Integer Flows and Circuit Covers of Signed Graphs

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Abstract

In graph theory, many classic concepts and results in graphs have been frequently discussed and studied in signed graphs during the past few decades. For instance, graph chromatic number, graph chromatic index, graph embeddings, graphical matroids, etc have been generalized to signed graphs. In this dissertation, we investigate two more topics in signed graphs: integer flows and the shortest circuit covers.

The work on integer flows is motivated by Tutte and Jaeger's pioneering work on converting modulo flows into integer-valued flows for ordinary graphs. Given a signed graphs (G, σ), we first prove that for each $k \in \{2, 3\}$, if (G, σ) is $(k-1)$-edge-connected and contains an even number of negative edges when $k = 2$, then every modulo k-flow f_1 of (G, σ) can be converted into an integer-valued $(k+1)$-flow f_2 of (G, σ) with $\text{supp}(f_1) \subseteq \{e \in E(G) \colon 1 \leq |f_2(e)| \leq k-1\}$. This generalizes two previous results by Xu and Zhang (DM2005). We also prove that if (G, σ) is odd-$(2p+1)$-edge-connected, then (G, σ) admits a modulo circular $(2 + \frac{1}{p})$-flows if and only if it admits an integer-valued circular $(2 + \frac{1}{p})$-flows, which improves all previous results respectively proved by Xu and Zhang (DM2005), Schubert and Steffen (EJC2015), and Zhu (JCTB 2015).

The Shortest Circuit Cover Conjecture is one of the major open problems in graph theory. It states that every bridgeless graph G contains a set of circuits $\mathcal{F}$ such that each edge is covered in at least one member of $\mathcal{F}$ and the length of $\mathcal{F}$ is at most $\frac{7}{5}|E(G)|$. This concept was recently generalized to signed graphs by Máčajová et al. (JGT2015). In this dissertation, we improve their upper bound from $11|E(G)|$ to $\frac{14}{3}|E(G)|$ ($\approx 4.667|E(G)|$), and if G is 2-edge-connected and has even negativeness, then it can be further reduced to $\frac{11}{3}|E(G)|$ ($\approx 3.667|E(G)|$).