



ALGEBRA SEMINAR - A MINI COURSE

Talk 1: Friday, January 24, 2020, 4:00 pm - 5:30 pm
Talk 2: Monday, January 27, 2020, 4:30 pm - 6:00 pm

Room: 315 Armstrong Hall

Introduction to Triangulated Categories in Commutative Algebra

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The theory of triangulated categories, introduced by Verdier and Dold-Puppe in 1960's, improves classical Homological Algebra of abelian categories. Although they initially stem from Algebraic Geometry and Algebraic Topology, triangulated categories are now indispensable tools in many different fields of mathematics. For example, in Commutative Algebra, triangulated categories appear as derived categories of modules, or stable categories of maximal Cohen-Macaulay modules over Gorenstein rings. A recent important development in the theory of triangulated categories is Balmer's tensor triangular geometry: it concerns triangulated categories with a tensor structure, and provides a way to study such triangulated categories from algebro-geometric and commutative-algebraic methods.

In this series of lectures, I will give an introduction to triangulated categories, and discuss Balmer's tensor triangular geometry with applications to Commutative Algebra. Denote by $D^-(\text{mod } R)$ the derived category of co-chain complexes X of finitely generated R -modules with $H^i(X) = 0$ for all $i \gg 0$. It is known that $D^-(\text{mod } R)$ has a structure of a tensor triangulated category with respect to derived tensor product $-\otimes_R^L$ and unit R . We will discuss thick tensor ideals of $D^-(\text{mod } R)$, i.e., thick subcategories which are closed under the tensor action by each object in $D^-(\text{mod } R)$. Moreover, we will investigate the Balmer spectrum of $D^-(\text{mod } R)$, that is, the set of prime thick tensor ideals of $D^-(\text{mod } R)$.