Math 155 – Review Worksheet for Exam 1

1: Evaluate the following limits by limit laws or show that the limit does not exist, as appropriate.

(1A) \( \lim_{x \to -1} (3 - x + x^3) \).

(1B) \( \lim_{x \to -2} \frac{x + 2}{x^2 + x - 2} \).

(1C) \( \lim_{x \to -3} \left( \frac{x^2 + 6x + 9}{9 - x^2} \right)^{\frac{4}{3}} \).

(1D) \( \lim_{x \to 2^+} \frac{2 - x}{\sqrt{4 - 4x - x^2}} \).

(1E) \( \lim_{x \to 4^+} \frac{|4 - x|}{4 - x} \).

(1F) \( \lim_{x \to 2^+} \frac{2}{2 - x} \).

(1G) \( \lim_{x \to 0} \frac{\tan(5x)}{x} \).

(1H) \( \lim_{x \to 0} \frac{1 - \cos(3x)}{2x^2} \).

2 For each of the function \( f(x) \) given below, apply the definition of the derivative to compute \( f'(x) \) (that is, use the 4-step process).

(2A) \( f(x) = x - \frac{1}{x} \).

(2B) \( f(x) = \frac{1}{\sqrt{x + 1}} \).

3 Given a function \( f(x) = \frac{2x - 1}{x^2 - 1} \), do each of the following.

(3A) Find the domain of \( f(x) \).

(3B) Apply limit laws and the continuity of polynomial to show that the function is continuous in its domain.

(3C) Find discontinuities of \( f(x) \). Does \( f(x) \) have a removable discontinuity? If yes, where?
4 Given a function

\[ f(x) = \begin{cases} 
1 + x - x^2 & \text{if } x > 1, \\
3x^2 - 4x + 2 & \text{if } x < 1
\end{cases} \]

find a point \( a \) at which \( f(x) \) is not continuous. Is \( a \) a removable discontinuity of \( f(x) \)?

5 Find a value \( c \) such that the function

\[ f(x) = \begin{cases} 
  c - x^2 & \text{if } x \geq 1, \\
  x^2 - c & \text{if } x < 1
\end{cases} \]

such that \( f(x) \) is a continuous for all \( x \).

6 Use the intermediate value property of continuous function to show that the equation \( \sqrt{x^3 + x^5} + 9 = 5 \) has a solution in \([0, 2] \).

7 Given the position function \( x = 100t^2 + 40 \) of a particle moving in a horizontal straight line, find its location \( x \) when the velocity is zero.

8 The height (in feet at time \( t \) seconds) of a ball thrown vertically upward is \( y(t) = -16t^2 + 256t + 50 \). Find the maximum height that the ball attains.

9 Compute the derivative of the functions below.

(9A) \( f(x) = (4x - 7)^2 \).

(9B) \( f(x) = x^3 + \frac{1}{x^4 + 1} \).

(9C) (Simplify your answer) \( f(x) = \frac{x^6 + 4x^3 + 2x^2 - 1}{x^3} \).

(9D) \( f(x) = \frac{1}{x(x^2 + x)} \).

(9E) \( f(x) = \frac{4 - 7x + x^2}{x^3 - 2} \).

10 Find an equation of the line tangent to the curve \( y = (3x^2 + x + 1)(3 - 2x^3) \) at \((1, 5)\).