1. (8 points) Draw the algebraic tangle (3,32) \cdot (233).

2. (10 points) Calculate the linking number of the two links below.

![Diagram of two links](image)

3. (6 points) Show that no knot is two-colorable. Also, show that any link with more than one component is two-colorable.

Points earned: [ ]/24
4. (a) (10 points) Find a picture of $8_{10}$ that shows that it is at most a three-bridge knot. (In fact, this is a three-bridge knot, but you don’t need to prove the lower bound. In other words, *don’t* show that it is *not* a two-bridge knot.)

(b) (10 points) Show that the bridge number $b(K)$ of a nontrivial knot $K$ is always less than or equal to the least number of crossings in any projection of the knot. (*Hint:* It may help to think about the cases where the projection is alternating or nonalternating separately.)
5. (10 points) Show that a link projection is alternating if and only if all of the edges in the corresponding signed planar graph have the same sign.

6. (8 points) Show that the mutation of a knot is always another knot, rather than a link.
7. (a) (8 points) Determine for which primes $p$ the following knot is $p$-colorable and provide such a $p$-coloring.

(b) (10 points) Determine for which primes $p$ the following knot is $p$-colorable and provide such a $p$-coloring.
8. (10 points) Show that every link is two-equivalent to the trivial link with the same number of components.

9. (a) (8 points) Use induction to show that the Euler characteristic of a surface of genus $g$ is $2 - 2g$.

(b) (8 points) Show that a compression of a surface always increases Euler characteristic.