West Virginia University
DEPARTMENT OF MATHEMATICS

ALGEBRA SEMINAR WEEK

Tuesday, Nov 14 – 4:00pm - 5:00pm – **313 Armstrong Hall**
Wednesday, Nov 15 – 4:00pm - 5:00pm – **315 Armstrong Hall**
Thursday, Nov 16 – 4:00pm - 5:00pm – **313 Armstrong Hall**

Tate resolutions and deviations of graded algebras

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When considering complexes of modules, elements appearing in different homological degrees do not talk to each other: there is no prescribed way to multiply them together. Consider the Koszul complex on n elements of a Noetherian ring S:

$$0 \to \bigwedge^{n} S^n \xrightarrow{d} \bigwedge^{n-1} S^n \xrightarrow{d} \cdots \to \bigwedge^{1} S^n \xrightarrow{d} \bigwedge^{0} S^n$$

We multiply elements in Koszul complex looking at exterior algebras. Moreover, for all $x \in \wedge^i S^n$ and $y \in \wedge^j S^n$,

$$d(xy) = (dx)y + (-1)^i x(dy).$$

How do we generalize multiplication in Koszul complexes? For this, we study differential graded algebras (DG-algebras) which induce a multiplicative structure on resolutions.

In the first talk on Tuesday, we will start with a brief introduction of DG-algebras and see some examples of resolutions with DG-algebra structure.

For $I \subset S$ an ideal of S and $R = S/I$, it is always possible to construct a DG-algebra resolution of R over S due to a result of Tate. If R is the residue field of S, then Gulliksen proved that such a DG-algebra resolution is minimal. In the second talk on Wednesday, we will discuss a construction of the Tate resolution and its minimality.

If R is the residue field, the i-th deviation of R, denoted as $\varepsilon_i(R)$, counts how many variables of degree i we have to introduce while constructing a Tate resolution of R. Finally, in the last talk on Thursday, we will discuss classification of rings using vanishings of i-th deviations.