal vector $\vec{n} = \langle a, b, c \rangle$ has equation

$$\langle a, b, c \rangle \bullet \langle x - x_0, y - y_0, z - z_0 \rangle = 0.$$

The angle between two planes is the angle between the two normal vectors.

8. (Basic facts on vector functions, Section 14.4) Vector functions and their limits, derivatives, and antiderivatives.

One key thing to remember: do each of these componentwise. Note that the product rules for derivatives are very similar to the product rule for ordinary functions, with exceptions in vector products.

9. (Basic techniques in motions, Section 14.5) Given $r(\vec{t})$, find the velocity, the speed, the acceleration, the unit tangent vector, the principal unit normal vector, the tangent component and the normal component of the acceleration, and the curvature. (See your notes for routines).

Conversely, given the velocity and the acceleration together with some initial conditions, find the position vector $r(\vec{t})$. (See your notes for routines).

10. (Basic techniques of studying graphs of equations, Section 14.6) Given the equation of a surface and the cutting planes, one can describe the the graph by using *traces, layers, and cylinders*, according to the nature of the given surface. One can also use the traces, layers and/or cylinders to sketch the graph.

11. (A technique of obtaining equations from descriptions, Section 14.6) Given the equation of a plane curve C and an axis L , find an equation of the surface generated by revolving C about L by using distance formula.

12. (Section 14.7) The Cylindrical and the spherical coordinates.
The fomulae about cylindrical coordinates:

$$\begin{cases} x = r \cos \theta \\ y = r \sin \theta \\ z = z \end{cases} \quad \text{and } x^2 + y^2 = r^2, \tan \theta = \frac{y}{x}.$$

The fomulae about spherical coordinates:

$$\begin{cases} x = \rho \cos \theta \sin \phi \\ y = \rho \sin \theta \sin \phi \\ z = \rho \cos \phi \end{cases} \quad \text{and } x^2 + y^2 + z^2 = \rho^2.$$