



21 April, 1988

## TO MY COLLEAGUES IN MATHEMATICS

Have you ever seen any expansion resembling the following?

Special expansion of a function  $F$  by means of a function  $f$ :

$$(1) \quad F(x) = \sum_{n=0}^{\infty} (-1)^n \frac{A_n}{n!} D_x^n f(x)$$

where

$$(2) \quad A_n = \sum_{k=0}^n \binom{n}{k} D_t^k \left( \frac{1}{\int_{-\infty}^{+\infty} f(x) e^{tx} dx} \right) \Big|_{t=0} \int_{-\infty}^{+\infty} x^{n-k} F(x) dx$$

Can you determine any conditions sufficient to allow such an expansion?

I ran into this expansion many years ago in a statistics journal, I see from my old notes, where I had written that the author attributed it to A. C. Aitken (Univ. of Edinburgh). However a few years before his death I asked Aitken about it in a letter and he denied any association with it and did not volunteer any remarks to shed light on it. I have, unfortunately, been unable to determine where I ran into the expansion. It seems to be unusual and I offer it to you for study. I have mentioned it to several other colleagues in letters recently, but no one has added anything. I would like to determine what functions,  $F$ , allow such an expansion using another function,  $f$ .

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