Hamiltonian properties , branch number and k-tree related graphs

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Abstract

Investigating Hamiltonian properties of graphs is a very hot topic in graph theory and k-trees (a generalization of trees for an integer $k \geq 2$) are important graphs with some special structures. A long standing conjecture of Chvátal(1973) states that there exists a finite constant t_0 such that every graph G with $\tau(G) \geq t_0$ is Hamiltonian, where $\tau(G) = min\{\frac{|S|}{\omega(G-S)} \mid \text{for any cut-set } S \subset V(G)\}$ denotes the toughness of a graph G and $\omega(G-S)$ is the number of components of $G-S$. Chen et al.(1998) showed that every 18-tough chordal graph, which is a graph with no induced cycle of length greater than 3, is Hamiltonian. Notice that to determine the toughness of a graph is very difficult and k-trees are special chordal graphs, we introduce a new parameter, the branch number of G, which is easier to calculate and work with than the toughness for k-trees. Some results on the relationships between the branch number and other graph parameters will be presented. Applying the branch number, we will explore some Hamiltonian properties for k-trees and its related graphs. Further research problems will also be proposed.