Name: $\qquad$

## Math 378 Spring 2011 Bonus Questions 1

1. Find a sequence of 16 distinct real-numbers with neither an increasing subsequence of length 5 nor a decreasing subsequence of length 5 .
2. Show $K_{17} \rightarrow K_{3}, K_{3}, K_{3}$.
3. Show $K_{10} \rightarrow K_{3}, K_{4}$.
4. Show $K_{9} \rightarrow K_{3}, K_{4}$. The two cases from the previous part apply here, but there is also another case. Show that the extra case is actually impossible (this may require a little idea from graph theory).
5. Use Ramsey's theorem to prove that there exists a positive integer $N$ such that every sequence of $N$ distinct real numbers has either an increasing subsequence of length $n$ or a decreasing subsequence of length $n$.
