Guidelines: You may use books and notes. You can't discuss the problems with anyone other than myself. Make sure to read, write and sign the last page. Assign each of the questions you solve a point value from $\{3 \cdot 7,7 \cdot 8,3 \cdot 9\}$ such that they total exactly 56 points. If you don't assign a point value to a question it will be assigned a value of 7 points.
These 7 questions will count for a maximum of 56 points.

Name:
6. Give a combinatorial argument for the following identity:

$$
D_{n}=n!-\sum_{k=1}^{n}\binom{n}{k} D_{n-k}
$$

7. How many numbers can be obtained as the product of two or more of the numbers $3,4,4,5,5,6,7,7,7$ ?

No credit will be given for answers that simply enumerate all possibilities.
8. Let $h_{n}=8 h_{n-1}-16 h_{n-2}$ for $n \geqslant 2$ where $h_{0}=-1$ and $h_{1}=0$. Use generating functions to find a (nice) formula for $h_{n}$.
9. How many 10 digit phone numbers contain at least one of each odd digit?
10. There are 17 students seated in a room with 30 seats. The instructor is not satisfied with the seating arrangement and requests that everyone move to a new seat. How many new seating arrangements are possible?
11. Fix a regular hexagon and regular pentagon in the plane centered on the same point precisely as shown in the image below. Let $\mathcal{S}$ denote their vertices along with the center.

How many equilateral triangles can be formed with at least two vertices in $\mathcal{S}$ ?
No credit will be given for answers that simply enumerate all triangles.

12. A bakery sells chocolate, cinnamon, plain and crueler doughnuts. Suppose they are attempting to make doughnut eating more exciting by putting the doughnuts in an opaque bag that stacks the doughnuts up on top of each other. To eat them you must take a doughnut out of the bag (off the top of the stack) and eat it, they then repeat until finished. These "Grab Bags" are created so that they contain an even number of chocolate doughnuts, an odd number of plain doughnuts and any number of the other two types.
How many different grab bags of $n$ doughnuts could be made?
Hint: Find a generating function for the "Grab Bags". Then find a formula from the generating function.
13. (5 points) Bonus: Recall that for the in class portion of the exam, you were to clearly mark 5 questions to solve for full credit and the other two would be bonus questions worth 5 points each. On the take-home you were to assign values to all of the problems from the set $\{3 \cdot 7,7 \cdot 8,3 \cdot 9\}$ such that the total points was exactly 56 points. Assuming we assigned point values to all questions on the take-home, how many possible ways of completing the two exams and assigning points to each problem on the take home are there?

Once you have finished, please write, in the space provided, "I have neither given nor received help on this test through any means other than using books, notes or asking Ryan Hansen." and sign your name.
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