

Factorizations of Power Series as Generalized Power Product Expansions

August 31, 2017

Abstract: I will discuss the factorization of generating functions that are given in the form of Power Series $f(x) = 1 + \sum_{n=1}^{\infty} a_n x^n$. We define the *Generalized Power Product Expansion* or GPPE of f as

$$f(x) = \prod_{n=1}^{\infty} (1 + g_n x^n)^{r_n} = (1 + g_1 x^1)^{r_1} (1 + g_2 x^2)^{r_2} (1 + g_3 x^3)^{r_3} \dots, \quad (1)$$

and the *Generalized Inverse Power Product Expansion* or GIPPE of f as

$$f(x) = \prod_{n=1}^{\infty} (1 - h_n x^n)^{r_n} = (1 - h_1 x^1)^{r_1} (1 - h_2 x^2)^{r_2} (1 - h_3 x^3)^{r_3} \dots, \quad (2)$$

where $\{g_n\}_{n=1}^{\infty}$, $\{h_n\}_{n=1}^{\infty}$, and $\{r_n\}_{n=1}^{\infty}$ are arbitrary nonzero complex numbers. Algebraic, Combinatorial and Analytic properties of these expansions will be reviewed and I will outline applications to Cryptography. The talk will be accessible to graduate students.