MATH 251 AND MATH 261
SUMMARY OF MATERIAL COVERED AND RELATIONSHIP

Math 251

1 Linear Algebra
  A. Matrix Algebra
  B. Vector Space Theory. (**Vector Space**, **Subspace**, **Linear Operator**, **Linear Independence**, **Span**, **Basis**, **Dimension**)
  C How to solve Ax = b.
2. Multivariate Calculus (Lines, Planes, Curves, Surfaces, Sketching, Partial Derivatives, Gradient, Multiple integrals)
3. Vector Calculus (Green's Theorem, Stokes' Theorem, Gauss's Theorem)

Math 261

1. First Order ODE's
2. Linear Mapping Problem(s)
   Consider the linear equation

$$T(\vec{x}) = \vec{b}. \qquad (*)$$

where $T:V\to W$ is a **linear operator** and V and W are **vector spaces** so that the nullspace of L, $N_{L,}$ is a **subspace** of V. If **Dim**(N_L) = k and B = $\{\vec{x}_1, \vec{x}_2, \vec{x}_3, \ldots, \vec{x}_k\} \subseteq N_L$, is a **linearly independent** set, then B is a **basis** for N_L (and hence **spans** N_L) and the (set of) solution(s) of (*) is

$$\vec{x} = \vec{x}_p + c_1\vec{x}_1 + c_2\vec{x}_2 + \bullet\bullet\bullet + c_k\vec{x}_k$$

where $\vec{x}_p$ is any particular solution of (*).

Problem	Linear Operator	Dim(N_L)
1. Ax = b	T(x) = Ax	Depends on A
2. $y' + p(x)y = g(x)$	$L[y] = y' + p(x)y$	1
3. $y'' + p(x)y' + q(x)y = g(x)$	$L[y] = y'' + p(x)y' + q(x)y$	2
4. $y^{(n)} + p_{n-1}(x)y^{(n-1)} + \ldots + p_1(x)y' + p_0(x)y = g(x)$	$L[y] = y^{(n)} + p_{n-1}(x)y^{(n-1)} + .. + p_1(x)y' + p_0(x)y$	n
5. $\mathcal{L}(f) = F(s)$	$\mathcal{L}(f) = \int_0^\infty f(t)e^{-st}\,dt$	0
6. $x' = Ax - b + g(t)$ (A is nxn)	$L[x] = x' - Ax$	n
7. $u_t = \alpha^2 u_{xx} - b(x) + g(x,t)$	$L[u] = u_t - \alpha^2 u_{xx}$	∞
8. $u_t = \alpha^2 u_{xx} - b(x) + g(x,t)$, $u(t,0) = T_1$, $u(t,\ell) = T_2$	$L_B[u] = u_t - \alpha^2 u_{xx}$	∞

Hence we need the concepts of **Vector Space**, **Subspace**, **Linear Operator**, **Linearly Independent**, **Spans**, **Basis,** and **Dimension**.