Math 155 – Review Worksheet for Exam 4

**Short Answer Problems:** There will be 5 Short Answer Problems (4 point each) in Exam 4 for which no partial credit will be given. Students are expected to know how to work on these problems with high level of computational accuracy and good comprehension of the related concepts. There is no need to simplify the answers in this group of problems.

1. Compute the sum \( \sum_{j=1}^{3} \frac{1}{j+1} \).

2. Write the sum 1 + 4 + 9 + 16 in summation notation.

3. Compute the Riemann sum for \( f(x) = x^2 \) on \([0, 1]\) and a regular partition with \( n = 3 \), using \( x^*_i = x_i \), the right-hand end point of the \( i \)th interval \([x_{i-1}, x_i]\).

4. Evaluate \( \int_{2}^{3} (x^3 - x^2) \, dx \).

5. Compute the limit \( \lim_{n \to \infty} \frac{1^2 + 2^2 + \cdots + n^2}{n^3} \).

6. Compare the integral \( \int_{0}^{1} (1 + x) \, dx \) and the Riemann sum \( R_3 \) for \( f(x) = 1 + x \) on \([0, 1]\) and a regular partition with \( n = 3 \), using \( x^*_i = x_i \), the right-hand end point of the \( i \)th interval \([x_{i-1}, x_i]\). Which quantity is bigger?

**Traditional Problems:** Other problems in Exam 4 will be graded in the traditional way in which partial credit will be given. Students are expected to use their knowledge and to demonstrate a sufficiently high level of their skills on the related subjects to handle these problems.

6. Evaluate the following integrals.

   (6A) \( \int_{-1}^{2} (3x^2 + 2x + 4) \, dx \).

   (6B) \( \int_{-1}^{2} (x^2 + 1)^2 \, dx \).

   (6C) \( \int_{1}^{2} \frac{x^3 + 1}{x^3} \, dx \).

   (6D) \( \int_{0}^{2} \frac{1}{(x + 2)^3} \, dx \).

   (6E) \( \int_{1}^{2} (1 + \sqrt[3]{x})^2 \, dx \).

   (6F) \( \int_{0}^{\frac{\pi}{4}} \cos(2x) \sin(2x) \, dx \).
(6G) \[ \int_0^{\pi} \sec^2(2t) \, dt. \]

(6H) \[ \int_{-2}^{3} |1-x| \, dx. \]

(6I) \[ \int_0^4 x \sqrt{x^2 + 9} \, dx. \]

(6J) \[ \int_0^{2\pi} t \sin(t^2) \, dt. \]

(6K) \[ \int_{-2}^{2} |x^2 - 1| \, dx. \]

7. Find the average of the function \( f(x) = \sin(2x) \) on the interval \([0, \pi]\).

8. Evaluate these antiderivatives:

(8A) \[ \int (x + 1)^{10} \, dx. \]

(8B) \[ \int x^2 \cos(2x^3) \, dx. \]

(8C) \[ \int x \sqrt{x^2 + 9} \, dx. \]

9. Find a function \( y = y(x) \) satisfying \[ \frac{dy}{dx} = \frac{1}{\sqrt{x + 2}} \] and \( y(2) = -1 \).

10. Find the area of the region bounded above by the graph of \( y = x^2 \) and below by the graph of \( y = x^4 \) over the interval \([0, 1]\).

11. Find the area of the region \( R \) between the graphs \( x = 8 - y^2 \) and \( x = y^2 - 8 \).

12. Find the area of the region \( R \) between bounded by the curves \( x = y^2 \) and \( x = y + 6 \).

13. Find the area of the region \( R \) between bounded by the curves \( y = x^2 \) and \( y = 2x + 3 \).

14. Compute \( F'(x) \), given

(14A) \[ F(x) = \int_0^x \sin(t^2) \, dt. \]

(14B) \[ F(x) = \int_0^{x^2 + 1} \sin(t^2) \, dt. \]