This Exam is being given under the guidelines of the **Honor Code**. You are expected to respect those guidelines and to report those who do not. Answer the questions in the spaces provided. If you run out of room for an answer, continue on the back of the page. There are 8 questions for a total of 66 points.

Name:

- 1. A group of mn people are to be arranged into m teams each with n players.
 - (a) (5 points) Determine the number of ways if each team has a different name.

(b) (5 points) Determine the number of ways if the teams don't have names.

2. (4 points) Find the coefficient of $x^{15}y^{12}$ in the expansion of $(4x^3 - 3y^2)^{11}$.

3. (6 points) Determine the number of ways to distribute 10 orange drinks, 1 lemon drink and 1 lime drink to 5 (very) thirsty students (Sarah, Clara, Tucker, Daryl, and Joe) so that each student gets at least one drink and the lemon and lime drinks go to different students.

4. Simplify

(a) (4 points)
$$5^{20} - {\binom{20}{1}}5^{19} \cdot 3^1 + {\binom{20}{2}}5^{18} \cdot 3^2 - {\binom{20}{3}}5^{17} \cdot 3^3 + \dots + {\binom{20}{20}}3^{20}.$$

(b) (4 points)
$$\binom{30}{30} + \binom{30}{29} + \binom{30}{28} + \dots + \binom{30}{1} + \binom{30}{0}$$

5. Prove
$$\binom{n}{k} = \binom{n-2}{k-2} + 2\binom{n-2}{k-1} + \binom{n-2}{k}$$
 two ways:

(a) (5 points) Using Pascal's identity. NO CREDIT WILL BE GIVEN FOR SOLUTIONS USING FACTORIALS!

- (b) (5 points) By giving a combinatorial argument.
 - (Hint: Consider a and b as special elements)

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6. (a) (6 points) Consider the Boolean lattice when n = 14, how many symmetric chains of length 9 go through the set $\{4, 5, 8, 9, 10\}$?

(b) (3 points) How many chains use $\{6, 7, 8, 12, 13\}$ in a symmetric chain decomposition of the Boolean lattice when n = 14?

- 7. (6 points) Find an expression for the number of ways to form a committee of 7 people including a chairman from a group of 14 men and 16 women if all of the following must hold:
 - 1. A man must be the chair of the committee.
 - 2. There must be a total of at least two men on the committee.
 - 3. There must be a total of at least three women on the committee.
 - (Note: The same 7 people with a different chairman is considered a different committee.)

8. (a) (3 points) What does $K_{332} \longrightarrow K_3, K_3, K_3, K_3, K_3$ mean?

(b) (4 points) Using "arrow notation" (partially exemplified when I asked part (a)), explain fully what the notation r(3,9) = 36 means.