1. Compute the integral \( \int \frac{\sec(\ln x) \tan(\ln x)}{x} \, dx \).

2. Compute the integral \( \int_{0}^{4} 2x\sqrt{x^2 + 9} \, dx \).

3. Compute the integral \( \int_{0}^{1} x^3 e^{x^4} \, dx \).

4. Compute the integral \( \int_{0}^{2} \frac{3x}{(x^2 + 1)^2} \, dx \).

5. Find the area between the curves \( y = x^2 - 3 \), \( y = x - 1 \) on the interval \( 0 \leq x \leq 3 \).
6. Find the area between the curves $y = x^2, y = e^{-x}$ on the interval $1 \leq x \leq 4$.

7. Sketch and find the area of the region determined by the intersection of the curves $y = x^2 - 2$ and $y = x^2$.

8. Sketch and find the area of the region determined by the intersection of the curves $y = 2\sqrt{2x}$ and $y = x^2$. 
Math 155 – Spring 2004 WORKSHEET 13, Solutions

1. Compute the integral \( \int \frac{\sec(\ln x) \tan(\ln x)}{x} \, dx \).

**Solution:** Set \( u = \ln(x) \), then \( du = \frac{1}{x} \, dx \) and so
\[
\int \frac{\sec(\ln x) \tan(\ln x)}{x} \, dx = \int \sec(u) \tan(u) \, du = \sec(u) + C = \sec(\ln(x)) + C.
\]

2. Compute the integral \( \int_0^4 2x \sqrt{x^2 + 9} \, dx \).

**Solution:** Set \( u = x^2 + 9 \). Then \( du = 2x \, dx \), and when \( x = 0 \), \( u = 9 \), and when \( x = 4 \), \( u = 16 + 9 = 25 \). Thus
\[
\int_0^4 2x \sqrt{x^2 + 9} \, dx = \int_9^{25} u^{\frac{1}{2}} \, du = \left[ \frac{2}{3} u^{\frac{3}{2}} \right]_9^{25} = \frac{250}{3}.
\]

3. Compute the integral \( \int_0^1 x^3 e^{x^4} \, dx \).

**Solution:** Set \( u = x^4 \). Then \( du = 4x^3 \, dx \), and when \( x = 0 \), \( u = 0 \) and when \( x = 1 \), \( u = 1 \). Thus
\[
\int_0^1 x^3 e^{x^4} \, dx = \frac{1}{4} \int_0^1 e^u \, du = e - 1.
\]

4. Compute the integral \( \int_0^2 \frac{3x}{(x^2 + 1)^2} \, dx \).

**Solution:** Set \( u = x^2 + 1 \). Then \( du = 2x \, dx \), and when \( x = 0 \), \( u = 1 \) and when \( x = 2 \), \( u = 5 \). Thus
\[
\int_0^2 \frac{3x}{(x^2 + 1)^2} \, dx = \frac{3}{2} \int_1^5 u^{-2} \, du = \frac{3}{2} \left[ \frac{-1}{u} \right]_1^5 = \frac{3}{2} \left[ 1 - \frac{1}{5} \right] = \frac{6}{5}.
\]

5. Find the area between the curves \( y = x^2 - 3 \), \( y = x - 1 \) on the interval \( 0 \leq x \leq 3 \).

**Solution:** Sketch the graph of the region. The region has two parts, one is above \( y = x^2 - 3 \) and below \( y = x - 1 \), between \( 0 \leq x \leq 2 \), and the other is above \( y = x - 1 \) and below \( y = x^2 - 3 \), between \( 2 \leq x \leq 3 \). Thus the area is
\[
\text{Area} = \int_0^2 [(x - 1) - (x^2 - 3)] \, dx + \int_2^3 [(x^2 - 3) - (x - 1)] \, dx
\]
\[
= \int_0^2 (x + 2 - x^2) \, dx + \int_2^3 (x^2 - x - 3) \, dx
\]
\[
= \left[ \frac{x^2}{2} + 2x - \frac{x^3}{3} \right]_0^2 + \left[ \frac{x^3}{3} - \frac{x^2}{2} - 2x \right]_2^3 = \frac{10}{3} - 3 - \frac{10}{3} = 31.
\]
6. Find the area between the curves $y = x^2, y = e^{-x}$ on the interval $1 \leq x \leq 4$.

**Solution:** Note that $y = x^2$ is above the curve $y = e^{-x}$ on $[1, 4]$. Thus the area is

$$\int_0^4 (x^2 - e^{-x})dx = \left[ \frac{x^3}{3} + e^{-x} \right]_0^4 = \frac{64}{3} + e^{-4} - 1 = \frac{61}{3} + e^{-4}.$$

7. Sketch and find the area of the region determined by the intersection of the curves $y = 2 - x^2$ and $y = x^2$.

**Solution:** (Omit the graph). Solve the system $y = 2 - x^2$ and $y = x^2$ for $x$ to get $x^2 = 1$. Thus the integration bounds are $a = -1$ and $b = 1$. Therefore, the area is

$$\int_{-1}^1 |(2 - x^2) - x^2|dx = 2 \left[ x - \frac{x^3}{3} \right]_{-1}^1 = \frac{8}{3}.$$

8. Sketch and find the area of the region determined by the intersection of the curves $y = 2\sqrt{2x}$ and $y = x^2$.

**Solution:** (Omit the graph). Solve the system $y = 2\sqrt{2x}$ and $y = x^2$ for $x$ to get $x^4 = 8x$. Thus the integration bounds are $a = 0$ and $b = 2$. Therefore, the area is

$$\int_0^2 [2\sqrt{2x} - x^2]dx = \left[ 2\sqrt{2} \cdot \frac{x^3}{3} + \frac{x^3}{3} \right]_0^2 = \frac{4\sqrt{2} (\sqrt{2})^3}{3} - 0 - \frac{8}{3} + 0 = \frac{8}{3}.$$