Listen very carefully to the examiner as testing procedures are explained. Before working on any problems, write your student ID# on the exam as well as at the top right portion (marked by the words: WRITE ID NUMBER HERE) on the scantron card, and fill in the appropriate bubbles in that area. Also write your name in the indicated space of the scantron card, and the exam version (A, B) after your name.

PART 1. This portion of the exam is to test basic knowledge and calculation skills to a correct solution. The questions are multiple choice with “None of these” as a possible valid choice. The choice “DNE” stands for the answer “does not exist”. No partial credit is given in this section, so work very carefully. Value: 6 points each

1. The \( \lim_{x \to 1} \frac{x^2 + 3x - 4}{x^2 - 5x + 4} \) is equal to
   (A) 0  (B) -1  (C) \( \frac{5}{3} \)  (D) \( \frac{-5}{3} \)  (E) \( \frac{-3}{5} \)  (AB) none of these.

2. If \( f(x) = \begin{cases} 3x^2 - 2 & \text{if } x < 2 \\ 3 - 2x^2 & \text{if } x \geq 2 \end{cases} \), then the \( \lim_{x \to 2^-} f(x) \) is equal to
   (A) -5  (B) 10  (C) 12  (D) 4  (E) -8  (AB) none of these.

3. The \( \lim_{x \to \infty} \frac{4 - x^2}{8 + 5x + x^2} \) is equal to
   (A) 0  (B) \( \frac{1}{2} \)  (C) \( \frac{1}{5} \)  (D) -1  (E) 1  (AB) none of these.

4. An equation of the line tangent to the curve \( y = 5x^4 - 2x \) at the point (1, 3) is
   (A) \( y - 3 = 18x - 1 \)  (B) \( y = 18x - 15 \)  (C) \( y = 18x + 3 \)
   (D) \( y - 3 = x - 1 \)  (E) \( y + x = 3 \)  (AB) none of these.
5. Let \( f(x) = x^2 + 2x \) be a function on \([0, 2]\). Partition the interval \([0, 2]\) into 3 subintervals of equal length and choose \( c_i \) to be the right-endpoint of the \( i \)th subinterval. Then the Riemann sum \( \sum_{i=1}^{3} f(c_i) \Delta x \) is

\[
(A) \frac{112}{27} \quad (B) \frac{36}{9} \quad (C) \frac{8}{27} \quad (D) \frac{72}{27} \quad (E) \frac{36}{9} \quad (AB) \text{ none of these.}
\]

6. The slope of the line tangent to the curve \( x^3 + y^3 + xy + 1 = 0 \) at the point \((1, -1)\) is equal to

\[
(A) 1 \quad (B) 2 \quad (C) \frac{1}{2} \quad (D) \frac{-1}{2} \quad (E) \frac{5}{6} \quad (AB) \text{ none of these.}
\]

7. The integral \( \int_{1}^{2} (x^2 - x) \, dx \) equals

\[
(A) -\frac{1}{2} \quad (B) \frac{1}{2} \quad (C) -\frac{1}{3} \quad (D) -\frac{1}{6} \quad \quad (E) \frac{4}{3} \quad (AB) \frac{5}{6} \quad (AC) \text{ none of these.}
\]

8. The derivative of \( F(x) = \int_{2}^{x} \sqrt{t^4 - 1} \, dt \) is equal to

\[
(A) \frac{2x^3}{\sqrt{x^4 - 1}} \quad (B) \frac{2x^3}{\sqrt{x^4 - 1}} - 2 \quad (C) \sqrt{x^4 - 1} - 2 \quad (D) \sqrt{x^4 - 1} \quad (E) 4x^3 \sqrt{x^4 - 1} \quad (AB) \text{ none of these.}
\]
9. The antiderivative \( \int \frac{x^3 - 1}{\sqrt{x}} \, dx \) is equal to

(A) \( \frac{2x^{3/2} - 2\sqrt{x}}{7} + C \)  
(B) \( \frac{x^4 - x}{2x^{3/2}} + C \)  
(C) \( \frac{2x^{5/2}}{7} - 2\sqrt{x} + C \)  
(D) \( \frac{x^2}{2} - \ln |x| + C \)  
(E) \( \frac{x^2}{2} + \frac{1}{x^2} + C \)  (AB) none of these.

10. The \( \lim_{x \to 0} \frac{2 - 2 \cos x}{x^2} \) is equal to

(A) \( \frac{1}{2} \)  
(B) 2  
(C) 1  
(D) 0  
(E) \(-\frac{1}{2}\)  (AB) none of these.

11. A rectangle is increasing in size in such a way that its width is always half of its length. When the width of this rectangle is 5 inches and is increasing at the rate of 2 inches per minute, at what rate is its area increasing? The correct answer is:

(A) 40 in\(^2\)/min  
(B) 20 in\(^2\)/min  
(C) 5 in\(^2\)/min  
(D) 2 in\(^2\)/min  
(E) 10 in\(^2\)/min  (AB) none of these.

12. Given the function \( f(x) = \ln x + x^3 \), the second derivative \( f''(x) \) is equal to

(A) \( 3x^2 + 1 \)  
(B) \( 6x + 1 \)  
(C) \( 6x - \frac{1}{x} \)  
(D) \( 6x - \frac{1}{x^2} \)  
(E) \( 3x^2 - \frac{1}{x} \)  (AB) \( 3x^2 - \frac{1}{x^2} \)  (AC) \( 6x \)  (AD) none of these.
PART 2: This portion of the exam will be graded on a partial credit basis. Answers without supporting work shown on the paper will receive NO credit.

13. (10 points) Find the absolute maximum and the absolute minimum of the function $f(x) = x^3 - 12x + 1$ on the closed interval $[-3, 5]$.

14. (10 points) A farmer with 120 feet of fencing wants to fence in a rectangular area and then divide it into two pens with a fence parallel to one side of the rectangle. What is the largest possible total area of the two pens?
15. \((10 \text{ points})\) Given the function \(f(x) = x^2 + 3x\), apply the formal definition of the derivative to compute \(f'(x)\). (No credit for answers not using the formal definition of the derivative).

16. \((8 \text{ points each})\) Compute the following derivatives. Do not simplify your answer.

(a) \(f(x) = (e^x - \sin x)(\cos^2 x + \sqrt{x})\).

(b) \(f(x) = \frac{\ln(2x - 3)}{x^2 + 5x}\).

(c) \(f(x) = (2x^3 + \tan(3x))^4\).
17. (4 points each) Given the function \( f(x) = \frac{1}{x^2 + x - 2} \), do each of the following.

(a) Determine the domain of \( f(x) \).

(b) Determine the vertical asymptote(s) of the graph of \( y = f(x) \), if there are any.

(c) Determine the horizontal asymptote(s) of the graph of \( y = f(x) \), if there are any.
18. (8 points each) Given the function \( f(x) = x^4 - 6x^2 \), and given

\[
f'(x) = 4x^3 - 12x, \quad \text{and} \quad f''(x) = 12x^2 - 12,
\]

do each of the following.

(a) Find the interval(s) on which \( f \) is increasing and the interval(s) on which \( f \) is decreasing. Determine the \( x \)-coordinate of each local maximum and each local minimum.

(b) Find the interval(s) on which the graph of \( y = f(x) \) is concave upward and the interval(s) on which it is concave downward. Determine the \( x \)-coordinate of each inflection point.

(c) Sketch the graph of \( y = f(x) \). In your graph, you should accurately present the following features of the curve: \( x \) and \( y \) intercepts, local maxima/minima, increasing/decreasing, concavity, and inflection points.
19. \((10 \text{ points})\) Compute the following definite integral.
\[
\int_{1}^{e} \frac{\ln x}{x} \, dx
\]

20. \((12 \text{ points})\) Sketch the region bounded by the curves \(y = 3 - x^2\) and \(y = x^2 - 5\). Then find the area of this region.
21. (8 points each) Compute the following antiderivatives.

(a) $\int (e^{3x} + 2\sin x + x^{\frac{1}{3}})dx$

(b) $\int \frac{x + 1}{(x^2 + 2x)^2}dx$
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