

This Exam is being given under the guidelines of the **Honor Code**. You are expected to respect those guidelines and to report those who do not. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page.

Clearly mark 5 of the seven questions to solve for full credit!

If you do more, each extra question will be worth 5 points.

Your 5 full credit questions will count 9 points each for a total of 45 points.

Name: \_\_\_\_\_

1. How many circular permutations of  $\{3 \cdot a, 4 \cdot b, 2 \cdot c, 2 \cdot d\}$  are there which use all of the elements and in which not all of the same letter appears consecutively?
2. A bakery sells chocolate, cinnamon and plain doughnuts. At a particular time, they have on hand 6 chocolate, 6 cinnamon and 3 plain doughnuts. How many boxes of 12 doughnuts can be made with the doughnuts available?

3. A bakery sells chocolate, cinnamon, plain and crueller doughnuts. Suppose they sell a “Surprise Box” of doughnuts which consists of an even number of chocolate doughnuts, at most one cinnamon doughnut, a multiple of 5 plain doughnuts and at most 4 crueller doughnuts.

How many different “Surprise Boxes” can be made of size  $n$ ?

*Hint:* Find a generating function for the “Surprise Boxes”. Then find a formula from the generating function.

4. Find a recurrence relation for the number of ternary strings of length  $n$  containing at least one pair of consecutive 0's. (ie. 200121000 is OK, but 022010101 is not)

5. How many ternary strings of length  $n$  contain at least one pair of consecutive symbols that are the same? (This is not the same as problem 4.)

6. How many ways can four nonattacking rooks be placed on the following board?

		×	
×			
×			×
	×		

7. At the annual Worthington Hooker School\* Dance-Till-You-Drop dance-a-thon, 30 couples are “dancing”. Of the 30 couples 12 are jock-cheerleader couples. The principal arrives and decides that things are getting a little too steamy. He asks that everyone switch to a new partner. Of course, the jocks again end up with the cheerleaders.

How many new configurations are possible?

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\* Actual high school located in New Haven Connecticut named after former Yale University professor and physician Dr. Worthington Hooker.